

# A Note on HA Models with Aggregate Shocks

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In this note, I quantitatively solve three heterogeneous-agent models with aggregate risk using simple forecasting rules à la Krusell and Smith (1998) written in terms of endogenous market-clearing objects. The models are solved numerically using standard iterative procedures which run in a few minutes on a standard modern laptop.<sup>1</sup> The exercise here is not new: this same forecasting rule approach has been used in many papers.

The three models are: a Huggett (1993) exchange economy, a version with Imrohoroğlu (1989) income dynamics, and a HANK model with nominal rigidities. The key step as already shown in Kaplan, Mitman, and Violante (2020), Bayer, Luetticke, Pham-Dao, and Tjaden (2019), Fernández-Villaverde, Marbet, Nuño, and Rachedi (2025), and emphasized in Moll (2025), is the use of endogenous market-clearing aggregates (prices in Huggett complemented with output in HANK) to summarize the relevant information about the evolution of the distribution of wealth. For each model, a forecasting rule based on interest rates, wages, or output achieves a near-perfect fit. In addition, in the HANK model, the impulse responses closely match those of a representative-agent New Keynesian model.

The first model is a version of Huggett (1993) with aggregate shocks. The setup of this model is discussed in Maliar and Maliar (2020) and Yang, Wang, Schaab, and Moll (2025), which highlight the difficulty of getting a Krusell–Smith type forecasting rule to work in this relatively simple economy given the lack of an obvious moment of the wealth distribution to use as a forecasting variable. Maliar and Maliar (2020) instead use neural networks to solve this model. Yang et al. (2025) show that a reinforcement learning algorithm can also generate a solution by restricting the household policies to be functions of just the real rate and the aggregate shock (and the individual state variables). Endowing the households with a simple forecasting rule that assumes that next-period’s real rate is just a function of the

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<sup>1</sup>The Julia code for this paper can be found at [https://github.com/manuelamador/Solving\\_HA](https://github.com/manuelamador/Solving_HA). Claude Code Opus 4.6 was used in designing and implementing the code.

current real rate, and the current and next-period aggregate shock solves this model with a near-perfect fit.

One may think that part of the reason for this success is that, in the calibrated model, the wealth distribution is mostly invariant to the aggregate shocks. To address this, I next consider a version of the model with a different income process following Imrohoroğlu (1989), where the wealth distribution is no longer invariant. This version of the model can still be solved just as accurately with the same simple forecasting rule.

The final model is a heterogeneous agent model with endogenous labor supply, nominal rigidities and aggregate TFP and monetary policy shocks (HANK). This model is significantly more complex. There are firms that set prices subject to adjustment costs, and generate dividends that flow back to households. There is also a monetary authority that follows a Taylor rule, and a fiscal authority that maintains a real level of debt. However, even with this complexity, a simple forecasting rule that assumes that next period wage and output are just functions of their current values and the current and next-period aggregate shocks achieves a near-perfect fit. With this solution method at hand, I explore three specifications of the model with different levels of government debt and inflation costs. In all cases, the equilibrium of the model is well approximated by a corresponding 3-equation New Keynesian model with a representative agent, although the accuracy in the low-debt case is somewhat reduced.

There is a large literature on solving heterogeneous agent models with aggregate shocks. An interested reader can find a review of the early contributions in Den Haan (2010a) and the recent summary in Moll (2025). For other recent contributions, see Auclert, Bardóczy, Rognlie, and Straub (2021) who develop a first order perturbation method using the sequence space Jacobian to analyze impulse responses, and Bhandari, Bourany, Evans, and Golosov (2023) who propose a perturbation method that generalizes to higher orders. Fernández-Villaverde, Nuño, and Perla (2024) in their discussion of deep learning methods in economics state that "Deep learning makes it possible for the representation space to be learned and approximated rather than "engineered" by the economist". The approach here is one example of an "engineered" representation space, and it is likely that a change in the models will necessitate a different approach.

## 1 Model 1: Huggett's Model with Aggregate Shocks

I start with the Huggett (1993) model with aggregate shocks. Time is discrete and infinite. The economy is an exchange economy where a continuum of households face a stochastic endowment process. Households have preferences over consumption, and they can save and

borrow through a one-period uncontingent asset, which is in zero net supply.

The endowment process for each household is given by:

$$\tilde{y}_{i,t} = y_{i,t}z_t,$$

where  $y_{i,t}$  is an idiosyncratic shock, and  $z_t$  is an aggregate shock common to all households. Both shocks follow a Markov process. Let  $\pi(y'|y)$  and  $\pi(z'|z)$  be the transition probabilities for the idiosyncratic and aggregate shocks, respectively.

A household holding assets  $a_{i,t}$  at the beginning of period  $t$  chooses consumption  $c_{i,t}$  and next period's assets  $a_{i,t+1}$  subject to the budget constraint:

$$c_{i,t} + \frac{1}{R_t}a_{i,t+1} = a_{i,t} + y_{i,t}z_t$$

where  $R_t$  is the real interest rate. The household is subject to a borrowing constraint  $a_{i,t+1} \geq \underline{a}$ .

Household's preferences are given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t}),$$

where  $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ .

The household problem is:

$$\begin{aligned} V(a, y, z, \mathbf{X}) = \max_{c, a'} & \left\{ u(c) + \beta \sum_{y'|y} \sum_{z'|z} \pi(y'|y)\pi(z'|z)V(a', y', z', \hat{\mathbf{X}}(z')) \right\} & (1) \\ \text{s.t. } & c + \frac{1}{R(\mathbf{X})}a' = a + yz, \\ & a' \geq \underline{a}, \\ & \hat{\mathbf{X}}(z') = f(z, z', \mathbf{X}) \end{aligned}$$

where  $\mathbf{X}$  represents a vector of endogenous aggregate variables observed by the households. The notation  $R(\mathbf{X})$  is short-hand for stating that the current real rate,  $R$ , is an element of the vector  $\mathbf{X}$ . The last equation specifies that households form their forecasts for the next period using a forecasting rule  $f$  that depends on the current and next-period aggregate shocks, and the current  $\mathbf{X}$  as in Kaplan et al. (2020). Let  $g(a, y, z, \mathbf{X})$  be the policy function for next period's assets.

