Online Appendix to

Tariff Wars and Net Foreign Assets

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This appendix contains three main sections: (i) model extensions, (ii) quantification appendix, and (iii) currency composition of assets and liabilities in the data.

I Model Extensions

In this section we introduce four extensions of the analysis: (i) Balanced-trade equilibria with positive trade, extending the results of Proposition 2; (ii) The multiple goods extension based on Dixit and Norman (1980); (iii) The introduction of trade costs into this last environment; and (iv) The dynamic and risk re-interpretation, which highlights the role of the international partity condition and the exchange rate in the adjustment; and

I.1 Balanced trade equilibria with positive trade

Proposition 2 restricts attention to cases in which tariffs are high enough that autarky emerges as an equilibrium. We now show that for intermediate tariff levels, balanced trade is also attainable away from autarky.

We do so under Assumptions 1 and 2, with $a_A^* > 0$ and $a_B > 0$, so that the characterization in Section 4.2 (Lemma B.1) applies. As shown in equation (1), with positive trade, the budget constraint for Home becomes:

$$c_A + \rho c_B = (Y_A - a_A^*) + \rho(Y_B + a_B)$$

where $\rho = p_B^*/p_A$ are the equilibrium terms of trade. Balanced trade occurs when $Y_A - c_A + \rho(Y_B - c_B) = 0$. From the budget constraint, this requires the net foreign assets positions to be zero:

$$\rho a_B = a_A^*.$$

Therefore, the terms of trade must be as in the autarky case, $\rho = \rho^{BT} \equiv a_A^*/a_B$.

Let us define

$$\underline{\theta} \equiv \frac{\overline{Y}_B - h(\rho^{BT})\left(Y_A - \rho^{BT} Y_B^*\right)}{h(\rho^{BT})\rho^{BT}\overline{Y}_A + Y_A^* - \rho^{BT} Y_B}.$$

As we show below, $\underline{\theta}$ reflects the Foreign consumption ratio, c_B^*/c_A^*, that satisfies balanced trade and the resource conditions when Home's tariff is zero. Any Foreign relative consumption ratio that is weakly greater than this and less than Y_B^*/Y_A^* can be implemented with an appropriate weakly positive Home tariff. We have the following result (proved at the end of this appendix):

Proposition I.1 (Balanced Trade Equilibria)**.** *Suppose that Assumptions 1 and 2 hold. Suppose also that $g(Y_B^*/Y_A^*) < \rho^{BT} < g(Y_B/Y_A)$. Then $\underline{\theta} > 0$, so that*

$$T^* \equiv \left[\rho^{BT}/g(\underline{\theta}) - 1,\ \rho^{BT}/g(Y_B^*/Y_A^*) - 1\right)$$

is well defined, and

(i) T^* is non-empty.

(ii) For any $\tau^* \in T^*$ there exists a $\tau \geq 0$ such that there exists a tariff equilibrium with active trade such that trade is balanced:

$$Y_A - c_A = -\rho^{BT}(Y_B - c_B),$$

and Home exports good A.

Proof. We first check that $\underline{\theta} > 0$. Note that

$$g^{-1}(\rho^{BT})\rho^{BT}\overline{Y}_A + Y_A^* - \rho^{BT}Y_B > (Y_B/Y_A)\rho^{BT}\overline{Y}_A + Y_A^* - \rho^{BT}Y_B = \rho^{BT}Y_B(\overline{Y}_A/Y_A - 1) + Y_A^* > 0,$$

where the first inequality follows from $g^{-1}(\rho^{BT}) > Y_B/Y_A$, a premise of the Proposition. For the numerator, Assumption 2 makes Home a net debtor at free-trade prices, so $\rho^{FT}a_B < a_A^*$ and hence $\rho^{FT} < \rho^{BT}$. As h is decreasing, $h(\rho^{BT}) < h(\rho^{FT}) = \overline{Y}_B/\overline{Y}_A$. If $Y_A - \rho^{BT}Y_B^* \leq 0$ the numerator is at least $\overline{Y}_B > 0$; otherwise

$$\overline{Y}_B - h(\rho^{BT})\left(Y_A - \rho^{BT}Y_B^*\right) > \overline{Y}_B - \frac{\overline{Y}_B}{\overline{Y}_A}\left(Y_A - \rho^{BT}Y_B^*\right) = \overline{Y}_B\frac{Y_A^* + \rho^{BT}Y_B^*}{\overline{Y}_A} > 0.$$

Hence $\underline{\theta} > 0$. Now, let $t_1 = \frac{\rho^{BT}}{g(\underline{\theta})} - 1$ and $t_2 = \frac{\rho^{BT}}{g(Y_B^*/Y_A^*)} - 1$. Given $\underline{\theta} > 0$, after some algebra, we get that

$$\underline{\theta} < \frac{Y_B^*}{Y_A^*} \Leftrightarrow Y_B(Y_A^* + \rho^{BT}Y_B^*) < g^{-1}(\rho^{BT})Y_A(Y_A^* + \rho^{BT}Y_B^*) \Leftrightarrow Y_B < g^{-1}(\rho^{BT})Y_A$$

which holds by $g(Y_B/Y_A) > \rho^{BT}$, and thus $t_2 > t_1$.

Consider a $\tau^* \in [t_1, t_2)$, and let us construct a tariff equilibrium for some $\tau \geq 0$. Given that there is trade in equilibrium, $g(c_B^*/c_A^*) = \rho^{BT}/(1 + \tau^*)$. Define

$$\theta^* \equiv g^{-1}\left(\frac{\rho^{BT}}{1 + \tau^*}\right) = \frac{c_B^*}{c_A^*} < \frac{Y_B^*}{Y_A^*}, \tag{1}$$

where the last inequality follows from $\tau^* < t_2$: $g(Y_B^*/Y_A^*) < \rho^{BT}/(1+\tau^*)$. Using the fact that $a_A^* - \rho^{BT}a_B =$

0 by definition of ρ^{BT}, the Foreign budget constraint at world prices implies

$$c_A^* = \frac{Y_A^* + \rho^{BT} Y_B^*}{1 + \rho^{BT}\theta^*},$$

where we have used $c_B^* = \theta^* c_A^*$. The proposed equilibrium allocation for Foreign is thus:

$$(c_A^*, c_B^*) = \left(\frac{Y_A^* + \rho^{BT} Y_B^*}{1 + \rho^{BT}\theta^*}, \frac{\theta^* \left(Y_A^* + \rho^{BT} Y_B^*\right)}{1 + \rho^{BT}\theta^*} \right). \tag{2}$$

Note that $c_A^* > 0$ and $c_B^* > 0$. From the resource conditions, we can obtain the implied consumption allocation for Home:

$$\begin{aligned} c_A &= \overline{Y}_A - c_A^* = \frac{Y_A + \rho^{BT}(\theta^* \overline{Y}_A - Y_B^*)}{1 + \rho^{BT}\theta^*} = Y_A - \frac{\rho^{BT}(Y_B^* - \theta^* Y_A^*)}{1 + \rho^{BT}\theta^*} \\ c_B &= \overline{Y}_B - c_B^* = \frac{\overline{Y}_B - \theta^* Y_A^* + \rho^{BT}\theta^* Y_B}{1 + \rho^{BT}\theta^*} = Y_B + \frac{Y_B^* - \theta^* Y_A^*}{1 + \rho^{BT}\theta^*}. \end{aligned} \tag{3}$$

As $Y_B^* > \theta^* Y_A^*$ from (1), we have $c_B > Y_B > 0$ and $c_A < Y_A$ as required. That is, Home exports good A. To ensure that $c_A \geq 0$, we require

$$0 \leq Y_A + \rho^{BT}(\theta^* \overline{Y}_A - Y_B^*) \iff \frac{\rho^{BT} Y_B^* - Y_A}{\rho^{BT} \overline{Y}_A} \leq \theta^* = g^{-1}\left(\frac{\rho^{BT}}{1 + \tau^*}\right).$$

If $\rho^{BT} Y_B^* \leq Y_A$, then this condition is satisfied immediately. Otherwise, the following condition ensures $c_A \geq 0$:

$$\tau^* \geq \frac{\rho^{BT}}{g\left(\frac{\rho^{BT} Y_B^* - Y_A}{\rho^{BT} \overline{Y}_A}\right)} - 1. \tag{4}$$

Next, we need to construct a tariff τ such that the allocation in (3) is optimal for Home. Consider a candidate τ given by

$$\tau = \frac{g\left(c_B/c_A\right)}{\rho^{BT}} - 1,$$

where c_A and c_B are given by (3). With this tariff rate, the proposed allocation would be optimal for Home but we need to verify $\tau \geq 0$:

$$\tau \geq 0 \iff (1+\tau)\rho^{BT} \geq \rho^{BT} \iff \frac{c_B}{c_A} = g^{-1}((1+\tau)\rho^{BT}) \leq g^{-1}(\rho^{BT}).$$

Substituting for c_B/c_A using (3), we require

$$g^{-1}(\rho^{BT}) \geq \frac{\overline{Y}_B + \rho^{BT}\theta^* Y_B - \theta^* Y_A^*}{Y_A + \rho^{BT}(\theta^* \overline{Y}_A - Y_B^*)}$$
$$\iff$$
$$\left(g^{-1}(\rho^{BT})\rho^{BT}\overline{Y}_A + Y_A^* - \rho^{BT} Y_B\right)\theta^* \geq \overline{Y}_B - g^{-1}(\rho^{BT})\left(Y_A - \rho^{BT} Y_B^*\right). \tag{5}$$

Dividing through (5) by the left-hand-side coefficient, shown strictly positive at the start of the proof, we obtain the following condition as requirement for the Home tariff to be non-negative:

$$\theta^* \geq \frac{\overline{Y}_B - g^{-1}(\rho^{BT})\left(Y_A - \rho^{BT} Y_B^*\right)}{g^{-1}(\rho^{BT})\rho^{BT}\overline{Y}_A + Y_A^* - \rho^{BT} Y_B} \equiv \underline{\theta}. \tag{6}$$

Using $g(\theta^*) = \rho^{BT}/(1+\tau^*)$, the condition is equivalent to

$$\tau^* \geq \frac{\rho^{BT}}{g(\underline{\theta})} - 1. \tag{7}$$

And thus $\tau^* \in T^*$ implies that the Home tariff is non-negative.

Finally, let us come back to the restriction we needed for $c_A \geq 0$, (4).

If $\rho^{BT} Y_B^* \leq Y_A$, then (4) holds vacuously. Otherwise the argument of g below is strictly positive, and since $\frac{\rho^{BT} Y_B^* - Y_A}{\rho^{BT}\overline{Y}_A} < \underline{\theta}$ with g decreasing, we have that

$$\frac{\rho^{BT}}{g(\underline{\theta})} - 1 > \frac{\rho^{BT}}{g\left(\frac{\rho^{BT} Y_B^* - Y_A}{\rho^{BT}\overline{Y}_A}\right)} - 1$$

and (4) is implied by (7). And thus $\tau^* \in T^*$ is sufficient to guarantee $c_A \geq 0$.

Summarizing this: for any $\tau^* \in [t_1, t_2)$ condition (6) holds, and thus there exists a $\tau \geq 0$ such that the allocation in (2) and (3) is a balanced trade equilibrium. In this equilibrium, Home exports good A. □

I.2 Multiple Goods

Suppose there are $N \geq 1$ traded goods, and M and M^* non-traded goods in Home and Foreign, respectively. Let $\mathcal{I} \equiv \{1, \ldots, N+M\}$ and $\mathcal{I}^* \equiv \{1, \ldots, N+M^*\}$ denote the set of goods in each of the two countries. Home endowments are denoted by $\boldsymbol{Y} = \{Y_1, \ldots, Y_N, Y_{N+1}, \ldots, Y_{N+M}\} = \{Y_i\}_{i\in\mathcal{I}}$. Foreign endowments are $\boldsymbol{Y}^* \equiv \{Y_i^*\}_{i\in\mathcal{I}^*}$. It is straightforward to extend the environment to allow for resource costs of trade for traded goods, and we do so in Appendix I.3.