The initial state of the economy is given by a distribution of wealth  $\mu_0(a, y|z^0)$ . Consider

a history of aggregate shocks  $z^t = \{z_0, z_1, \dots, z_t\}$ . Let  $\mathbf{X}(z^t)$  denote the equilibrium evolution given the history of shocks. The distribution evolves according to the following law of motion induced by the households policy function  $g$ :

$$\mu_{t+1}(a', y' | z^{t+1}) = \int \int \pi(y' | y) \mathbf{1}_{\{g(a, y, z_t, \mathbf{X}(z^t)) = a'\}} \mu_t(a, y | z^t) da dy. \quad (2)$$

Market clearing requires that the total demand for assets equals the total supply, which is zero in this case:

$$\int \int g(a, y, z, \mathbf{X}(z^t)) \mu_t(a, y | z^t) da dy = 0 \quad (3)$$

**The “Best” Forecasting Rule.** Given the stochastic processes for  $\mathbf{X}(z^t)$ , I assume that the best forecasting rule is the one that minimizes the mean squared error of the forecast within a class:

$$\boldsymbol{\alpha} = \arg \min_{\boldsymbol{\alpha}} \sum_{t \in [\underline{T}, \bar{T}]} \sum_{z^{t+1}} \pi(z^{t+1} | z_0) [\mathbf{X}(z^{t+1}) - f(z, z', \mathbf{X}(z^t) | \hat{\boldsymbol{\alpha}})]^2. \quad (4)$$

for some pre-defined interval  $[\underline{T}, \bar{T}]$ .

With this, I am ready to define an equilibrium. I define the equilibrium directly inclusive of the forecast rule as follows:

**Equilibrium Definition.** Given an initial distribution,  $\mu_0$ , and a class of forecasting rules  $f$ , an equilibrium consists of a forecasting rule for the interest rate  $\boldsymbol{\alpha}$ , a value function  $V(a, y, z, \mathbf{X})$ , a policy function  $g(a, y, z, \mathbf{X})$ , an equilibrium interest rate  $R(z^t)$ , a wealth distribution  $\mu_t(a, y | z^t)$ , and an evolution for the forecasting variables  $\mathbf{X}(z^t)$ , such that  $R(z^t) = R(\mathbf{X}(z^t))$  and:

- Given the forecasting rule, the value function and policy function solve the household’s problem, (1).
- The distribution of wealth evolves according to the law of motion induced by the policy function and the transition probabilities of the shocks, as in (2).
- The market clearing condition, (3), holds for all  $t$  and all history  $z^t$ .
- The forecasting rule is the best forecasting rule,  $f(z, z', \mathbf{X}) = f(z, z', \mathbf{X} | \boldsymbol{\alpha})$ , where  $\boldsymbol{\alpha}$  solves (4).

At this point, I have not yet specified the class of forecasting rules, nor the forecasting variables  $\mathbf{X}$ . Part of the difficulty in solving the model is that there is no obvious moment

of the wealth distribution to use as a forecasting variable, as the mean of the distribution equals zero by market clearing. Note that from the household's problem, the only variable that needs to be forecasted is the real rate tomorrow, and this variable needs to be included in  $\mathbf{X}$ . I assume that the households just use the real rate itself as the only forecasting variable, that is,  $\mathbf{X}(z^t) = \log R(z^t)$ , and I restrict the class of forecasting rules to be log-linear:

$$f_R(z, z', X|\boldsymbol{\alpha}) = \alpha_0 + \alpha_1 X + \alpha_2 \log z' + \alpha_3 \log z.$$

where  $\boldsymbol{\alpha} = (\alpha_0, \alpha_1, \alpha_2, \alpha_3)$  are the coefficients to be estimated.

## 1.1 Parameters and Numerical Algorithm

The parameterization follows Yang et al. (2025) closely. As I will show, the resulting equilibrium policies look quite similar to theirs.

The Markov chains are approximations of the following continuous processes using Tauchen's method:

$$\begin{aligned} \log y_{i,t+1} &= \rho_y \log y_{i,t} + \varepsilon_{i,t+1}^y, & \varepsilon_{i,t+1}^y &\sim N(0, \sigma_{\varepsilon_y}^2), \\ \log z_{t+1} &= \rho_z \log z_t + \varepsilon_{t+1}^z, & \varepsilon_{t+1}^z &\sim N(0, \sigma_{\varepsilon_z}^2), \end{aligned}$$

where  $\sigma_{\varepsilon_y} = \sigma_y \sqrt{1 - \rho_y^2}$  and  $\sigma_y$  is the *unconditional* standard deviation of  $\log y$ , and where  $\sigma_{\varepsilon_z} = \sigma_z \sqrt{1 - \rho_z^2}$  and  $\sigma_z$  is the *unconditional* standard deviation of  $\log z$ .

The bond grid uses  $n_a = 200$  points over  $[a, 50]$  with quadratic spacing (denser near the borrowing constraint). The interest rate grid uses  $n_r = 40$  evenly spaced points over  $[-0.04, 0.10]$ .

The parameters are summarized in Table 1.

**Numerical Algorithm.** The iteration is initialized with a constant forecast rule  $\boldsymbol{\alpha}^{(0)} = (\log(1/\beta), 0, 0, 0)$  and a cold-start policy  $g_{-1} \equiv 0$  (all agents save nothing). The guess  $\log R = \log(1/\beta)$  is the starting point.

### *Numerical Algorithm*

For  $n = 0, 1, 2, \dots$ :

1. Solve the household problem given forecast rule  $\boldsymbol{\alpha}^{(n)}$  to obtain policy  $g_n$ .
2. Simulate the economy for  $\bar{T}$  periods using  $g_n$ , find the interest rate that clears the bond market each period.

Table 1: Baseline calibration (quarterly).

Parameter	Description	Symbol	Value
Discount factor		$\beta$	0.96
Risk aversion	CRRA coefficient	$\gamma$	2
Idiosyncratic persistence		$\rho_y$	0.6
Idiosyncratic std (unconditional)	of $\log y$	$\sigma_y$	0.2
Number of $y$ states	Tauchen grid	$n_y$	3
Aggregate persistence		$\rho_z$	0.9
Aggregate std (unconditional)	of $\log z$	$\sigma_z$	0.02
Number of $z$ states	Tauchen grid	$n_z$	30
Borrowing constraint		$\underline{a}$	-1
Upper asset bound		$\bar{a}$	50
Asset grid points	quadratic spacing	$n_a$	200
Interest rate grid	$r \in [-0.04, 0.10]$	$n_r$	40
Asset supply	zero net supply	$A$	0

- Run OLS on the simulated data for  $t \in [\underline{T}, \bar{T}]$  to estimate the coefficients  $\hat{\boldsymbol{\alpha}}$  in the regression:

$$\log R_{t+1} = \hat{\alpha}_0 + \hat{\alpha}_1 \log R_t + \hat{\alpha}_2 \log z_{t+1} + \hat{\alpha}_3 \log z_t + \text{residual}.$$

- Compute the update direction  $G^{(n)} = \hat{\boldsymbol{\alpha}} - \boldsymbol{\alpha}^{(n)}$ .
- Update with damping:  $\boldsymbol{\alpha}^{(n+1)} = \boldsymbol{\alpha}^{(n)} + \lambda G^{(n)}$ , where  $\lambda = 0.3$ .
- Stop when  $\|G^{(n)}\| < \text{tol}_{\text{KS}} = 10^{-5}$ .