The utility functions of the corresponding representative households in Home and Foreign are $u(\boldsymbol{c})$ and $u^*(\boldsymbol{c}^*)$, respectively, where $\boldsymbol{c} \equiv \{c_i\}_{i\in\mathcal{I}}$ and $\boldsymbol{c}^* \equiv \{c_i^*\}_{i\in\mathcal{I}^*}$ are the consumption vectors. Let $\boldsymbol{p} \equiv \{p_i\}_{i\in\mathcal{I}}$ and $\boldsymbol{p}^* \equiv \{p_i^*\}_{i\in\mathcal{I}^*}$ denote the domestic price vectors in Home and Foreign, all quoted in a numeraire unit.

As before, the representative household in each country starts the period with gross claims on the other country to be settled at the respective issuers' prices. For non-traded goods, we assume that the payouts are delivered in the prices of their respective countries. Specifically, let $\boldsymbol{a} \equiv \{a_i\}_{i \in \mathcal{I}^*}$ denote Home ownership of assets (or liabilities, if negative) for delivery in Foreign, denominated in each of the Foreign goods $i \in \mathcal{I}^*$ (potentially including claims indexed on non-traded goods). Similarly, we denote by $\boldsymbol{a}^* \equiv \{a_i^*\}_{i \in \mathcal{I}}$ Foreign ownership of assets denominated in goods for delivery at Home.

Home households maximize utility subject to their budget constraint:

$$\max_{\boldsymbol{c}} u(\boldsymbol{c}) \quad \text{subject to} \quad \boldsymbol{p} \cdot \boldsymbol{c} \leq \boldsymbol{p} \cdot \boldsymbol{Y} + \boldsymbol{p}^* \cdot \boldsymbol{a} - \boldsymbol{p} \cdot \boldsymbol{a}^* + T.$$

where T is Home's tariff revenue, and the operator $\cdot$ denotes the dot product. Similarly, the Foreign households' problem is:

$$\max_{\boldsymbol{c}^*} u^*(\boldsymbol{c}^*) \quad \text{subject to} \quad \boldsymbol{p}^* \cdot \boldsymbol{c}^* \leq \boldsymbol{p}^* \cdot \boldsymbol{Y}^* + \boldsymbol{p} \cdot \boldsymbol{a}^* - \boldsymbol{p}^* \cdot \boldsymbol{a} + T^*.$$

We allow for u and u^* to be different and potentially non-homothetic. We only impose that they are concave and differentiable functions with finite derivatives at the autarkic allocations:

Assumption I.1. *The utility functions u and u^* are strictly increasing, concave, differentiable, and have finite marginal utilities evaluated at the endowment: $u_i(\boldsymbol{Y}) < \infty$ and $u_j^*(\boldsymbol{Y}^*) < \infty$ for $i \in \mathcal{I}$ and $j \in \mathcal{I}^*$.*

The requirement of finite derivatives is satisfied if the endowment of each good is strictly positive in both countries (as we assumed in the preceding section), or if the marginal utilities are finite when consumption is zero (for example, if the utility delivers a demand with a choke price). As before, the key is that domestic relative prices for all goods are well defined in autarky.

International goods arbitrage requires that for all traded goods, $i \in \{1, \ldots, N\}$, we have:

$$\frac{p_i}{p_i^*} = \begin{cases} (1+\tau) & \text{for Home imports: } c_i > Y_i \\ 1/(1+\tau^*) & \text{for Foreign imports: } c_i < Y_i \end{cases}$$
$$\text{with } p_i/p_i^* \in [1/(1+\tau^*), (1+\tau)] \text{ if } c_i = Y_i.$$

The governments' budget constraints are given by equating transfers to the tariffs raised on the goods actively imported:

$$T = \tau \sum_{i=1}^{N} p_i^* \max\{c_i - Y_i, 0\}, \quad \text{and} \quad T^* = \tau^* \sum_{i=1}^{N} p_i \max\{c_i^* - Y_i^*, 0\}.$$

Finally, the resource constraints are:

$$\begin{aligned} c_i + c_i^* &= Y_i + Y_i^* && \text{for all } i \in \{1, \ldots, N\} \\ c_i &= Y_i && \text{for all } i \in \{N+1, \ldots, N+M\} \\ c_i^* &= Y_i^* && \text{for all } i \in \{N+1, \ldots, N+M^*\}, \end{aligned}$$

where the last two constraints follow from the non-traded nature of these goods.

We now define a competitive equilibrium for a given set of tariff rates, $\{\tau, \tau^*\}$, which is standard:

Definition 1. *Given a pair of tariff rates, (τ, τ^*), an equilibrium is a pair of consumption allocations for Home and Foreign, $\boldsymbol{c}$ and $\boldsymbol{c}^*$, and domestic prices in Home and Foreign, $\boldsymbol{p}$ and $\boldsymbol{p}^*$, such that: (i) households optimize their consumption levels subject to their budget constraints; (ii) prices are consistent with international goods arbitrage; (iii) the government budget constraints hold; and (iv) the aggregate resource constraints hold.*

Free Trade As in the two-good case, it is useful to establish the free trade equilibrium as a benchmark. In a free trade equilibrium, prices are equalized across countries for all traded goods:

$$p_i = p_i^* \text{ for all } i \in \{1, \ldots, N\}.$$

Prices and consumption levels are related through the optimality condition of the respective households. For the traded goods, it must be that

$$\frac{p_i}{p_1} = \frac{u_i(\boldsymbol{c}_{FT})}{u_1(\boldsymbol{c}_{FT})} = \frac{u_i^*(\boldsymbol{c}_{FT}^*)}{u_1^*(\boldsymbol{c}_{FT}^*)} \text{ for all } i \in \{1, \ldots, N\}.$$

For the non-traded goods in each country we have that

$$\frac{p_i}{p_1} = \frac{u_i(\boldsymbol{c}_{FT})}{u_1(\boldsymbol{c}_{FT})} \text{ for } i \in \{N+1, \ldots, N+M\}, \quad \text{and} \quad \frac{p_i^*}{p_1^*} = \frac{u_i^*(\boldsymbol{c}_{FT}^*)}{u_1^*(\boldsymbol{c}_{FT}^*)} \text{ for } i \in \{N+1, \ldots, N+M^*\}.$$

Note that if $\boldsymbol{p}^*_{FT} \cdot \boldsymbol{a} - \boldsymbol{p}_{FT} \cdot \boldsymbol{a}^* < 0$, then Home is a net debtor under free trade prices. Home is a net creditor for the reverse inequality.

Autarky. We define the *autarkic allocation* to be the one where each country's consumption corresponds to their *physical* endowment. That is, $\boldsymbol{c} = \boldsymbol{Y}$ and $\boldsymbol{c}^* = \boldsymbol{Y}^*$. We proceed next to extend Proposition 2 to this more general environment by providing conditions on assets and tariffs such that the autarkic allocation is an equilibrium.

Towards this, denote by $\boldsymbol{\rho}_{aut}$ and $\boldsymbol{\rho}^*_{aut}$ the vectors of relative prices with respect to good 1 consistent with autarky for both Home and Foreign. That is, $\boldsymbol{\rho}_{aut} = \{1, \rho_{aut,2}, \ldots, \rho_{aut,N+M}\}$ and $\boldsymbol{\rho}^*_{aut} = \{1, \rho^*_{aut,2}, \ldots, \rho^*_{aut,N+M^*}\}$ such that:

$$\rho_{aut,i} \equiv \frac{u_i(\boldsymbol{Y})}{u_1(\boldsymbol{Y})} \text{ for all } i \in \mathcal{I}, \qquad \text{and} \qquad \rho^*_{aut,j} \equiv \frac{u^*_j(\boldsymbol{Y}^*)}{u^*_1(\boldsymbol{Y}^*)} \text{ for all } j \in \mathcal{I}^*.$$

Note that these relative prices are completely determined by endowments and preferences.

Then, we have the following result which generalizes Proposition 2 from the two-good environment:

Proposition I.2. *Suppose Assumption I.1 holds, and that $\boldsymbol{a}$ and $\boldsymbol{a}^*$ are such that*

$$(\boldsymbol{\rho}_{aut} \cdot \boldsymbol{a}^*) \times (\boldsymbol{\rho}^*_{aut} \cdot \boldsymbol{a}) > 0. \tag{8}$$

Then there exist $\underline{\tau}$ and $\underline{\tau}^*$ such that for all tariffs such that $\tau \geq \underline{\tau}$ and $\tau^* \geq \underline{\tau}^*$, the autarky allocation $\boldsymbol{c} = \boldsymbol{Y}$ and $\boldsymbol{c}^* = \boldsymbol{Y}^*$ is an equilibrium.

The proof is by construction, and it follows below.

Condition (8) may hold independently of whether Home is a net debtor or a net creditor under free trade. It generalizes and relaxes Assumption 1 introduced in the two-good environment. What is important is that the gross positions of each country are non-zero and of the same sign when evaluated at the autarkic relative prices. This nests both the case of simple "positive claims" and the case of symmetric negative positions from the two-good model. A simple case where this will occur is when all of the elements of $\boldsymbol{a}$ and $\boldsymbol{a}^*$ are of the same sign, with at least one of each non-zero.

Given that consumption allocations must equal endowments, we know that equilibrium Home and Foreign consumption levels can be supported by domestic relative prices that equal the ratios of the marginal utilities in autarky:

$$\boldsymbol{p} = p_1 \times \boldsymbol{\rho}_{aut}, \qquad \text{and} \qquad \boldsymbol{p}^* = p_1^* \times \boldsymbol{\rho}^*_{aut}.$$

for some finite non-zero scalars p_1 and p_1^*. Assumption I.1 guarantees that these prices are well defined. Since endowments pin down relative prices within each region, the ratio p_1^*/p_1 determines the ratio of the price *level* in Foreign relative to Home. Hence, p_1^*/p_1 will be proportional to the real exchange rate. For convenience, we therefore define

$$e \equiv p_1^*/p_1$$

to be our notion of a real exchange rate. This will be our counterpart to the relative price ρ that featured in the analysis of Section 3.

In an autarkic equilibrium, there is no trade; and no tariff revenue, $T = T^* = 0$. Hence, given that the budget constraints of both Home and Foreign hold with equality for $\boldsymbol{c} = \boldsymbol{Y}$ and $\boldsymbol{c}^* = \boldsymbol{Y}^*$, it must be that $\boldsymbol{p} \cdot \boldsymbol{a}^* - \boldsymbol{p}^* \cdot \boldsymbol{a} = 0$. For the net foreign asset positions to zero out we require:

$$\boldsymbol{p} \cdot \boldsymbol{a}^* = \boldsymbol{p}^* \cdot \boldsymbol{a} \quad \Rightarrow \quad p_1(\boldsymbol{\rho}_{Aut} \cdot a^*) = p_1^*(\boldsymbol{\rho}^*_{Aut} \cdot \boldsymbol{a}), \tag{9}$$

which will hold for a particular level of the real exchange rate:

$$e^{BT} \equiv \frac{\boldsymbol{\rho}_{Aut} \cdot \boldsymbol{a}^*}{\boldsymbol{\rho}^*_{Aut} \cdot \boldsymbol{a}} > 0,$$

where the inequality follows from the condition in the proposition. It is in this way that *asset positions pin down the real exchange rate* when tariffs are sufficiently high. This is the generalization of ρ^{BT} from Proposition 2.

To verify the rest of the equilibrium conditions, concavity of the utility functions guarantee that, given that the optimality condition holds by the construction of $\boldsymbol{p}$ and $\boldsymbol{p}^*$, the autarkic consumption bundles solve the households' problem at Home and Foreign. Moreover, the autarky allocations satisfy the resource constraints by construction. The only equilibrium condition left to check is the international goods arbitrage. For that, we need to ensure that the relative prices for each traded good across countries are consistent with no trade:

$$\frac{p_i}{p_i^*} = \frac{p_1}{p_1^*} \frac{u_i(\boldsymbol{Y})}{u_1(\boldsymbol{Y})} \frac{u_1^*(\boldsymbol{Y}^*)}{u_i^*(\boldsymbol{Y}^*)} \in \left[\frac{1}{1+\tau^*}, 1+\tau\right] \text{ for all } i \in \{1, \dots, N\}.$$

The following (finite) lower bounds on tariffs ensure this is true for all goods:

$$\begin{aligned} \tau \geq \underline{\tau} &= \left(\frac{\boldsymbol{\rho}^*_{Aut} \cdot \boldsymbol{a}}{\boldsymbol{\rho}_{Aut} \cdot \boldsymbol{a}^*}\right) \max_{i \in \{1,\ldots,N\}} \left\{\frac{u_i(\boldsymbol{Y})}{u_1(\boldsymbol{Y})} \frac{u^*_1(\boldsymbol{Y}^*)}{u^*_i(\boldsymbol{Y}^*)}\right\} - 1, \\ \tau^* \geq \underline{\tau}^* &= \left(\frac{\boldsymbol{\rho}_{Aut} \cdot \boldsymbol{a}^*}{\boldsymbol{\rho}^*_{Aut} \cdot \boldsymbol{a}}\right) \max_{i \in \{1,\ldots,N\}} \left\{\frac{u^*_i(\boldsymbol{Y}^*)}{u^*_1(\boldsymbol{Y}^*)} \frac{u_1(\boldsymbol{Y})}{u_i(\boldsymbol{Y})}\right\} - 1. \end{aligned} \tag{10}$$

If tariffs satisfy these bounds, then the autarky allocation is an equilibrium.

It is instructive to consider a case where Assumption I.1 fails, such as an Armington model with fully specialized endowments. If the marginal utility of the imported good approaches infinity at zero consumption, Assumption I.1 is violated, rendering autarky unattainable via finite tariffs. However, introducing a choke price (implying finite marginal utility at zero) restores the validity of Assumption I.1.[1]

I.3 Trade costs

Modern quantitative trade theory emphasizes the role of trade costs in generating the "gravity" relation between bilateral expenditure shares, economic sizes, and distance. It is straightforward to incorporate such costs into the multi-good environment of Section I.2. We focus now on an environment where all goods can potentially be traded (so $M = M^* = 0$). Let G be an "export" technology that takes as input a vector of goods in Home and generates a vector of goods in Foreign. Specifically, for $G(\boldsymbol{x})$ to reach Foreign, a vector of goods $\boldsymbol{x}$ needs to be allocated. Let G^* be the analogous "export" technology for Foreign. These technologies capture all feasible methods of transporting goods across countries. We assume that G and G^* are weakly increasing, weakly concave, and differentiable functions in $\mathbb{R}^N_{\geq 0}$, both are constant returns to scale, and satisfy $\boldsymbol{0} = G(\boldsymbol{0}) = G^*(\boldsymbol{0})$. The case with no trade costs corresponds to $G(\boldsymbol{x}) = \boldsymbol{x}$ and $G^*(\boldsymbol{x}^*) = \boldsymbol{x}^*$. The standard case of good-specific iceberg trade costs corresponds to $G(\boldsymbol{x}) = \Lambda\boldsymbol{x}$ and $G^*(\boldsymbol{x}^*) = \Lambda^*\boldsymbol{x}^*$, where Λ and Λ^* are diagonal matrices with elements in $(0, 1]$, reflecting the survival rate of goods in transit (that is, the inverse of the iceberg trade costs).

The resource constraints are modified as follows:

$$\boldsymbol{Y} = \boldsymbol{c} + \boldsymbol{x} - G^*(\boldsymbol{x}^*), \quad \text{and} \quad \boldsymbol{Y}^* = \boldsymbol{c}^* + \boldsymbol{x}^* - G(\boldsymbol{x}),$$

where $\boldsymbol{x} \geq 0$ is the vector of Home goods used as inputs (or lost in transit for the iceberg case) to

[1] A related question is whether the autarkic prices $\boldsymbol{p}$ and $\boldsymbol{p}^*$ are unique under Assumption I.1. Provided all autarkic consumption levels are strictly positive, the differentiability of the utility functions ensures uniqueness. However, if consumption of some goods is zero and marginal utilities remain bounded, the supporting prices are not unique; any price exceeding the marginal valuation at zero would support autarky.

produce Foreign goods (exports), and $\boldsymbol{x}^* \geq 0$ is the corresponding vector for Foreign.

Competitive firms operate these technologies. Their final outputs are taxed with tariffs at Home and in Foreign. The optimization problems of the representative exporting firms are:[2]

$$\max_{\{\boldsymbol{x}\}} \left\{ \frac{1}{1+\tau^*} \boldsymbol{p}^* \cdot G(\boldsymbol{x}) - \boldsymbol{p} \cdot \boldsymbol{x} \right\}, \quad \text{subject to: } \boldsymbol{x} \geq 0.$$

$$\max_{\{\boldsymbol{x}^*\}} \left\{ \frac{1}{1+\tau} \boldsymbol{p} \cdot G^*(\boldsymbol{x}^*) - \boldsymbol{p}^* \cdot \boldsymbol{x}^* \right\}, \quad \text{subject to: } \boldsymbol{x}^* \geq 0.$$

International goods arbitrage is encapsulated in these optimization problems. Constant returns to scale in the exporting technologies imply that exporting firms make zero profits in equilibrium, and thus the households' budget constraints remain as before. The governments' tariff revenues become:

$$T = \frac{\tau}{1+\tau} \boldsymbol{p} \cdot G^*(\boldsymbol{x}^*), \quad \text{and} \quad T^* = \frac{\tau^*}{1+\tau^*} \boldsymbol{p}^* \cdot G(\boldsymbol{x}).$$

The definition of equilibrium is analogous to the baseline definition, Definition 1, but includes $\boldsymbol{x}$ and $\boldsymbol{x}^*$ as equilibrium objects, have the updated resource constraints and government budget constraints, and requires that exporting firms optimize.

Consider the autarky allocation where $\boldsymbol{Y} = \boldsymbol{c}$, $\boldsymbol{Y}^* = \boldsymbol{c}^*$, and $\boldsymbol{x} = \boldsymbol{x}^* = \boldsymbol{0}$. Is this an equilibrium outcome for sufficiently high tariffs? This allocation satisfies the resource constraints. Domestic relative prices are determined by the respective autarkic allocations (endowments) exactly as in the frictionless case. Furthermore, the international relative price that zeros out the net foreign asset positions remains uniquely determined by the ratio of asset valuations at these autarkic prices, given that the households budget constraints are unchanged as there is no tariff revenue. This pins down $\boldsymbol{p}$ and $\boldsymbol{p}^*$.

The only remaining equilibrium condition to check is whether $\boldsymbol{x} = \boldsymbol{x}^* = \boldsymbol{0}$ is optimal for the exporting firms given these prices and tariffs. This holds if the marginal products of G and G^* are bounded above at $\boldsymbol{0}$, a natural restriction satisfied, for example, by linear iceberg costs. Under this restriction, a sufficiently high bilateral tariff pair (τ, τ^*) renders exporting unprofitable for any good, sustaining autarky as an equilibrium.[3]

[2] In our notation, domestic prices are measured inclusive of tariffs. As a result, the revenue of exporting firms must be scaled by $1/(1+\tau)$ and $1/(1+\tau^*)$ to reflect the pre-tariff revenue retained by firms.

[3] The exact condition, which follows from the concavity of the export technologies, is

$$\max_{i \in \{1,\dots,N\}} \sum_{j=1}^{N} \frac{p_j}{p_i^*} \left. \frac{\partial G_j^*(\boldsymbol{x}^*)}{\partial x_i^*} \right|_{\boldsymbol{x}^*=\boldsymbol{0}} - 1 = \underline{\tau} \leq \tau, \quad \text{and} \quad \max_{i \in \{1,\dots,N\}} \sum_{j=1}^{N} \frac{p_j^*}{p_i} \left. \frac{\partial G_j(\boldsymbol{x})}{\partial x_i} \right|_{\boldsymbol{x}=\boldsymbol{0}} - 1 = \underline{\tau}^* \leq \tau^*.$$

This collapses to (10) when $G(\boldsymbol{x}) = G^*(\boldsymbol{x}) = \boldsymbol{x}$, the case without trade frictions.

I.4 Time and uncertainty

In this appendix we extend our analysis to a dynamic, stochastic economy. The key outcome is to show how interest rates and exchange rates adjust so that desired inter-temporal risk-sharing becomes compatible with autarky.

Interpreting different goods as different times or states, Proposition I.2 tells us that, in a dynamic environment, a permanent increase in bilateral tariffs that is high enough makes autarky an equilibrium as long as the total value of the asset positions of each country have the same sign *when evaluated at the respective autarky prices.* More explicitly, consider a deterministic dynamic economy starting at $t = 0$. For simplicity, we abstract from uncertainty, but the addition of one more index indicating the state of nature is straightforward. Suppose households' utilities are given by:

$$\sum_{t=0}^{\infty} \beta^t u(C_t), \qquad \text{and} \qquad \sum_{t=0}^{\infty} \beta^{*t} u^*(C_t^*),$$

where C_t and C_t^* are the consumption aggregates at date t. That is,

$$C_t = C(c_1(t), c_2(t), \cdots, c_{N+M}(t)), \qquad \text{and} \qquad C_t^* = C^*(c_1^*(t), c_2^*(t), \cdots, c_{N+M^*}^*(t)),$$

where $c_i(t)$ is consumption of good i at time t in Home, $c_j^*(t)$ is consumption of good j at t in Foreign, and C and C^* are the respective aggregator functions. Let $Y_i(t)$ and $Y_j^*(t)$ be the endowments of good $i \in \mathcal{I}$ in Home and $j \in \mathcal{I}^*$ in Foreign at time t.

Let $p_i(t)$ be the time-zero price of good $i \in \mathcal{I}$ in Home at time t, and let $p_j^*(t)$, $j \in \mathcal{I}^*$ be the price of good $j \in \mathcal{I}^*$ in Foreign. That is, $p_i(t)$ is the period-zero Arrow-Debreu price of a unit of good i delivered (at Home) in period t. All prices are expressed in terms of a common numeraire. Autarky relative prices are pinned down by endowments. In particular, relative prices in autarky at Home satisfy

$$\frac{p_i(t)}{p_j(t')} = \beta^{t-t'} \frac{u_i(C_t)}{u_j(C_{t'})},$$

where $u_i(C_t)$ is the marginal utility of good i in period t evaluated at the autarkic allocation. A similar condition determines relative prices in Foreign.

Let $a_j(t)$ denote Home's initial claim to good $j \in \mathcal{I}^*$ in period t in Foreign. Let $a_i^*(t)$ denote Foreign's initial claim to good $i \in \mathcal{I}$ in Home. The condition corresponding to (9), i.e. the condition which zeros out net foreign assets, is:

$$\sum_{i \in \mathcal{I}} \sum_{t} p_i(t) a_i^*(t) = \sum_{j \in \mathcal{I}^*} \sum_{t} p_j^*(t) a_j(t). \tag{11}$$

As relative prices in each region are determined by endowments, equation (11) pins down the relative value of a numeraire in each country. It will be useful to consider as our numeraire the value of a basket consisting of weighted sum of the goods in period zero in each location:

$$\overline{p}_0 \equiv \sum_{i \in \mathcal{I}} \gamma_i p_i(0) = p_1(0) \sum_{i \in \mathcal{I}} \gamma_i \frac{p_i(0)}{p_1(0)}$$
$$\overline{p}_0^* \equiv \sum_{j \in \mathcal{I}^*} \gamma_j^* p_j^*(0) = p_1^*(0) \sum_{j \in \mathcal{I}^*} \gamma_j^* \frac{p_j^*(0)}{p_1^*(0)},$$

where $\gamma_i, \gamma_j^* > 0$ and $\sum_i \gamma_i = \sum_j \gamma_j^* = 1$ are the weights for the numeraire price index. The second equality in each line above establishes that, as in the benchmark, there is only one degree of freedom given that relative prices are given by endowments. Let $q_0 \equiv \overline{p}_0^*/\overline{p}_0$ be the initial exchange rate (i.e., relative value of the numeraire baskets) that zeros out initial asset positions in (11).

Proposition I.2 states that q_0 defined in this way supports an autarkic equilibrium for high enough (permanent) tariffs. The initial exchange rate is such that at period zero each country finds that the value of their initial assets equals their initial liabilities. From this period on, the financial positions can be netted out and there is no further need for additional trades.