## 1.2 Results

The converged forecast rule coefficients are:

$$\log R_{t+1} = 0.0046 + 0.8073 \log R_t - 0.2003 \log z_{t+1} + 0.1636 \log z_t, \quad R^2 = 0.9997.$$

The mean equilibrium interest rate is around 0.0243. The  $R^2$  of 0.9997 indicates a near-perfect forecasting rule: the triple  $(R_t, \log z_{t+1}, \log z_t)$  explains virtually all of the variation in  $R_{t+1}$ . This is comparable to the  $R^2 > 0.99$  typically found in Krusell–Smith production economies. The solver runs in approximately 52 seconds on a 10-core MacBook Pro M4

using Julia with 10 threads.

Figure 1 plots the consumption policy function  $c(a)$  under different  $(y, z, r = R - 1)$  combinations.

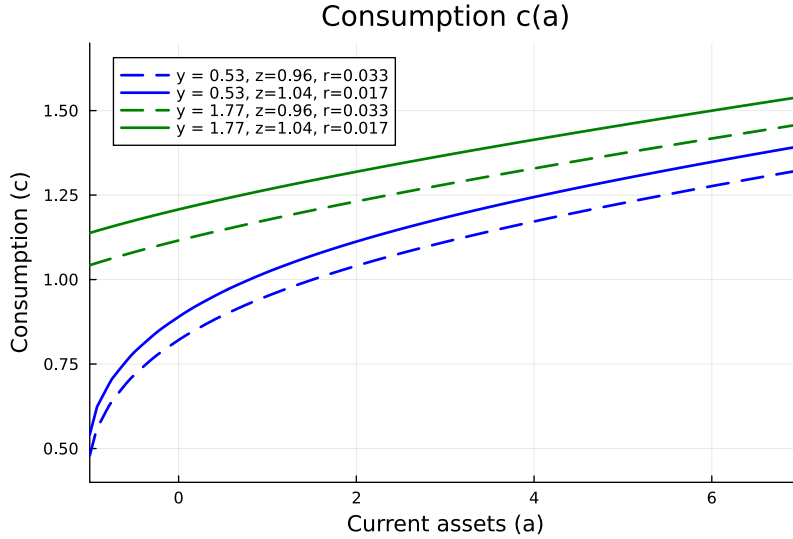


Figure 1: Consumption  $c(a)$  for low and high income agents under different  $(z, r)$  combinations. Dashed: low  $z$ , high  $r$ . Solid: high  $z$ , low  $r$ .

Figure 2 plots the CDF of bond holdings under three scenarios, all using the equilibrium policy from the full model: the ergodic distribution (averaging over aggregate shocks), and the conditional ergodic distributions when low or high  $z$  is permanently realized. All three curves are nearly indistinguishable, indicating that the wealth distribution is insensitive to the aggregate shock.

## 2 Model 2: Imrohoroglu's Version

In this section, I consider the same model as in the previous section but with a different income process following Imrohoroglu (1989). Idiosyncratic income takes two values,  $y \in \{1, \theta\}$ , where employed agents receive  $y = 1$  and unemployed agents receive  $y = \theta = 0.25$  (replacement rate), both normalized so that  $\mathbb{E}[y] = 1$  under the ergodic distribution.

The aggregate state  $z \in \{G, B\}$  (Good, Bad) evolves according to

$$\Pi^z = \begin{pmatrix} 0.9375 & 0.0625 \\ 0.0625 & 0.9375 \end{pmatrix},$$

so both states are highly persistent:  $\Pr(G' | G) = \Pr(B' | B) \approx 0.94$ . The idiosyncratic transition  $\pi(y' | y, z')$  depends on the next-period aggregate state  $z'$ . When the economy

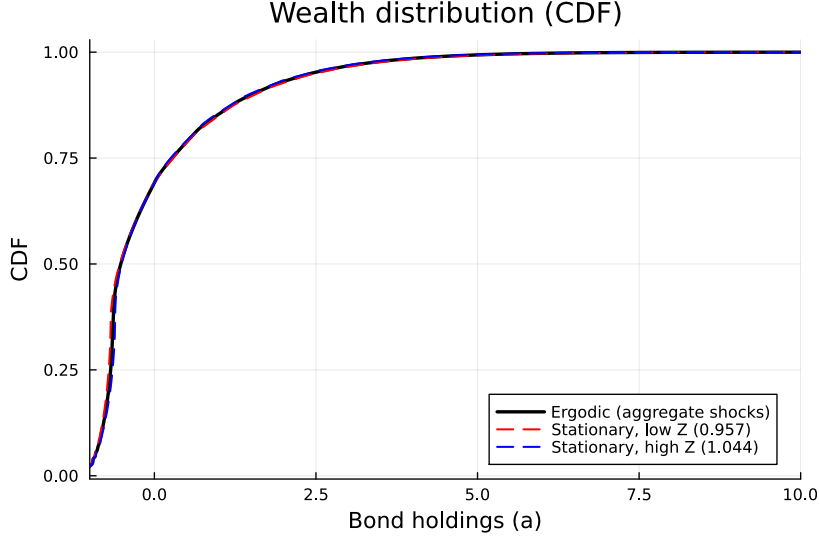


Figure 2: Wealth distribution (CDF of bond holdings). Black: ergodic distribution under aggregate shocks. Red/blue dashed: conditional ergodic distributions when low/high aggregate productivity is permanently realized, using the equilibrium policy from the full model. All three distributions are nearly indistinguishable.

transitions to the good state,

$$\Pi^y(\cdot \mid z' = G) = \begin{pmatrix} 0.9750 & 0.0250 \\ 0.6000 & 0.4000 \end{pmatrix},$$

and when it transitions to the bad state,

$$\Pi^y(\cdot \mid z' = B) = \begin{pmatrix} 0.9400 & 0.0600 \\ 0.4300 & 0.5700 \end{pmatrix},$$

where rows correspond to current employment status (employed, unemployed) and columns to next-period status. Unemployment is both more likely and more persistent in bad times.

The remaining parameters are summarized in Table 2. Note that the time period is now interpreted as monthly (matching Imrohoroglu's original calibration), so the discount factor is higher ( $\beta = 0.995$ ).

Table 2: Imrohoroğlu (1989) calibration (monthly).

Parameter	Description	Symbol	Value
Discount factor	monthly	$\beta$	0.995
Risk aversion	CRRA coefficient	$\gamma$	1.5
Replacement rate	unemployed income	$\theta$	0.25
Number of $y$ states	employed / unemployed	$n_y$	2
Number of $z$ states	Good / Bad	$n_z$	2
Borrowing constraint		$\underline{a}$	-1
Upper asset bound		$\bar{a}$	1.5
Asset grid points	quadratic spacing	$n_a$	200
Interest rate grid	$r \in [-0.04, 0.10]$	$n_r$	40
Asset supply	zero net supply	$A$	0

## 2.1 Results

Since  $z$  and  $z'$  take only discrete, non-cardinal values (Good/Bad), I use a fully interacted specification that effectively allows separate intercepts and slopes for each  $(z, z')$  bin:<sup>2</sup>

$$\begin{aligned} \log R_{t+1} = & \alpha_0 + \alpha_1 \log R_t + \alpha_2 z_t + \alpha_3 z_{t+1} + \alpha_4 z_t z_{t+1} \\ & + \alpha_5 z_t \log R_t + \alpha_6 z_{t+1} \log R_t + \alpha_7 z_t z_{t+1} \log R_t, \end{aligned}$$

where  $z \in \{-0.5, 0.5\}$  are binary indicators for Bad and Good. The converged coefficients are:

$$\begin{aligned} \log R_{t+1} = & -0.0134 + 0.2067 \log R_t - 0.0169 z_t \\ & + 0.0242 z_{t+1} + 0.0061 z_t z_{t+1} - 0.0398 z_t \log R_t \\ & - 0.1500 z_{t+1} \log R_t + 0.0122 z_t z_{t+1} \log R_t, \quad R^2 = 0.9973. \end{aligned}$$

The mean equilibrium interest rate is now  $-0.0158$ , reflecting the tight borrowing constraint and high precautionary savings demand in this calibration. The  $R^2$  of 0.9973 indicates a good fit, comparable to the baseline ( $R^2 = 0.9997$ ). The solver runs in approximately 17 seconds on the same machine.

Figure 3 plots the consumption policy function  $c(a)$  under the four joint states  $(y, z)$  at the mean equilibrium interest rate. Figure 4 plots the CDF of bond holdings under the ergodic distribution (averaging over aggregate shocks) and the conditional ergodic distribu-

<sup>2</sup>The simulation results are very similar if I instead use the linear forecasting rule from the previous section.

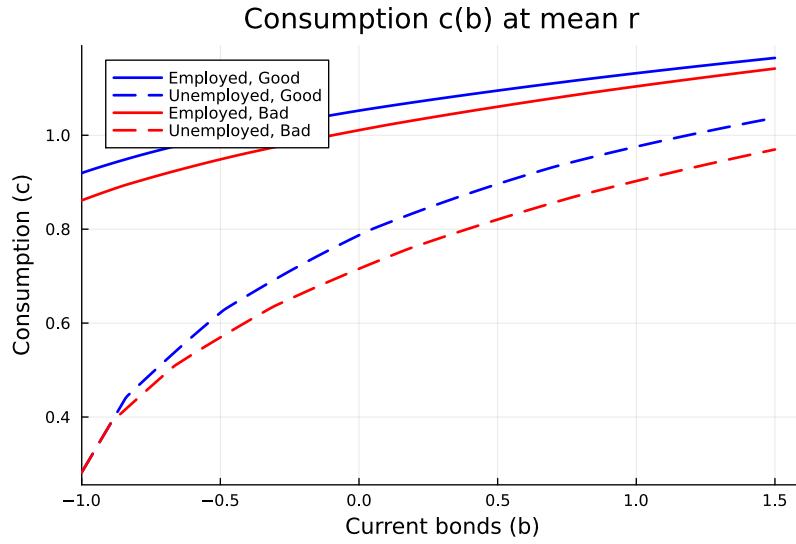


Figure 3: Consumption  $c(a)$  for employed and unemployed agents under Good and Bad aggregate states, evaluated at the mean equilibrium interest rate. Solid: employed. Dashed: unemployed. Blue: Good. Red: Bad.

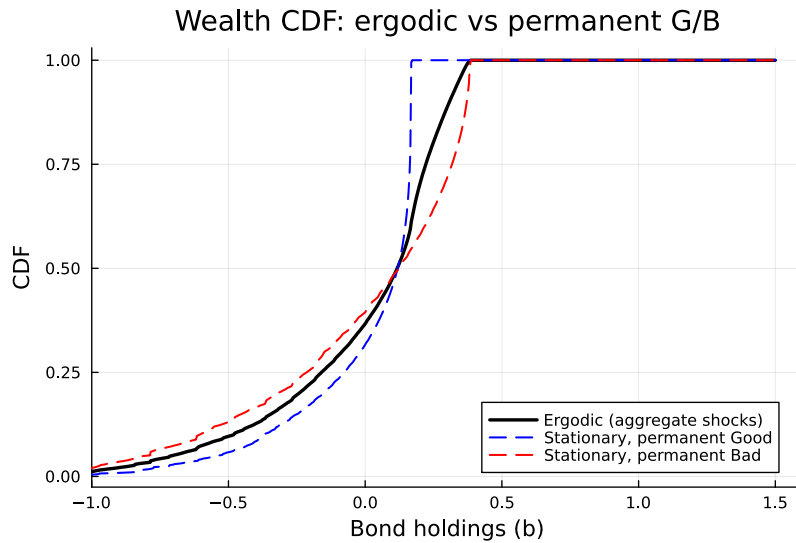


Figure 4: Wealth distribution (CDF of bond holdings). Black: ergodic distribution under aggregate shocks. Blue/red dashed: conditional ergodic distributions when Good/Bad is permanently realized, using the equilibrium policy from the full model.

tions when Good or Bad is permanently realized. Differently from the baseline model, the wealth distribution is now affected by the realization of the aggregate shock. However, the model still can be solved with a simple forecasting rule that depends only on the current and next-period aggregate states, without needing to track the entire distribution of wealth.

### 3 Model 3: HANK

Having established that the forecasting-rule approach works for exchange economies with different income processes, I now test whether it extends to a production economy with nominal rigidities and multiple aggregate shocks. A continuum of households choose consumption, labor supply, and how much to save in a nominal one-period bond.