We can shed more light on the dynamic behavior of the exchange rate by analyzing the corresponding sequence formulation of the autarkic equilibrium. Consider the two one-period risk-free bonds that promise at time t to pay one unit of the numeraire basket at $t+1$ in each country. The time-zero value of a such a claim in Home is $\overline{p}_t \equiv \sum_{i \in \mathcal{I}} \gamma_i p_i(t)$, while in Foreign it is: $\overline{p}_t^* \equiv \sum_{j \in \mathcal{I}^*} \gamma_j^* p_j^*(t)$. The one-period interest rate at time t for each bond can be defined as the relative value of the promised basket for period $t+1$ relative to t:

$$R_t \equiv \frac{\overline{p}_t}{\overline{p}_{t+1}} \quad \text{and} \quad R_t^* \equiv \frac{\overline{p}_t^*}{\overline{p}_{t+1}^*}.$$

The interest rate on each bond ensures the residents in the respective countries do not wish to borrow or save in their own bonds. Defining the sequential exchange rate at time t as $q_t \equiv \frac{\overline{p}_t^*}{\overline{p}_t}$, we have

$$R_t = R_t^* \frac{q_{t+1}}{q_t},$$

which is the usual parity condition relating the differences in rates of return across countries to the expected depreciation of the exchange rate. More generally, interest rates and exchange rates are such that the risk-sharing condition of Backus and Smith (1993) holds, despite no trade occurring. That is, the exchange rate and interest rates are such that neither country wishes to borrow or save in the other country's bonds. Hence, the autarkic interest rates and the changes in the exchange rate ensure zero asset net positions after the initial period, while the *the level* of the initial exchange rate q_0 ensures that budgets balance in autarky.

II Quantification Appendix

II.1 Calibration of Armington parameters and productivity processes

The model has four parameters, $\{\beta, \mu, \eta, \sigma\}$. We set $\beta = 0.96$ (discounting), $\mu = 0.5$ (materials share), $\eta = 4$ (Armington elasticity), and $\sigma = 1/1.01$ (inverse of risk aversion, so utility close to log). We then need to take a stand on labor supply, trade costs, tariffs, productivity and foreign asset portfolios in date 0, as well as the process for labor supply, trade costs, tariffs and productivity going forward from date 0. We also need to calibrate the preference shifter θ_0^*.

We first recover historical productivity and trade costs consistent with the model for the period 1970-2023 using the approach of Fitzgerald (2025).[4] This requires data on labor supply (we use population), national income accounts and expenditure PPPs for the US and ROW aggregate. Given these data and $\{\mu, \eta\}$, model inversion yields time series for $\left\{z_t, z_t^*\right\}$ and $\left\{d_t, d_t^*\right\}$ for 1970-2023 such that the model exactly matches the data. This procedure is invariant to intertemporal preferences and the nature of the asset market. Details of data construction and the model inversion procedure are below.

Going forward from date 0, we assume that labor supply and iceberg trade costs are fixed at their levels in 2023 (i.e. $L = L_{2023}$, $L^* = L^*_{2023}$, $d = d_{2023}$, $d^* = d^*_{2023}$). In the baseline, we set tariffs permanently equal to zero (i.e. $\tau = \tau^* = 0$) consistent with how we recover productivity and trade costs. We use the 54-year panel of data on productivity for the US and ROW to estimate a process for productivity. We assume the US is the world productivity leader, and that $\Delta \ln z_t$ follows:

$$\Delta \ln z_t = g + \varepsilon_t$$

where

$$\varepsilon_t = \psi \varepsilon_{t-1} + \sigma_z \nu_t$$

and $\nu_t \sim N(0, 1)$. Meanwhile $\Delta \ln z_t^*$ follows:

$$\Delta \ln z_t^* = g + \gamma^* \left(\ln z_{t-1} - \ln z_{t-1}^*\right) + \varepsilon_t^*$$

[4] We assume that there are only resource costs of trade, and no tariffs throughout this period. Given data on total tariff revenues for the US, and ROW revenues from tariffs levied on imports from the US for the period 1970-2023, it would be straightforward to allow for tariffs.

where

$$\varepsilon_t^* = \psi^* \varepsilon_{t-1}^* + \chi^* \varepsilon_{t-1} + \sigma_z^* \nu_t^* + \omega^* \nu_t$$

and $\nu_t^* \sim N(0,1)$. We estimate this process by maximum likelihood. Parameter estimates are reported in Table 1.

Table 1: Parameters of estimated productivity process

Parameter		Value	95% CI
US			
US long-run growth rate	g	0.0083	[0.0054, 0.0119]
Autocorr. of deviations from trend, US	ψ	0.24	[−0.06, 0.46]
SD of innovations to deviations from trend, US	σ_z	0.0106	[0.0080, 0.0127]
ROW			
Rate of convergence to US	γ^*	0.0044	[−0.0011, 0.0150]
Autocorr. of deviations from trend	ψ^*	0.21	[−0.12, 0.44]
Corr. with lagged US deviations from trend	χ^*	−0.02	[−0.45, 0.42]
SD of innovations to deviations from trend	σ_z^*	0.0143	[0.0114, 0.0175]
Corr. with US innovations to deviations from trend	ω^*	0.0012	[−0.0024, 0.0044]

Notes: ROW coefficients are estimated by maximum likelihood conditional on coefficients for US. Bootstrapped 95% confidence intervals based on 100 draws with replacement.

Our estimates imply a similar long-run growth rate for the US and ROW (ROW convergence is very slow), but that productivity is more volatile in ROW than in the US. In making use of this estimated process, we condition on $\{\ln \mathbf{z}_0, \Delta \ln \mathbf{z}_0\} = \{\ln \mathbf{z}_{2023}, \Delta \ln \mathbf{z}_{2023}\}$.

II.2 Data used to recover historical productivity and trade costs

We use World Development Indicators (WDI) data on US population to proxy for US labor supply. We construct ROW labor supply by subtracting US population from world population from WDI. We take the value of US GDP expressed in current dollars from WDI. We construct ROW GDP by subtracting US GDP from world GDP. US imports from ROW are given by total US imports in current dollars from WDI, and ROW imports from the US are given by total US exports. We use Penn World Tables (PWT) data on expenditure PPPs to construct a ROW PPP by weighting expenditure PPPs for all countries other than the US by their share in ROW gross expenditure (using the WDI data to construct the weights). For this purpose, we augment the WDI national income accounts data using OECD national income accounts data, and national income accounts data from the Taiwanese statistical agency as described in Fitzgerald (2025).

II.3 Model inversion to recover historical productivity and trade costs

Gross output in US is (ROW is analogous):

$$OUT_t = \frac{GDP_t}{1-\mu}$$

Expenditure on domestic production for the US is (ROW is analogous):

$$IMS_t = OUT_t - EX_t$$

where IMS_t is expenditure on domestic production and EX_t is total exports. Gross expenditure (absorption) for the US is (ROW is analogous):

$$ABS_t = OUT_t - EX_t + IM_t$$

where IM_t is total imports. The expenditure PPP for ROW is constructed as a weighted average of PPPs in all countries other than the US:

$$PPP_t^* = \sum_{i=1, i \neq US}^{N} \frac{ABS_t^i}{ABS_t^{ROW}} \frac{PPP_t^i}{PPP_t^{US}}$$

where

$$ABS_t^* = \sum_{i=1, i \neq US}^{N} ABS_t^i$$

Then the model tells us:

$$\left(\frac{p_t}{p_t^*}\right)^{1-\eta} = \frac{IMS_t}{ABS_t} \frac{ABS_t^*}{IMS_t^*} \left(\frac{PPP_t}{PPP_t^*}\right)^{1-\eta}$$

and

$$(d_t)^{1-\eta} = \frac{IMP_t}{ABS_t} \frac{ABS_t^*}{IMS_t^*} \left(\frac{PPP_t}{PPP_t^*}\right)^{1-\eta}$$

where IMP_t is US imports from ROW, and

$$(d_t^*)^{1-\eta} = \frac{EXP_t}{ABS_t^*} \frac{ABS_t}{IMS_t} \left(\frac{PPP_t^*}{PPP_t}\right)^{1-\eta}$$

where EXP_t is US exports to ROW.

Time-series variation in the US output price is given by:

$$\frac{p_t}{p_{1970}} = \frac{OUT_t/Realout_t}{OUT_{1970}/Realout_{1970}}$$

Normalize $p_{1970} = 1$. The output prices used to calculate ROW productivity are:

$$\left(\hat{p}_t^*\right)^{1-\eta} = \left(\frac{p_t^*}{p_t}\frac{p_t}{p_{1970}}\right)^{1-\eta}$$

Expenditure prices used to calculate US productivity are (ROW is analogous):

$$\hat{P}_t = \left(\frac{\left(\hat{p}_t\right)^{1-\eta} + \left(d\hat{p}_t^*\right)^{1-\eta}}{\left(\hat{p}_{1970}\right)^{1-\eta} + \left(d\hat{p}_{1970}^*\right)^{1-\eta}}\right)^{\frac{1}{1-\eta}}$$

Real output in the US is then (ROW is analogous):

$$y_t = \frac{OUT_t}{\hat{p}_t}$$

Materials inputs to production in the US are (ROW is analogous):

$$M_t = \mu\frac{\hat{p}_t y_t}{\hat{P}_t}$$

and productivity in the US is (ROW is analogous):

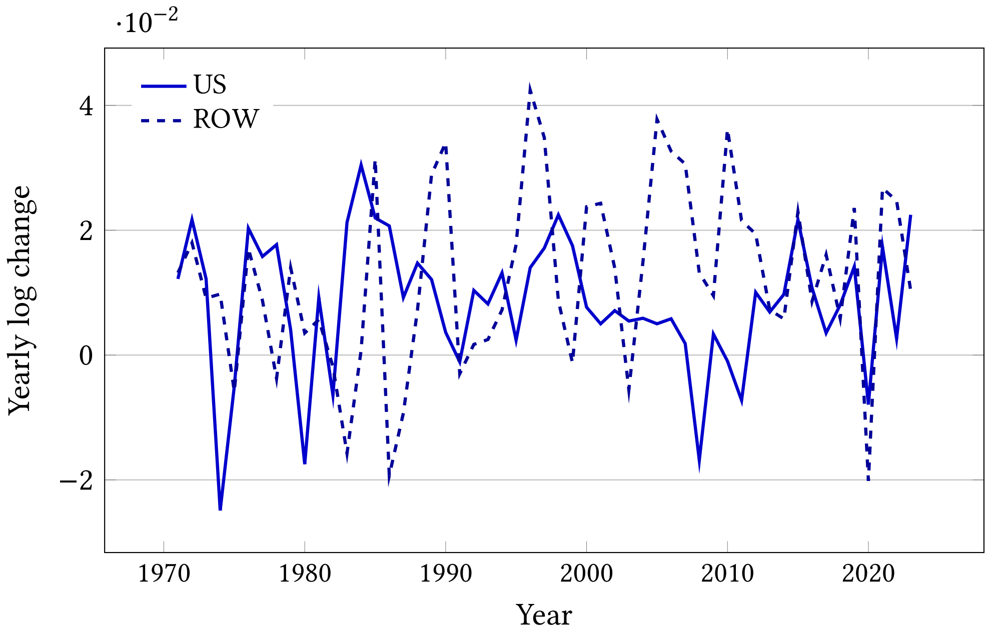
$$z_t = \frac{y_t}{\left(L_t\right)^{1-\mu}\left(M_t\right)^{\mu}}$$

Figure 1 plots the log change in the resulting productivity series for the US and ROW.

II.4 Data for construction of portfolio shares

For this exercise we restrict attention to FDI, equities and debt, and ignore financial derivatives and foreign exchange reserves on both sides of the balance sheet. US FDI & equity assets and liabilities relative to US GDP and US debt assets and liabilities relative to US GDP are taken from the External Wealth of Nations data. ROW assets are set equal to US liabilities, and ROW liabilities are set equal to US assets. The dollar share of US debt assets is taken from the US Treasury's International Capital (TIC) annual report on U.S. Portfolio Holdings of Foreign Securities (Table 14 in report dated December 29, 2023). The dollar share of US debt liabilities is from the US Treasury's International Capital (TIC) annual report on Foreign Portfolio Holdings of U.S. Securities (constructed from Table 12 in report dated June 30, 2023).