The aggregate exogenous state is denoted by  $s$  and follows a Markov process with transition probabilities  $\pi(s'|s)$ . Let  $s^t$  denote the history of the aggregate shocks. At history  $s^t$ , a household that starts the period with  $a$  nominal claims and receives an idiosyncratic productivity shock  $e$  faces the following sequential budget constraint for their choice of  $(c, a', n)$ :

$$c + a' = (1 + i(s^{t-1})) \frac{P(s^{t-1})}{P(s^t)} a + w(s^t)en + d(s^t) + T(s^t),$$

where  $i$  is the nominal interest rate decided in the previous period,  $P$  is the price level,  $w$  is the real wage,  $n$  is labor supply,  $d$  is aggregate dividends from firms, and  $T$  is lump-sum transfers. I also assume that  $a' \geq 0$ , the borrowing constraint. Note that in this formulation, the realized real rate is random if the price level is stochastic.

Households have preferences over consumption and labor given by

$$u(c, n) = \frac{c^{1-\gamma}}{1-\gamma} - \frac{n^{1+\varphi}}{1+\varphi}.$$

and discount the future at a rate  $\beta$ .

The government keeps the real debt constant,  $B(s^t) = B$ . This means that the government budget constraint is

$$T(s^t) = B - (1 + i(s^{t-1})) \frac{P(s^{t-1})}{P(s^t)} B = -(R(s^t) - 1)B.$$

where the realized real rate is

$$R(s^t) = (1 + i(s^{t-1})) \frac{P(s^{t-1})}{P(s^t)}.$$

Setting  $\tilde{a} = a - B$ , the household sequential constraint is given by:

$$\begin{aligned} c + \tilde{a}' &= R(s^t)\tilde{a} + w(s^t)en + d(s^t) \\ \tilde{a} &\geq -B. \end{aligned}$$

And as expected, higher government debt  $B$  is equivalent to a relaxation of the borrowing limits for households.

**Firms.** There are a continuum of firms that produce output using labor as the only input,  $y = z(s^t)l$ , where  $z(s^t)$  is an aggregate productivity shock. Each of these firms is a monopolistic competitor and produces a differentiated good. Firms set prices subject to adjustment costs as in Rotemberg (1982). Their demand is

$$y = \left( \frac{p}{P(s^t)} \right)^{-\epsilon} Y(s^t),$$

where  $p$  is the price set by the firm,  $Y(s^t)$  is aggregate output, and  $\epsilon$  is the elasticity of substitution between goods.

The current flow profits of a firm which inherits a price  $p_{-1}$  and chooses a price  $p$  is given by

$$d = \frac{p}{P(s^t)}y - w(s^t)l - \frac{\kappa}{2} \left( \frac{p}{p_{-1}} - 1 \right)^2 Y(s^t)$$

I assume that the firm optimizes in order to maximize the present discount value of the profits (net of inflation costs) using  $\beta$  as the discount factor: <sup>3</sup>

$$\begin{aligned} W(p_{-1}, s, \mathbf{X}) = \max_p \left\{ \left[ \left( \frac{p}{P(\mathbf{X})} \right)^{1-\epsilon} - \frac{w(\mathbf{X})}{z(s)} \left( \frac{p}{P(\mathbf{X})} \right)^{-\epsilon} - \frac{\kappa}{2} \left( \frac{p}{p_{-1}} - 1 \right)^2 \right] Y(\mathbf{X}) \right. \\ \left. + \beta \sum_{s'} \pi(s'|s) W(p, s', \hat{\mathbf{X}}(s')) \right\} \end{aligned}$$

where  $\hat{\mathbf{X}}(s')$  represents the firm's forecast of the future state variables. Let  $g_p(p_{-1}, s, \mathbf{X})$  be the policy function for the price.

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<sup>3</sup>It is not clear what the appropriate (stochastic) discount factor for the firm is in this setting where stockholders are not allowed to trade their shares. I decided for a simple path and made firms risk-neutral with a discount factor of  $\beta$ .

The first order condition for the firm can be written (using the envelope condition) as

$$\left[ (1 - \epsilon) \left( \frac{p}{P(\mathbf{X})} \right)^{1-\epsilon} + \epsilon \frac{w(\mathbf{X})}{z(s)} \left( \frac{p}{P(\mathbf{X})} \right)^{-\epsilon} - \kappa \left( \frac{p}{p-1} - 1 \right) \frac{p}{p-1} \right] + \kappa \beta \sum_{s'} \pi(s'|s) \frac{Y(\hat{\mathbf{X}}(s'))}{Y(\mathbf{X})} \left( \frac{p'}{p} - 1 \right) \frac{p'}{p} = 0.$$

where  $p'$  is the policy function for the price in the next period.

In a symmetric equilibrium, all firms set the same price. Denoting by  $\Pi$  the inflation policy function, the first order condition becomes:

$$(1 - \epsilon) + \epsilon \frac{w(\mathbf{X})}{z(s)} - \kappa (\Pi(s, \mathbf{X}) - 1) \Pi(s, \mathbf{X}) + \kappa \beta \sum_{s'} \pi(s'|s) \frac{Y(\hat{\mathbf{X}}(s'))}{Y(\mathbf{X})} \left( \Pi(s', \hat{\mathbf{X}}(s')) - 1 \right) \Pi(s', \hat{\mathbf{X}}(s')) = 0. \quad (5)$$

Note that written this way,  $w$  and  $Y$  are required to be in the forecasting vector  $\mathbf{X}$ , as the firm's optimal price depends on the current realization of these variables.

Given the equilibrium inflation policy, the dividend policy is then<sup>4</sup>

$$d(s, \mathbf{X}) = \left[ 1 - \frac{w(\mathbf{X})}{z(s)} \right] Y(\mathbf{X}). \quad (6)$$

**Monetary Policy.** Finally, the nominal interest rate follows a Taylor rule:

$$1 + i(s^t) = \bar{R} \Pi(s_t, \mathbf{X}(s^t))^\phi \varepsilon(s^t),$$

where  $\bar{R}$  is a parameter,  $\phi$  is the Taylor rule coefficient on inflation, and  $\varepsilon(s^t)$  is the monetary policy shock (with  $\varepsilon = 1$  in steady state).

Together with the Fisher equation, the Taylor rule pins down the realized return:

$$R(s^{t+1}) = \bar{R} \frac{\Pi(s^t, \mathbf{X}(s^t))^\phi \varepsilon(s^t)}{\Pi(s^{t+1}, \mathbf{X}(s^{t+1}))}. \quad (7)$$

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<sup>4</sup>In here I am assuming that the inflation costs do not reduce real profits for the firms. This keeps the dividend policy simple and not dependent on the inflation realization. The exclusion or not of inflation costs is probably important for normative analysis of counterfactual policies. In any case, Appendix B presents the simulation when the inflation costs are included in the dividend policy, and the results are very similar.