Figure 1: Log change in productivity



II.5 Computation of competitive equilibrium without tariffs

Set $\theta_0 = 1$. Fix a value for θ_0^*. Then guess a vector of output prices (i.e. $p_0\left(s^t\right)_0^{FT}$ and $p_0^*\left(s^t\right)_0^{FT}$ for all t, s^t), normalizing one value to 1. Calculate consumption prices using:

$$P_0\left(s^t\right)_0^{FT} = \left(\left(p_0\left(s^t\right)_0^{FT}\right)^{1-\eta} + \left(dp_0^*\left(s^t\right)_0^{FT}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}$$

(and similarly for $P_0^*\left(s^t\right)_0^{FT}$). Then output is

$$y\left(s^t\right)_0^{FT} = L\left(z\left(s^t\right)\right)^{\frac{1}{1-\mu}}\left(\frac{\mu p_0\left(s^t\right)_0^{FT}}{P_0\left(s^t\right)_0^{FT}}\right)^{\frac{\mu}{1-\mu}}$$

and net foreign assets are

$$b_0\left(s_0\right)_0^{FT} = \left(\chi_0 - \chi_0^*\right)\left(1-\mu\right)p_0\left(s_0\right)_0^{FT} y\left(s_0\right)_0^{FT}$$

where χ_0 is US foreign assets as a share of US GDP in period 0, and χ_0^* is ROW foreign assets as a share of US GDP in period 0. Define

$$\Pi_0\left(s_0\right)_0^{FT} =$$

$$\left(\frac{1-\beta}{\theta_0\left(1-\beta\right)+\beta\left(1-\beta^T\right)}\right)^{\frac{\sigma}{1-\sigma}}\left[\begin{array}{c}\left(\theta_0\right)^\sigma\left(P_0\left(s_0\right)_0^{FT}\right)^{1-\sigma}+\\ \sum_{t=1}^T\sum_{s^t}\left(\beta^t\pi\left(s^t\right)\right)^\sigma\left(P_0\left(s^t\right)_0^{FT}\right)^{1-\sigma}\end{array}\right]^{\frac{1}{1-\sigma}}$$

Then

$$M\left(s^t\right)_0^{FT}=\mu^{\frac{1}{1-\mu}}\left(\frac{p_0\left(s^t\right)_0^{FT}z\left(s^t\right)}{P_0\left(s^t\right)_0^{FT}}\right)^{\frac{1}{1-\mu}}L$$

and

$$C\left(s_0\right)_0^{FT}=$$

$$\left(\frac{\Pi_0\left(s_0\right)_0^{FT}\theta_0}{P_0\left(s_0\right)_0^{FT}}\frac{1-\beta}{\theta_0\left(1-\beta\right)+\beta\left(1-\beta^T\right)}\right)^\sigma\left(\frac{b_0\left(s_0\right)_0^{FT}+\sum_{t=0}^T\sum_{s^t}\left(\left(1-\mu\right)p_0\left(s^t\right)_0^{FT}y\left(s^t\right)_0^{FT}\right)}{\Pi_0\left(s_0\right)_0^{FT}}\right)$$

and for $t\geq 1$

$$C\left(s^t\right)_0^{FT}=$$

$$\left(\frac{\Pi_0\left(s_0\right)_0^{FT}\beta^t\pi\left(s^t\right)}{P_0\left(s^t\right)_0^{FT}}\frac{1-\beta}{\theta_0\left(1-\beta\right)+\beta\left(1-\beta^T\right)}\right)^\sigma\left(\frac{b_0\left(s_0\right)_0^{FT}+\sum_{t=0}^T\sum_{s^t}\left(\left(1-\mu\right)p_0\left(s^t\right)_0^{FT}y\left(s^t\right)_0^{FT}\right)}{\Pi_0\left(s_0\right)_0^{FT}}\right)$$

so

$$X\left(s^t\right)_0^{FT}=C\left(s^t\right)_0^{FT}+M\left(s^t\right)_0^{FT}$$

$$x_1\left(s^t\right)_0^{FT}=\left(\frac{P_0\left(s^t\right)_0^{FT}}{p_0\left(s^t\right)_0^{FT}}\right)^\eta X\left(s^t\right)_0^{FT}$$

and

$$x_2\left(s^t\right)_0^{FT}=\left(\frac{P_0\left(s^t\right)_0^{FT}}{dp_0^*\left(s^t\right)_0^{FT}}\right)^\eta X\left(s^t\right)_0^{FT}$$

and excess demand for the US good is given by (ROW is analogous):

$$ED\left(s^t\right)_0^{FT}=x_1\left(s^t\right)_0^{FT}+d^*x_1^*\left(s^t\right)_0^{FT}-y\left(s^t\right)_0^{FT}$$

Use excess demand to inform a new guess of output prices, i.e. $p_0\left(s^t\right)_1^{FT}$: If excess demand is positive, increase the output price, if it is negative, reduce the output price. Iterate to convergence. This gives market-clearing output prices conditional on the value of θ_0^*.

Next, solve for the value of θ_0^* that exactly matches observed data at date 0. For a guess of θ_0^*, check the date-0 US ratio of net exports to GDP:

$$\frac{NX_0\left(s_0\right)^{FT}}{GDP_0\left(s_0\right)^{FT}}=\frac{p_0\left(s_0\right)^{FT}y\left(s_0\right)^{FT}-P_0\left(s_0\right)^{FT}X(s_0)^{FT}}{\left(1-\mu\right)p_0\left(s_0\right)^{FT}y\left(s_0\right)^{FT}}=\kappa_0$$

If κ_0 is greater than the target, increase θ_0^*. If κ_0 is less than the target, reduce θ_0^*. Iterate to convergence. The value of θ_0^* that matches net exports is 0.78.

II.6 Computation of trade war equilibrium

Guess a vector of output prices (i.e. $p_0\left(s^t\right)_0^{TW}$ and $p_0^*\left(s^t\right)_0^{TW}$ for all t, s^t), normalizing one value to 1. Calculate consumption prices using:

$$P_0\left(s^t\right)_0^{TW} = \left(\left(p_0\left(s^t\right)_0^{TW}\right)^{1-\eta} + \left((1+\tau)\, dp_0^*\left(s^t\right)_0^{TW}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}$$

(and similarly for $P_0^*\left(s^t\right)_0^{TW}$). Then output is:

$$y\left(s^t\right)_0^{TW} = L\left(z\left(s^t\right)\right)^{\frac{1}{1-\mu}}\left(\frac{\mu p_0\left(s^t\right)_0^{TW}}{P_0\left(s^t\right)_0^{TW}}\right)^{\frac{\mu}{1-\mu}}$$

and

$$M\left(s^t\right)_0^{TW} = \mu^{\frac{1}{1-\mu}}\left(\frac{p_0\left(s^t\right)_0^{TW} z\left(s^t\right)}{P_0\left(s^t\right)_0^{TW}}\right)^{\frac{1}{1-\mu}} L$$

and net foreign assets are:

$$b_0\left(s_0\right)_0^{TW} = \sum_{t=0}^{\infty}\sum_{s^t} p_0^*\left(s^t\right)_0^{TW} \alpha_0\left(s^t\right) y^*\left(s^t\right)_0^{TW} - \sum_{t=0}^{\infty}\sum_{s^t} p_0\left(s^t\right)_0^{TW} \alpha_0^*\left(s^t\right) y\left(s^t\right)_0^{TW}$$

given the portfolio $\left\{\alpha_0\left(s^t\right), \alpha_0^*\left(s^t\right)\right\}$. Now define

$$\Pi_0\left(s_0\right)_0^{TW} =$$

$$\left(\frac{1-\beta}{\theta_0(1-\beta)+\beta\left(1-\beta^T\right)}\right)^{\frac{\sigma}{1-\sigma}}\left[\begin{array}{c}\left(\theta_0\right)^{\sigma}\left(P_0\left(s_0\right)_0^{TW}\right)^{1-\sigma} + \\ \sum_{t=1}^{T}\sum_{s^t}\left(\beta^t\pi\left(s^t\right)\right)^{\sigma}\left(P_0\left(s^t\right)_0^{TW}\right)^{1-\sigma}\end{array}\right]^{\frac{1}{1-\sigma}}$$

Guess that $C\left(s^t\right)_0^{TW} = 0$ for all t, s^t, i (i.e. demand only comes from materials), and similarly for ROW. Use this guess to calculate tariff revenue in the US (and similarly for $T_0^*\left(s^t\right)_{00}^{TW}$):

$$T_0\left(s^t\right)_{00}^{TW} = \frac{\left(C\left(s^t\right)_{00}^{TW} + M\left(s^t\right)_0^{TW}\right)}{\left(P_0\left(s^t\right)_0^{TW}\right)^{-\eta}}\left(\tau(1+\tau)^{-\eta}\left(dp_0^*\left(s^t\right)_0^{TW}\right)^{1-\eta}\right)$$

Then given the tariff revenue, calculate updated consumption:

$$C\left(s_0\right)_{01}^{TW} = \left(\frac{\Pi_0\left(s_0\right)_0^{TW}\theta_0}{P_0\left(s_0\right)_0^{TW}}\frac{1-\beta}{\theta_0\left(1-\beta\right)+\beta\left(1-\beta^T\right)}\right)^{\sigma}\times$$
$$\times\left(\frac{b_0\left(s_0\right)_0^{TW}+\sum_{t=0}^{\infty}\sum_{s^t}\left(\left(1-\mu\right)p_0\left(s^t\right)_0^{TW}y\left(s^t\right)_0^{TW}+T_0\left(s^t\right)_{00}^{TW}\right)}{\Pi_0\left(s_0\right)_0^{TW}}\right)$$

and for $t\geq 1$

$$C\left(s^t\right)_{01}^{TW} = \left(\frac{\Pi_0\left(s_0\right)_0^{TW}\beta^t\pi\left(s^t\right)}{P_0\left(s^t\right)_0^{TW}}\frac{1-\beta}{\theta_0\left(1-\beta\right)+\beta\left(1-\beta^T\right)}\right)^{\sigma}\times$$
$$\times\left(\frac{b_0\left(s_0\right)_0^{TW}+\sum_{t=0}^{\infty}\sum_{s^t}\left(\left(1-\mu\right)p_0\left(s^t\right)_0^{TW}y\left(s^t\right)_0^{TW}+T_0\left(s^t\right)_{00}^{TW}\right)}{\Pi_0\left(s_0\right)_0^{TW}}\right)$$

and similarly for ROW. Iterate until $C\left(s^t\right)_{0n}^{TW}=C\left(s^t\right)_{0n-1}^{TW}=C\left(s^t\right)_0^{TW}$. Then:

$$X\left(s^t\right)_0^{TW}=C\left(s^t\right)_0^{TW}+M\left(s^t\right)_0^{TW}$$

and

$$x_1\left(s^t\right)_0^{TW}=\left(\frac{P_0\left(s^t\right)_0^{TW}}{p_0\left(s^t\right)_0^{TW}}\right)^{\eta}X\left(s^t\right)_0^{TW}$$

and

$$x_2\left(s^t\right)_0^{TW}=\left(\frac{P_0\left(s^t\right)_0^{TW}}{\left(1+\tau\right)dp_0^*\left(s^t\right)_0^{TW}}\right)^{\eta}X\left(s^t\right)_0^{TW}$$

with similar expressions for ROW. Excess demand for the US good is given by (ROW is analogous):

$$ED\left(s^t\right)_0^{TW}=x_1\left(s^t\right)_0^{TW}+d^*x_1^*\left(s^t\right)_0^{TW}-y\left(s^t\right)_0^{TW}$$

Use excess demand to inform a new guess of output prices, i.e. $p_0\left(s^t\right)_1^{TW}$: If excess demand is positive, increase the output price, if it is negative, reduce the output price. Iterate to convergence.

II.7 Shadow terms of trade in autarky

If the gross asset positions of each country are non-zero and of the same sign when evaluated at the relevant country's autarkic relative prices, we can calculate the shadow terms of trade in autarky as follows. Home's net foreign assets evaluated at autarky prices (and production) is:

$$b_0 = \left(\alpha_0 \sum_{t=0}^{T} \sum_{s^t} p_0^* \left(s^t\right)^{aut} y^* \left(s^t\right)^{aut} + \beta_0 \sum_{t=0}^{T} \sum_{s^t} P_0^* \left(s^t\right)^{aut} \lambda^t\right) - \left(\alpha_0^* \sum_{t=0}^{T} \sum_{s^t} p_0 \left(s^t\right)^{aut} y \left(s^t\right)^{aut} + \beta_0^* \sum_{t=0}^{T} \sum_{s^t} P_0 \left(s^t\right)^{aut} \lambda^t\right)$$

Setting this equal to zero, and rearranging, we get:

$$\frac{p_0 \left(s_0\right)^{aut}}{p_0^* \left(s_0\right)^{aut}} = \frac{\left(\alpha_0 \sum_{t=0}^{T} \sum_{s^t} \frac{p_0^*\left(s^t\right)^{aut}}{p_0^*\left(s_0\right)^{aut}} y^* \left(s^t\right)^{aut} + \beta_0 \sum_{t=0}^{T} \sum_{s^t} \frac{P_0^*\left(s^t\right)^{aut}}{p_0^*\left(s_0\right)^{aut}} \lambda^t\right)}{\left(\alpha_0^* \sum_{t=0}^{T} \sum_{s^t} \frac{p_0\left(s^t\right)^{aut}}{p_0\left(s_0\right)^{aut}} y \left(s^t\right)^{aut} + \beta_0^* \sum_{t=0}^{T} \sum_{s^t} \frac{P_0\left(s^t\right)^{aut}}{p_0\left(s_0\right)^{aut}} \lambda^t\right)}$$

As long as the numerator and the denominator of the right hand side are non-zero and of the same sign, this object is a well-defined terms of trade.