**Households.** It is convenient to define the "exogenous cash-on-hand" as

$$m \equiv R\tilde{a} + d(s, \mathbf{X}),$$

which combines the return on assets and dividends. Using this, the household problem can be written as the following Bellman equation:

$$V(m, e, s, \mathbf{X}) = \max_{c, n, \tilde{a}'} \left\{ u(c, n) + \beta \sum_{e'|e} \sum_{s'|s} \pi(e'|e) \pi(s'|s) V(\hat{m}'(s'), e', s', \hat{\mathbf{X}}(s')) \right\}$$

s.t.  $c + \tilde{a}' = m + w(\mathbf{X})en, \quad \tilde{a}' \geq -B, \quad (8)$

where next period's (forecast) of cash-on-hand is

$$\hat{m}'(s') = \hat{R}(s')\tilde{a}' + d(s', \hat{\mathbf{X}}(s')).$$

And the value of  $\hat{R}(s')$  is determined using (7) and the forecasts:

$$\hat{R}(s') = \bar{R} \frac{\Pi(s, \mathbf{X})^\phi \varepsilon(s)}{\Pi(s', \hat{\mathbf{X}}(s'))}.$$

Let  $h_a(m, e, s, \mathbf{X})$  and  $h_n(m, e, s, \mathbf{X})$  be the policy functions for next period's assets and current labor supply.

Equations (5), (6) and (8) show that the vector  $\mathbf{X}$  must include the wage  $w$  and aggregate demand  $Y$ .

**The Distribution of Wealth.** Given an initial distribution of wealth  $\mu_0(\tilde{a}, e|s^0)$ , the distribution evolves according to the following law of motion induced by the household savings policy  $h_a$ :

$$\mu_{t+1}(\tilde{a}', e'|s^{t+1}) = \int \int \pi(e'|e) \mathbf{1}_{\{h_a(m(\tilde{a}, s^t), e, s_t, \mathbf{X}(s^t)) = \tilde{a}'\}} \mu_t(\tilde{a}, e|s^t) d\tilde{a} de, \quad (9)$$

where  $m(\tilde{a}, s^t) = R(\mathbf{X}(s^t))\tilde{a} + d(s_t, \mathbf{X}(s^t))$  is cash-on-hand.

**Market Clearing Conditions.** The market clearing condition for the bond market is given by

$$\int \int h_a(m(\tilde{a}, s^t), e, s, \mathbf{X}(s^t)) \mu_t(\tilde{a}, e|s^t) d\tilde{a} de = 0. \quad (10)$$

The market clearing condition for the labor market is given by

$$\int \int e \cdot h_n(m(\tilde{a}, s^t), e, s, \mathbf{X}(s^t)) \mu_t(\tilde{a}, e|s^t) d\tilde{a} de = \frac{Y(s^t)}{z(s^t)}. \quad (11)$$

**The best forecasting rules.** I assume, as before, that the vector  $\mathbf{X}$  just includes the minimal set of variables necessary to state the household and the firms' problem. In this case  $\mathbf{X} = (w, Y)$ .

As before, given the equilibrium evolution of the wage  $w(s^t)$ , and output  $Y(s^t)$ , the best forecasting rules are those that minimize the mean squared error of the forecasts:

$$\begin{aligned} \min_{\alpha_w} \sum_{t \in [\underline{T}, \bar{T}]} \sum_{s^t} \pi(s^t|s^0) [(\log w(s^t) - f_w(s_t, s_{t-1}, (w(s^{t-1}), Y(s^{t-1}))|\alpha_w))^2], \quad (12) \\ \min_{\alpha_Y} \sum_{t \in [\underline{T}, \bar{T}]} \sum_{s^t} \pi(s^t|s^0) [(\log Y(s^t) - f_Y(s_t, s_{t-1}, (w(s^{t-1}), Y(s^{t-1}))|\alpha_Y))^2]. \end{aligned}$$

And thus, the forecasts used by households and firms are:

$$\hat{\mathbf{X}}(s') = (\hat{w}(s'), \hat{Y}(s')) = (\exp(f_w(s', s, (w, Y)|\alpha_w)), \exp(f_Y(s', s, (w, Y)|\alpha_Y))).$$

I specialize the forecasting rules to be log-linear:

$$\begin{aligned} f_x(s, s_{-1}, w, Y | \alpha_x) = \alpha_0^x + \alpha_1^x \log z(s) + \alpha_2^x \log \varepsilon(s) + \alpha_3^x \log z(s_{-1}) \\ + \alpha_4^x \log \varepsilon(s_{-1}) + \alpha_5^x \log w + \alpha_6^x \log Y, \end{aligned}$$

for  $x \in \{w, Y\}$ .

An equilibrium can now be defined:

**Equilibrium Definition.** Starting from an initial wealth distribution,  $\mu_0(\tilde{a}, e|s^0)$ , and an initial nominal interest rate  $i_{-1}$ , an equilibrium consists of forecasting rules for the wage and aggregate demand,  $\alpha_w, \alpha_Y$ , a value function  $V(m, e, s, (w, Y))$ , policy functions  $h_a(m, e, s, (w, Y))$  and  $h_n(m, e, s, (w, Y))$  for next period's assets and current labor supply respectively, an inflation policy  $\Pi(s, (w, Y))$ , and an equilibrium evolution of the wage  $w(s^t)$ , aggregate demand  $Y(s^t)$ , and the wealth distribution  $\mu_t(\tilde{a}, e|s^t)$ , such that:

- Given the forecasting rules, the inflation policy solves the firm's problem (5).
- The realized return evolves according to the Taylor rule and the Fisher equation, given the inflation policy, that is equation (7).

- Given the forecasting rules, the value function and policy functions solve the household's problem (8) given the Taylor rule and the Fisher equation.
- The distribution of wealth evolves according to the law of motion (9).
- The market clearing conditions (10)–(11) hold for all  $t$  and all history  $s^t$ .
- The forecasting rules are the best forecasting rules (12) given the equilibrium evolution of the wage and aggregate demand.

### 3.1 Numerical Algorithm

The algorithm is more involved than before, but follows a similar iterative structure. The idea is the same: start with an initial guess for the forecasting rules. Given this guess, solve for the equilibrium inflation policy consistent with the NKPC (5). Then solve the household problem given the forecast rules. Next, simulate the economy using the equilibrium policies and use the market clearing conditions and the real rate equation to recover the equilibrium evolution of the real return, the wage and output. Finally, update the forecasting rules using OLS on the simulated data. Iterate until convergence.

#### *Numerical Algorithm for the HANK Model*

Set initial forecast rules  $\alpha_w^{(0)}$ ,  $\alpha_Y^{(0)}$  and inflation policy  $\Pi^{(0)}(s, w, Y)$ .

**Iteration.** Given forecast rules  $\alpha_w^{(n)}$ ,  $\alpha_Y^{(n)}$  and inflation policy  $\Pi^{(n)}(s, w, Y)$ :

1. **NKPC.** Update the inflation policy  $\Pi^{(n+1)}(s, w, Y)$  on the  $(z, \varepsilon, w, Y)$  grid. Working with  $\psi = \kappa(\Pi - 1)\Pi$ , iterate:

$$\psi(s, w, Y) = (1 - \epsilon) + \epsilon \frac{w}{z} + \beta \sum_{s'} \pi(s'|s) \frac{\hat{Y}'}{Y} \psi(s', \hat{w}', \hat{Y}'),$$

where  $\hat{w}' = \hat{w}(s'; s, w, Y)$  and  $\hat{Y}' = \hat{Y}(s'; s, w, Y)$  are given by the forecast rules. Recover  $\Pi$  from  $\psi$  via the quadratic root.

2. **Household.** Solve the household problem (8) via EGM using the forecast rules and the updated inflation policy  $\Pi^{(n+1)}$  to obtain the savings and employment policies  $h_a^{(n+1)}(m, e, s, w, Y)$  and  $h_n^{(n+1)}(m, e, s, w, Y)$ .
3. **Simulation.** Simulate the economy for  $\bar{T}$  periods. At each period  $t$ :
  - (a) Draw  $(z_t, \varepsilon_t)$  from the Markov chains.

(b) **Clear markets** by solving for  $(w_t, Y_t)$  such that bond and labor markets clear. That is, for each  $(w, Y)$ : obtain  $\Pi_t$  from the policy rule and set  $R_t = (1 + i_{t-1})/\Pi_t$ . Given this, compute dividends and cash-on-hand, integrate over the distribution  $\mu_t$ , and look for a root in the market clearing conditions (10)–(11). This determines  $(w_t, Y_t)$ .

(c) Set the nominal rate  $i_t$  from the Taylor rule:  $1 + i_t = \bar{R} \Pi_t^\phi \varepsilon_t$ .

(d) Advance the distribution  $\mu_{t+1}$  using Young (2010)'s lottery method.

4. **OLS.** Fit two log-linear forecast rules on the simulated data:

$$\log x_{t+1} \approx \alpha_0^x + \alpha_1^x \log z_{t+1} + \alpha_2^x \log \varepsilon_{t+1} + \alpha_3^x \log z_t + \alpha_4^x \log \varepsilon_t + \alpha_5^x \log w_t + \alpha_6^x \log Y_t,$$

for  $x \in \{w, Y\}$ .

5. **Update and check convergence.** Dampen:  $\alpha^{(n+1)} = \alpha^{(n)} + \lambda(\hat{\alpha} - \alpha^{(n)})$ . Stop when  $\|\hat{\alpha} - \alpha^{(n)}\|$  is small.

## 3.2 Results

Table 3 summarizes the model calibration. One period corresponds to one year. Preferences are CRRA with log utility ( $\gamma = 1$ ) and unit inverse Frisch elasticity ( $\varphi = 1$ ). The Taylor rule target  $\bar{R}$  is set slightly below  $1/\beta$  to ensure a stationary wealth distribution while keeping mean inflation close to zero. Table 4 reports the grid sizes and numerical parameters.

The converged forecast rule coefficients are:

$$\begin{aligned} \log w_{t+1} = & -0.034 + 0.782 \log z_{t+1} - 2.272 \log \varepsilon_{t+1} - 0.053 \log z_t + 1.320 \log \varepsilon_t \\ & + 0.945 \log w_t - 0.772 \log Y_t, \quad R_w^2 = 0.9999, \end{aligned}$$

$$\begin{aligned} \log Y_{t+1} = & -0.015 + 0.888 \log z_{t+1} - 1.104 \log \varepsilon_{t+1} - 0.392 \log z_t + 0.584 \log \varepsilon_t \\ & + 0.063 \log w_t + 0.386 \log Y_t, \quad R_Y^2 = 1.0000. \end{aligned}$$

Both forecast rules achieve near-perfect fit, with  $R^2$  values of 0.9999 and 1.0000 for the wage and output rules, respectively. The solver converges in approximately 500 seconds on a 10-core MacBook Pro M5 using Julia with 10 threads.

Figure 5 displays the household and firm policy functions at steady state. Panels (a) and (b) show consumption and labor supply as functions of wealth  $\tilde{a}$  for each idiosyncratic productivity level  $e$ . Panel (c) plots equilibrium inflation against log TFP, comparing the

Table 3: Calibration for the HANK model

Parameter	Description	Value
<i>Preferences</i>		
$\beta$	Discount factor	0.975
$\gamma$	Coefficient of relative risk aversion	1
$\varphi$	Inverse of Frisch elasticity	1
<i>Firms and monetary policy</i>		
$\epsilon$	Elasticity of substitution	10
$\kappa$	Price adjustment cost (Rotemberg)	100
$\phi$	Taylor rule coefficient	1.5
$\bar{R}$	Taylor rule target gross rate	1.023
$B$	Government bond supply	5
<i>Aggregate shocks</i>		
$\rho_z$	TFP persistence	0.9
$\nu_z$	TFP innovation volatility	0.07
$\rho_\epsilon$	Monetary policy persistence	0.9
$\nu_\epsilon$	Monetary policy innovation volatility	0.002
<i>Idiosyncratic income</i>		
$\rho_e$	Income persistence	0.6
$\nu_e$	Income innovation volatility	0.16

simulation average with the inflation policy evaluated at mean wages and output. Panel (d) shows the savings policy  $\tilde{a}'(\tilde{a})$  together with the ergodic wealth distribution (shaded CDF).

Following the same exercise as in Section 2, I compute the stationary wealth distribution when the TFP shock is permanently fixed at  $\pm 2$  standard deviations from its mean, while keeping the monetary policy shock at its mean. Figure 6 shows that the wealth distribution shifts. Under permanently low TFP, households are poorer and more mass concentrates near the borrowing constraint. Under permanently high TFP, the distribution shifts slightly to the right relative to the ergodic distribution.