II.8 The Path to Autarky: Terms of Trade and Net Foreign Assets

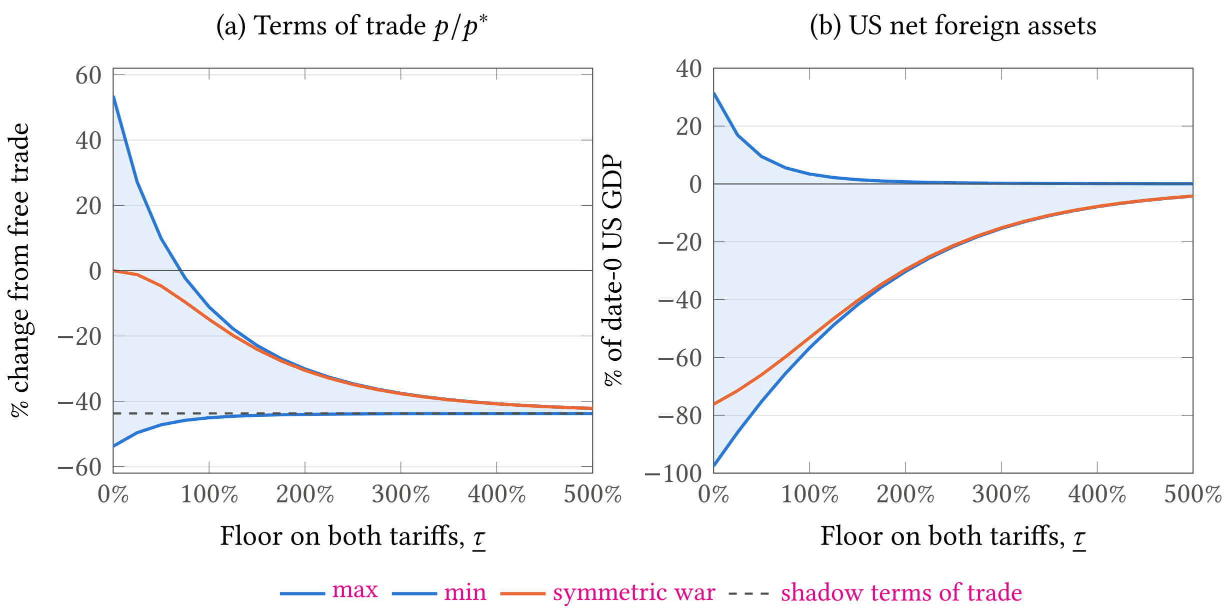
In the theoretical part of the paper, we discussed what happens as tariffs become large. We showed that, under the same sign restriction on the gross portfolios, the countries necessarily go to autarky at some finite tariff levels. In the quantitative model, this result does not hold, as the CES demand specification guarantees that there is always some strictly positive amount of trade. However, as we see below, even in this environment, the autarky result remains valid as an approximation of the outcome of a severe trade war.

We do the following exercise. We solve the benchmark quantitative model on a square grid of tariffs $\{0, 50\%, \ldots, 1000\%, \infty\}$ for each country (22×22 pairs), where a tariff of ∞ means that the country imports nothing and collects no tariff revenue (the price index is its own producer price). We show numerically that, for every $\tau, \tau^* \geq \underline{\tau}$, the largest and smallest values of the terms of trade and US net foreign assets are attained at the two corners $(\tau, \tau^*) = (\infty, \underline{\tau})$ and $(\underline{\tau}, \infty)$.

This suggests the following exercise. For each value of $\underline{\tau}$ on a finer grid, we compute the terms of trade and net foreign assets at the two corners; they should bound the values that can arise in any equilibrium with both tariffs at or above $\underline{\tau}$. Figure 2 plots these ranges as $\underline{\tau}$ rises from 0 to 500%; the orange line in each panel is the symmetric war $\tau = \tau^* = \underline{\tau}$.

As the lower bound increases, the range of equilibrium terms of trade converges to the shadow autarky terms of trade of the previous subsection (in this case, -43.7% relative to free trade), and the US net foreign asset position converges to zero: at $\underline{\tau} = 500\%$ the ranges are $[-43.7\%, -42.2\%]$

Figure 2: Tariff Floors and Convergence to Autarky



Notes: Benchmark portfolio. For each floor $\underline{\tau}$ on the horizontal axis, the shaded band is the range of the variable across trade-war equilibria with $\tau \geq \underline{\tau}$ and $\tau^* \geq \underline{\tau}$. The orange line is the symmetric war $\tau = \tau^* = \underline{\tau}$. Panel (a): date-0 terms of trade $p_0(s_0)/p_0^*(s_0)$ in percent of their free-trade value; the dashed line is the corresponding shadow autarky terms of trade, −43.7%. Panel (b): date-0 US net foreign assets in percent of date-0 US GDP.

and $[-4.2\%, 0.0\%]$ of GDP. This is the theoretical result in the limit: when no goods are traded, the portfolio can only be settled if net foreign assets valued at equilibrium prices are zero, and that condition pins the terms of trade at a/a^*.

II.9 Additional results

Tables 2, 3 and 4 contain additional results for the baseline tariff of 25% and the results for counterfactual tariffs set at 100% and 200%. For comparison, the tables also report results for a version of the model where we set initial net and gross foreign assets to zero, and recalibrate θ^* to match the trade balance in 2023.

II.10 Parameter robustness

We examine the robustness of our baseline results (Table 4) to changing key parameters in the model. First, rather than setting $\theta_0 = 1$ and choosing θ_0^* to match the observed data in 2023, we set $\theta_0^* = 1$, and choose θ_0 to match the data (Table 5). As noted in the paper, these choices are not without loss of generality, but they do not meaningfully affect the results. In addition, we examine what happens when we set $\theta_0 = \theta_0^* = 1$ (Table 6). In this case, since the US is a net debtor, it runs a trade surplus on average, including in the initial period. Terms-of-trade changes are bigger, but otherwise, results are consistent with the baseline.

We also examine what happens when we set σ, the inverse of risk aversion, equal to $1/2$ and $1/4$ in turn (Tables 7 and 8) compared with the baseline value of $1/1.01$. In each case, we also adjust θ_0^* so as to match exactly the 2023 data. Results are broadly consistent with the baseline.

Finally, we examine what happens when we set η, the elasticity of substitution between Home and ROW goods, equal to 3 and 6 in turn (Tables 9 and 10) compared with the baseline value of 4. This requires recomputing model-implied productivity and re-estimating the resulting process for productivity. In each case, we also adjust θ_0^* so as to match exactly the 2023 data. Results are broadly consistent with the baseline.

II.11 Welfare decomposition

We decompose the welfare losses from the benchmark 25%-25% trade war reported in the last two columns of Table 4 into three components: (i) production and consumption distortions due to the tariff wedge between world relative prices and the prices faced by consumers and firms, (ii) terms-of-trade effects, and (iii) valuation effects on the portfolio. The decomposition is reported in Table 11.

To calculate (i) we compare welfare in the tariff equilibrium with welfare for each country if they were to levy no tariffs, yet face the tariff equilibrium world price vector for the two intermediate goods, holding fixed the value of net foreign assets at their value under the tariff equilibrium expressed in terms of some numeraire. To calculate (ii), we compare this latter welfare with welfare if each country were to levy no tariffs, face the free trade equilibrium world price vector for the two intermediate goods, holding fixed the value of net foreign assets at their value under the tariff equilibrium expressed in terms of the same numeraire. To calculate (iii) we compare this latter welfare with welfare under free trade.

The decomposition into these three terms depends on the choice of numeraire. The top panel of Table 11 uses the Home price of the Home intermediate in the tariff equilibrium as the numeraire. The bottom panel of Table 11 uses the Foreign price of the Foreign intermediate in the tariff equilibrium as the numeraire. The exact decomposition does depend on the choice of numeraire, but the qualitative pattern is the same for both.

II.12 Optimal tariffs: best response and unilateral

Table 12 reports the best response and unilaterally optimal tariff counterfactuals for the four portfolios considered in the paper, as well as for the case of zero net and gross foreign assets.

III Currency Composition of Assets and Liabilities

This appendix provides details behind the construction of Figure 2. We use data from the following international datasets: (i) The External Wealth of Nations (EWN) database (Lane and Milesi-Ferretti, 2018); (ii) the Bank for International Settlements (BIS) International Debt Securities statistics (IDS); (iii) the BIS Locational Banking Statistics (LBS); and (iv) the International Monetary Fund (IMF) International Investment Position (IIP) database; and (v) the Sovereign Debt Investor Base for EMs constructed by Arslanalp and Tsuda (2014). We supplement this with a Turkey-specific dataset, the Gross External Debt Stock dataset compiled by the Ministry of Treasury and Finance of the Republic of Türkiye and disseminated by the Central Bank of the Republic of Türkiye (CBRT).

III.1 Assets

All EWN asset positions in foreign portfolio equity (PEA), FDI (FDIA), portfolio debt (PDA), and foreign reserves excluding gold (FR) are treated as fully denominated in foreign currency. The EWN's other investment asset (OIA) position is divided into foreign currency and domestic currency using the shares reported in the BIS LBS dataset for banks. Specifically, the LBS reports claims (assets) and liabilities of banks located in a reporting country vis-à-vis counterparties abroad broken down by currency. Let f_{OI}^{A} denote the foreign-currency share of resident banks' cross-border claims divided by all cross-border claims. This ratio is then applied to the entire OIA level to compute the foreign currency denominated other investment assets; that is, we assume that debt claims held by domestic residents on foreigners have the same currency composition as the claims of banks. The conventions used on the asset side are consistent with Allen, Gautam, and Juvenal (2023). The residual OIA are considered domestic currency assets. To summarize, foreign currency assets (FCA) and domestic currency assets (DCA) are computed as:

$$\begin{aligned} \text{FCA} &= \text{PEA} + \text{FDIA} + \text{PDA} + \text{FR} + f_{OI}^{A} \times \text{OIA} \\ \text{DCA} &= \text{EWN Assets} - \text{FCA}, \end{aligned}$$

where our measure of total EWN Assets excludes gold and financial derivatives.

III.2 Liabilities

EWN's portfolio equity liabilities (PEL) are considered 100 percent domestic currency.

For portfolio debt liabilities (PDL), the level is obtained from EWN. This consists of liabilities of both the public and private sectors. The total amount of government debt (G) held by foreigners is obtained from Arslanalp and Tsuda (2014). We assume this is the public component of EWN's PDL.[5] The Arslanalp-Tsuda dataset also reports foreign holding of domestic-currency public debt as share of total foreign holding of public debt $\left(s_{G}^{DC}\right)$. The product of the two provides the government portfolio liabilities held by foreign residents denominated in domestic currency. The remainder is the foreign currency external public portfolio liability position. For the currency composition of private portfolio liabilities, we use the BIS IDS dataset. The BIS IDS dataset measures bonds/notes issued by domestic residents in international markets by currency. From this, we compute the share of private portfolio liabilities denominated in foreign currency f_{PDL}^{BIS}, which is then used to convert the private component of portfolio debt liabilities.

For EWN's FDI liabilities (FDIL), we first use the IIP dataset to split the total into an equity and debt component. The IMF IIP dataset reports debt and equity FDI liabilities positions separately. Using IIP's measure of equity FDI divided by the total IIP FDI number we compute the share of total FDI that is equity $\left(s_{FDI}^{Eq}\right)$. The FDI equity position, computed as $s_{FDI}^{Eq} \times \text{FDIL}$, is considered domestic currency. The debt component of FDIL is considered to have the same share of foreign currency denomination as private portfolio debt (f_{PDL}^{BIS} computed above).

[5] If this amount is greater than the total portfolio debt liability reported in EWN, then we truncate total government debt at the latter level.

For EWN's other investment liabilities (OIL), we use the same procedure used to compute f^A_{OI}, but now using liabilities reported in the BIS LBS to obtain f^L_{OI}. The assumption for Mexico is that all other-investment liabilities have the same currency composition as banks. For Turkey, we first split OIL by bank and non-bank sectors. The IMF IIP dataset reports OIL from deposit-taking institutions, which we divide by the IIP's total OIL figure to obtain the share of OIL owed by banks (s^{Bank}_{OIL}). This share we decompose into foreign and domestic currency using f^L_{OI} obtained from the BIS LBS data. For the non-bank share, we use data on corporate borrowing from the Gross External Debt Stock dataset to compute the foreign currency share for that sector f_{Corp}. To summarize, foreign currency liabilities (FCL) for Mexico are computed as:

$$\text{FCL} = (1 - s^{DC}_G) \times G + f^{BIS}_{PDL} \times (PDL - G) + f^{BIS}_{PDL} \times (1 - s^{Eq}_{FDI}) \times FDIL + f^L_{OI} \times \text{OIL}.$$

For Turkey, we have

$$\begin{aligned}\text{FCL} =& (1 - s^{DC}_G) \times G + f^{BIS}_{PDL} \times (PDL - G) + f^{BIS}_{PDL} \times (1 - s^{Eq}_{FDI}) \times FDIL \\ &+ f^L_{OI} \times s^{Bank}_{OIL} \times \text{OIL} + f_{Corp} \times (1 - s^{Bank}_{OIL}) \times \text{OIL}.\end{aligned}$$

For both Mexico and Turkey, domestic currency liabilities (DCL) are total EWN liabilities minus FCL, where we omit financial derivatives from our measure of total external liabilities.

The difference with Allen, Gautam, and Juvenal (2023) for Mexico is the treatment of domestic-currency government debt. Allen, Gautam, and Juvenal (2023) uses issuance data from the BIS, which for government debt is almost exclusively foreign currency. We adjust for the fact that foreigners are holding an increasing share of domestic currency government bonds in Mexico purchased on the domestic market. This reduces our estimate of Mexico's foreign currency liabilities relative to Allen, Gautam, and Juvenal (2023). For Turkey, the difference is the treatment of non-bank OIL, which they treat in the same manner as we do for Mexico. The fact that corporations borrow more heavily in foreign currency than do banks increases our estimate of Turkey's foreign currency liabilities.

Table 2: Trade War Scenarios: 25% Tariffs

τ	τ^*	a	a^*	NX	$\Delta(p/p^*)$	$\Delta(P/P^*)$	ΔU	ΔU^*
				Benchmark Portfolio				
0	0	81%	157%	-2.9%	0	0	0	0
25%	25%	83%	154%	-2.2%	-1.20%	-0.33%	-1.04 %	-0.17 %
25%	0	72%	156%	-3.3%	12.4%	12.9%	0.20%	-0.21 %
0	25%	92%	156%	-2.3%	-11.5%	-11.0%	-1.00 %	0.14%
				No Gross Positions				
0	0	0	76%	-2.9%	0	0	0	0
25%	25%	0	75%	-2.1%	-1.47%	-0.60%	-1.19 %	-0.12 %
25%	0	0	75%	-3.6%	12.9%	13.4%	0.58%	-0.32 %
0	25%	0	75%	-1.9%	-12.1%	-11.6%	-1.51 %	0.28%
				Debt-Trap Portfolio				
0	0	-76%	0	-2.9%	0	0	0	0
25%	25%	-79%	0	-2.0%	-1.84%	-0.95%	-1.39 %	-0.06 %
25%	0	-68%	0	-3.9%	13.5%	13.9%	0.96%	-0.42 %
0	25%	-88%	0	-1.5%	-12.7%	-12.2%	-2.06 %	0.44%
				Contract Curve Portfolio				
0	0	323%	399%	-2.9%	0	0	0	0
25%	25%	328%	389%	-2.6%	-0.25%	0.58%	-0.53 %	-0.31 %
25%	0	293%	394%	-2.6%	11.1%	11.7%	-0.66 %	0.04%
0	25%	361%	394%	-3.4%	-9.91%	-9.53%	0.38%	-0.26 %
				Zero Initial Net and Gross Foreign Assets				
0	0	0	0	-2.9%	0	0	0	0
25%	25%	0	0	-3.2%	1.04%	1.82%	-0.88 %	-0.21 %
25%	0	0	0	-4.3%	14.2%	14.6%	0.90%	-0.43 %
0	25%	0	0	-2.3%	-11.5%	-11.0%	-1.51 %	0.31%

Notes: a, a^* and NX are date-0 values of US claims to ROW, ROW claims to US, and US net exports respectively, expressed as percentages of date-0 US GDP. $\Delta(p/p^*)$ and $\Delta(P/P^*)$ are percentage deviations of date-0 US terms of trade and real exchange rate from their levels at date 0 under free trade. ΔU and ΔU^* are percentage deviations of date-0 US and ROW welfare (in consumption-equivalent units) from their levels at date 0 under free trade.

Table 3: Trade War Scenarios: 100% Tariffs

τ	τ^*	a	a^*	NX	$\Delta(p/p^*)$	$\Delta(P/P^*)$	ΔU	ΔU^*
				Benchmark Portfolio				
0	0	81%	157%	-2.9%	0	0	0	0
100%	100%	98%	151%	0.2%	-14.9%	-13.5%	−3.01 %	−0.90 %
100%	0	61%	153%	-1.2%	36.1%	37.7%	−1.98 %	−0.30 %
0	100%	120%	153%	-1.5%	-31.2%	-30.4%	−1.57 %	−0.42 %
				No Gross Positions				
0	0	0	76%	-2.9%	0	0	0	0
100%	100%	0	73%	0.8%	-20.2%	-18.8%	−3.93 %	−0.56 %
100%	0	0	74%	-2.0%	40.0%	41.5%	−0.96 %	−0.60 %
0	100%	0	74%	-0.2%	-34.4%	-33.6%	−3.35 %	0.11%
				Debt-Trap Portfolio				
0	0	-76%	0	-2.9%	0	0	0	0
100%	100%	-107%	0	1.8%	-26.9%	-25.6%	−5.41 %	−0.01 %
100%	0	-54%	0	-2.9%	44.5%	46.0%	0.18%	−0.94 %
0	100%	-126%	0	1.4%	-38.0%	-37.1%	−5.52 %	0.76%
				Contract Curve Portfolio				
0	0	323%	399%	-2.9%	0	0	0	0
100%	100%	350%	376%	-0.7%	-5.13%	-3.65%	−1.64 %	−1.39 %
100%	0	256%	387%	0.2%	29.1%	30.9%	−3.93 %	0.29%
0	100%	430%	383%	-4.4%	-23.8%	-23.1%	2.11%	−1.51 %
				Zero Initial Net and Gross Foreign Assets				
0	0	0	0	-2.9%	0	0	0	0
100%	100%	0	0	-1.3%	2.15%	3.66%	−3.32 %	−0.93 %
100%	0	0	0	-4.1%	51.0%	52.3%	0.16%	−0.96 %
0	100%	0	0	-1.3%	-31.8%	-31.0%	−3.37 %	0.12%

Notes: a, a^* and NX are date-0 values of US claims to ROW, ROW claims to US, and US net exports respectively, expressed as percentages of date-0 US GDP. $\Delta(p/p^*)$ and $\Delta(P/P^*)$ are percentage deviations of date-0 US terms of trade and real exchange rate from their levels at date 0 under free trade. ΔU and ΔU^* are percentage deviations of date-0 US and ROW welfare (in consumption-equivalent units) from their levels at date 0 under free trade.

Table 4: Trade War Scenarios: 200% Tariffs

τ	τ^*	a	a^*	NX	$\Delta(p/p^*)$	$\Delta(P/P^*)$	ΔU	ΔU^*
				Benchmark Portfolio				
0	0	81%	157%	-2.9%	0	0	0	0
200%	200%	120%	150%	0.4%	-30.5%	-29.3%	−2.77 %	−1.48 %
200%	0	56%	152%	0.6%	48.0%	50.4%	−3.96 %	−0.26 %
0	200%	146%	151%	-1.3%	-43.2%	-42.4%	−1.14 %	−1.19 %
				No Gross Positions				
0	0	0	76%	-2.9%	0	0	0	0
200%	200%	0	73%	1.4%	-44.8%	-43.7%	−4.54 %	−0.64 %
200%	0	0	73%	-0.2%	56.5%	58.9%	−2.80 %	−0.65 %
0	200%	0	73%	0.7%	-51.0%	-50.2%	−4.09 %	−0.22 %
				Debt-Trap Portfolio				
0	0	-76%	0	-2.9%	0	0	0	0
200%	200%	-183%	0	4.2%	-57.1%	-56.2%	−8.78 %	1.03%
200%	0	-46%	0	-1.2%	69.4%	71.8%	−1.27 %	−1.15 %
0	200%	-192%	0	4.1%	-59.0%	-58.3%	−8.80 %	1.34%
				Contract Curve Portfolio				
0	0	323%	399%	-2.9%	0	0	0	0
200%	200%	364%	372%	-0.2%	-8.67%	-7.13%	−1.69 %	−2.02 %
200%	0	244%	384%	2.0%	35.8%	38.2%	−5.96 %	0.41%
0	200%	463%	377%	-4.7%	-28.9%	-28.3%	3.23%	−2.46 %
				Zero Initial Net and Gross Foreign Assets				
0	0	0	0	-2.9%	0	0	0	0
200%	200%	0	0	-0.4%	2.09%	3.78%	−4.37 %	−1.25 %
200%	0	0	0	-3.0%	91.5%	93.7%	−1.21 %	−1.19 %
0	200%	0	0	-0.7%	-45.7%	-44.9%	−4.16 %	−0.28 %

Notes: a, a^* and NX are date-0 values of US claims to ROW, ROW claims to US, and US net exports respectively, expressed as percentages of date-0 US GDP. $\Delta(p/p^*)$ and $\Delta(P/P^*)$ are percentage deviations of date-0 US terms of trade and real exchange rate from their levels at date 0 under free trade. ΔU and ΔU^* are percentage deviations of date-0 US and ROW welfare (in consumption-equivalent units) from their levels at date 0 under free trade.

Table 5: Trade War Scenarios: Set $\theta_0^* = 1$, choose θ_0

τ	τ^*	a	a^*	$a - a^*$	NX	$\Delta(p/p^*)$	$\Delta(P/P^*)$	ΔU	ΔU^*
				Contract Curve Portfolio					
0	0	244%	321%	-76%	-2.9%	0	0	0	0
25%	25%	250%	312%	-62%	-2.4%	-0.71%	0.14%	-0.48%	−0.32 %
				Benchmark Portfolio					
0	0	81%	157%	-76%	-2.9%	0	0	0	0
25%	25%	84%	154%	-71%	-2.0%	-1.69%	-0.81%	-1.02%	−0.17 %
				No Gross Positions					
0	0	0	76%	-76%	-2.9%	0	0	0	0
25%	25%	0	75%	-75%	-1.9%	-2.10%	-1.21%	-1.25%	−0.10 %
				Debt-Trap Portfolio					
0	0	-76%	0	-76%	-2.9%	0	0	0	0
25%	25%	-80%	0	-80%	-1.6%	-2.66%	-1.74%	-1.56%	−0.01 %

Notes: a, a^* and NX are date-0 values of US claims to ROW, ROW claims to US, and US net exports respectively, expressed as percentages of date-0 US GDP. $\Delta(p/p^*)$ and $\Delta(P/P^*)$ are percentage deviations of date-0 US terms of trade and real exchange rate from their levels at date 0 under free trade. ΔU and ΔU^* are percentage deviations of date-0 US and ROW welfare (in consumption-equivalent units) from their levels at date 0 under free trade.

Table 6: Trade War Scenarios: Set $\theta_0 = \theta_0^* = 1$

τ	τ^*	a	a^*	$a - a^*$	NX	$\Delta(p/p^*)$	$\Delta(P/P^*)$	ΔU	ΔU^*
				Contract Curve Portfolio					
0	0	289%	365%	-76%	7.2%	0	0	0	0
25%	25%	305%	368%	-63%	4.2%	-5.12%	-4.53%	-0.51%	−0.32 %
				Benchmark Portfolio					
0	0	81%	157%	-76%	7.2%	0	0	0	0
25%	25%	86%	159%	-73%	4.7%	-5.95%	-5.34%	-1.02%	−0.17 %
				No Gross Positions					
0	0	0	76%	-76%	7.2%	0	0	0	0
25%	25%	0	77%	-77%	4.9%	-6.26%	-5.64%	-1.21%	−0.12 %
				Debt-Trap Portfolio					
0	0	-76%	0	-76%	7.2%	0	0	0	0
25%	25%	-82%	0	-82%	5.1%	-6.65%	-6.01%	-1.45%	−0.05 %

Notes: a, a^* and NX are date-0 values of US claims to ROW, ROW claims to US, and US net exports respectively, expressed as percentages of date-0 US GDP. $\Delta(p/p^*)$ and $\Delta(P/P^*)$ are percentage deviations of date-0 US terms of trade and real exchange rate from their levels at date 0 under free trade. ΔU and ΔU^* are percentage deviations of date-0 US and ROW welfare (in consumption-equivalent units) from their levels at date 0 under free trade.

Table 7: Trade War Scenarios: $\sigma = 1/2$

τ	τ^*	a	a^*	$a - a^*$	NX	$\Delta(p/p^*)$	$\Delta(P/P^*)$	ΔU	ΔU^*
				Contract Curve Portfolio					
0	0	242%	318%	-76%	-2.9%	0	0	0	0
25%	25%	244%	305%	-61%	-3.0%	0.70%	1.49%	-0.47%	−0.32 %
				Benchmark Portfolio					
0	0	81%	157%	-76%	-2.9%	0	0	0	0
25%	25%	83%	153%	-70%	-2.6%	-0.37%	0.46%	-1.04%	−0.15 %
				No Gross Positions					
0	0	0	76%	-76%	-2.9%	0	0	0	0
25%	25%	0	74%	-74%	-2.4%	-0.82%	0.03%	-1.27%	−0.08 %
				Debt-Trap Portfolio					
0	0	-76%	0	-76%	-2.9%	0	0	0	0
25%	25%	-79%	0	-79%	-2.2%	-1.39%	-0.52%	-1.58%	0.01%

Notes: a, a^* and NX are date-0 values of US claims to ROW, ROW claims to US, and US net exports respectively, expressed as percentages of date-0 US GDP. $\Delta(p/p^*)$ and $\Delta(P/P^*)$ are percentage deviations of date-0 US terms of trade and real exchange rate from their levels at date 0 under free trade. ΔU and ΔU^* are percentage deviations of date-0 US and ROW welfare (in consumption-equivalent units) from their levels at date 0 under free trade.

Table 8: Trade War Scenarios: $\sigma = 1/4$

τ	τ^*	a	a^*	$a - a^*$	NX	$\Delta(p/p^*)$	$\Delta(P/P^*)$	ΔU	ΔU^*
				Contract Curve Portfolio					
0	0	173%	249%	-76%	-2.9%	0	0	0	0
25%	25%	173%	234%	-61%	-3.4%	1.71%	2.46%	-0.40%	−0.33 %
				Benchmark Portfolio					
0	0	81%	157%	-76%	-2.9%	0	0	0	0
25%	25%	82%	150%	-68%	-3.0%	0.59%	1.39%	-0.97%	−0.16 %
				No Gross Positions					
0	0	0	76%	-76%	-2.9%	0	0	0	0
25%	25%	0	73%	-73%	-2.7%	-0.10%	0.72%	-1.33%	−0.05 %
				Debt-Trap Portfolio					
0	0	-76%	0	-76%	-2.9%	0	0	0	0
25%	25%	-79%	0	-79%	-2.3%	-1.06%	-0.20%	-1.83%	0.10%

Notes: a, a^* and NX are date-0 values of US claims to ROW, ROW claims to US, and US net exports respectively, expressed as percentages of date-0 US GDP. $\Delta(p/p^*)$ and $\Delta(P/P^*)$ are percentage deviations of date-0 US terms of trade and real exchange rate from their levels at date 0 under free trade. ΔU and ΔU^* are percentage deviations of date-0 US and ROW welfare (in consumption-equivalent units) from their levels at date 0 under free trade.

Table 9: Trade War Scenarios: $\eta = 3$

τ	τ^*	a	a^*	$a - a^*$	NX	$\Delta(p/p^*)$	$\Delta(P/P^*)$	ΔU	ΔU^*
				Contract Curve Portfolio					
0	0	322%	398%	-76%	-2.9%	0	0	0	0
25%	25%	328%	390%	-62%	-3.2%	-0.36%	0.56%	-0.45%	-0.27 %
				Benchmark Portfolio					
0	0	81%	157%	-76%	-2.9%	0	0	0	0
25%	25%	83%	155%	-72%	-2.9%	-1.33%	-0.36%	-0.94%	-0.13 %
				No Gross Positions					
0	0	0	76%	-76%	-2.9%	0	0	0	0
25%	25%	0	75%	-75%	-2.8%	-1.60%	-0.62%	-1.07%	-0.09 %
				Debt-Trap Portfolio					
0	0	-76%	0	-76%	-2.9%	0	0	0	0
25%	25%	-79%	0	-79%	-2.6%	-2.00%	-1.00%	-1.28%	-0.04 %

Notes: a, a^* and NX are date-0 values of US claims to ROW, ROW claims to US, and US net exports respectively, expressed as percentages of date-0 US GDP. $\Delta(p/p^*)$ and $\Delta(P/P^*)$ are percentage deviations of date-0 US terms of trade and real exchange rate from their levels at date 0 under free trade. ΔU and ΔU^* are percentage deviations of date-0 US and ROW welfare (in consumption-equivalent units) from their levels at date 0 under free trade.

Table 10: Trade War Scenarios: $\eta = 6$

τ	τ^*	a	a^*	$a - a^*$	NX	$\Delta(p/p^*)$	$\Delta(P/P^*)$	ΔU	ΔU^*
				Contract Curve Portfolio					
0	0	327%	403%	-76%	-2.9%	0	0	0	0
25%	25%	331%	389%	-58%	-1.7%	-0.11%	0.57%	-0.57%	−0.37 %
				Benchmark Portfolio					
0	0	81%	157%	-76%	-2.9%	0	0	0	0
25%	25%	83%	153%	-70%	-1.3%	-1.16%	-0.45%	-1.14%	−0.20 %
				No Gross Positions					
0	0	0	76%	-76%	-2.9%	0	0	0	0
25%	25%	0	74%	-74%	-1.2%	-1.49%	-0.77%	-1.33%	−0.15 %
				Debt-Trap Portfolio					
0	0	-76%	0	-76%	-2.9%	0	0	0	0
25%	25%	-79%	0	-79%	-1.0%	-1.86%	-1.14%	-1.53%	−0.08 %

Notes: a, a^* and NX are date-0 values of US claims to ROW, ROW claims to US, and US net exports respectively, expressed as percentages of date-0 US GDP. $\Delta(p/p^*)$ and $\Delta(P/P^*)$ are percentage deviations of date-0 US terms of trade and real exchange rate from their levels at date 0 under free trade. ΔU and ΔU^* are percentage deviations of date-0 US and ROW welfare (in consumption-equivalent units) from their levels at date 0 under free trade.

Table 11: Decomposition of trade war welfare change relative to free trade

		Home				Foreign			
τ	τ^*	ΔU	dist	tot	val	ΔU^*	dist	tot	val
					Numeraire: Home intermediate				
					Contract Curve Portfolio				
25%	25%	-0.53%	-0.72%	-0.44%	0.64%	-0.31%	-0.31%	0.17%	−0.18 %
					Benchmark Portfolio				
25%	25%	-1.04%	-0.71%	-0.55%	0.22%	-0.17%	-0.31%	0.21%	−0.06 %
					No Gross Positions				
25%	25%	-1.19%	-0.70%	-0.59%	0.10%	-0.12%	-0.32%	0.22%	−0.03 %
					Debt-Trap Portfolio				
25%	25%	-1.39%	-0.70%	-0.63%	-0.07%	-0.06%	-0.32%	0.23%	0.02%
					Numeraire: Foreign intermediate				
					Contract Curve Portfolio				
25%	25%	-0.53%	-0.72%	-0.44%	0.64%	-0.31%	-0.31%	0.17%	−0.18 %
					Benchmark Portfolio				
25%	25%	-1.04%	-0.71%	-0.59%	0.25%	-0.17%	-0.31%	0.22%	−0.07 %
					No Gross Positions				
25%	25%	-1.19%	-0.70%	-0.63%	0.14%	-0.12%	-0.32%	0.23%	−0.04 %
					Debt-Trap Portfolio				
25%	25%	-1.39%	-0.70%	-0.69%	-0.01%	-0.06%	-0.32%	0.25%	0.00%

Notes: ΔU and ΔU^* are percentage deviations of date-0 US and ROW welfare (in consumption-equivalent units) from their levels at date 0 under free trade. Dist reports the contribution of distortions to consumption and production. Tot reports the contribution of the terms of trade. Val reports the contribution of the valuation effect on the portfolio. The decomposition in the top panel is calculated using the price of the Home intermediate in Home as the numeraire for the international portfolios. The decomposition in the bottom panel is calculated using the price of the Foreign intermediate in Foreign as the numeraire for the international portfolios.

Table 12: Nash trade war and unilateral optimal tariffs

τ	τ^*	a	a^*	NX/GDP	$\Delta(p/p^*)$	$\Delta(P/P^*)$	ΔU	ΔU^*
				Benchmark Portfolio				
0	0	81%	157%	-2.9%	0	0	0	0
11.5%	18%	84%	155%	-2.6%	-3.37%	-2.79%	-0.66 %	-0.02 %
15%	0	75%	156%	-3.3%	7.71%	8.01%	0.29%	-0.15 %
0	20%	90%	156%	-2.4%	-9.50%	-9.09%	-0.86 %	0.14%
				No Gross Positions				
0	0	0	76%	-2.9%	0	0	0	0
19%	37%	0	75%	-1.7%	-9.19%	-8.29%	-1.73 %	-0.03 %
23%	0	0	75%	-3.6%	12.0%	12.4%	0.59%	-0.30 %
0	37%	0	75%	-1.6%	-16.7%	-16.1%	-1.99 %	0.30%
				Debt-Trap Portfolio				
0	0	-76%	0	-2.9%	0	0	0	0
35%	400%	-376%	0	9.6%	-79.1%	-78.6%	-15.86 %	3.35%
35%	0	-65%	0	-3.9%	18.4%	18.9%	1.01%	-0.54 %
0	400%	-377%	0	9.6%	-79.1%	-78.6%	-15.87 %	3.38%
				Contract Curve Portfolio				
0	0	323%	399%	-2.9%	0	0	0	0
1.1%	0	322%	399%	-2.9%	0.56%	0.58%	0	0
1.1%	0	322%	399%	-2.9%	0.56%	0.58%	0	0
0	0	323%	399%	-2.9%	0	0	0	0
				Zero Initial Net and Gross Foreign Assets				
0	0	0	0	-2.9%	0	0	0	0
34.5%	37.5%	0	0	-2.8%	0.11%	1.08%	-1.44 %	-0.34 %
34.5%	0	0	0	-4.5%	19.3%	19.8%	0.95%	-0.54 %
0	38%	0	0	-2.1%	-16.2%	-15.6%	-2.02 %	0.33%

Notes: The tariffs in the second row of each panel are the Nash equilibrium tariffs on a grid of tariff pairs, the third row the unilaterally optimal US tariff (ROW at zero), and the fourth row the unilaterally optimal ROW tariff (US at zero). Search ranges: benchmark portfolio: US up to 17%, ROW up to 80%; no gross positions: US up to 24%, ROW up to 40%; debt-trap portfolio: US up to 50%, ROW up to 400%; contract curve portfolio: US up to 5%, ROW up to 5%; zero initial net and gross foreign assets: US up to 40%, ROW up to 40%. The optimum is at the top of its range for the ROW tariff in the Nash row of the debt-trap portfolio panel (35%, 400%); the ROW tariff in the ROW unilateral row of the debt-trap portfolio panel (0, 400%); a higher tariff would be chosen there if allowed, while every other optimum is interior. In the contract curve portfolio panel the Nash equilibrium and the US unilateral optimum coincide: ROW's best response to any US tariff is zero, so the two rows are the same equilibrium.

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