### 3.3 Impulse Responses

I compute Generalized Impulse Response Functions (GIRFs) following the method of Koop, Pesaran, and Potter (1996).<sup>5</sup> Each panel also displays the one-step-ahead forecast  $\mathbb{E}_{t-1}[\cdot]$

<sup>5</sup>These are created as follows. First, a baseline of uniform "draws" are generated for both exogenous aggregate processes  $(z, \epsilon)$ . With these, starting from an initial state  $(z_0, \epsilon_0)$ , a sequence of shocks is generated using the inverse CDF implied by the transition probability matrix given the previous state. This sequence of shocks is used to simulate the economy and obtain a baseline equilibrium sequence of variables,  $(w_t, Y_t, \Pi_t, i_t)$ . Then, for one of the processes (e.g.,  $z$ ), I change the initial shock state, while keeping the same uniform draws

### Household and Firm Policy Functions in HANK

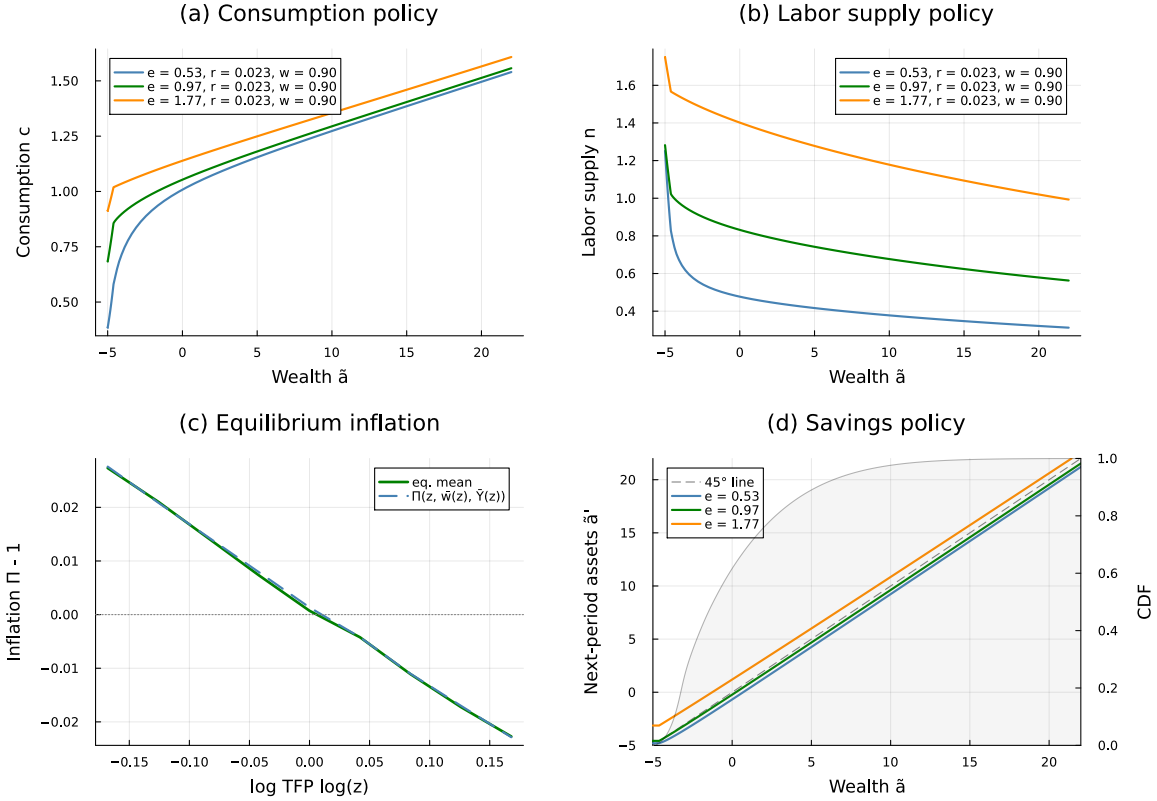


Figure 5: Household and firm policy functions in the HANK model at steady state. (a) Consumption  $c(\tilde{a})$  by idiosyncratic productivity  $e$ . (b) Labor supply  $n(\tilde{a})$  by  $e$ . (c) Equilibrium inflation  $\Pi - 1$  as a function of  $\log \text{TFP}$ : simulation mean (solid) vs. inflation policy evaluated at conditional mean  $(w, Y)$  (dashed). (d) Savings policy  $\tilde{a}'(\tilde{a})$  by  $e$ , with the ergodic wealth CDF (shaded).

Table 4: Grid sizes and numerical parameters

Grid / Parameter	Description	Value
$n_{\bar{a}}$	Asset grid (EGM, quadratic spacing)	100
$n_e$	Idiosyncratic productivity states	3
$n_z$	TFP states	11
$n_\varepsilon$	Monetary policy shock states	11
$n_w$	Wage grid points	10
$n_Y$	Output grid points	10
$n_{\bar{a}}^{\text{hist}}$	Distribution grid (quadratic spacing)	500
$\bar{T}$	Simulation length	2,000
$T$	Burn-in periods	200

implied by the model's forecast rules. At each period  $t$  in the simulation, the conditional expectation of next-period variables is computed by summing over all possible next-period aggregate states  $(z', \varepsilon')$ :

$$\mathbb{E}_t[x_{t+1}] = \sum_{z'} \sum_{\varepsilon'} \pi(z'|z_t) \pi(\varepsilon'|\varepsilon_t) f_x(z', \varepsilon', z_t, \varepsilon_t, w_t, Y_t),$$

where  $f_x$  is the forecast rule for variable  $x \in \{w, Y\}$ , and the forecast for inflation and the real rate follow from the inflation policy and the Fisher equation evaluated at the forecasted  $(w', Y')$ . The GIRF of these forecasts is then computed in the same way as for the actual variables (shocked minus baseline, averaged over paths).

Figures 7 and 8 display the results for TFP and monetary policy shocks, respectively. As can be seen in the figures, the close alignment between the actual GIRF and the forecast GIRF confirms the accuracy of the forecast rules.

For comparison, each panel also shows the analytical impulse responses from a linearized Representative Agent New Keynesian (RANK) model with the same preferences and firm parameters  $(\gamma, \varphi, \kappa, \phi, \beta)$ . Consider, for example, the TFP impulse response in Figure 7. The RANK model is fed a TFP shock of the same size as the one generated in the HANK model in the first period. The future TFP levels are obtained from an AR(1) process with the same persistence and volatility as in the HANK model (pre-discretization). The impulse responses for the equilibrium variables are then computed analytically using the sequence of exogenous shocks. The same procedure is used for the monetary policy shock in Figure 8.

for both processes. This generates a new sequence for this shock starting from this new initial state. This new sequence (together with the other baseline sequence) is used to simulate a new equilibrium sequence,  $(w'_t, Y'_t, \Pi'_t, i'_t)$ . The difference between this new sequence and the baseline sequence gives us one draw of the GIRF. By averaging this difference across many such draws, we obtain the estimated GIRF.

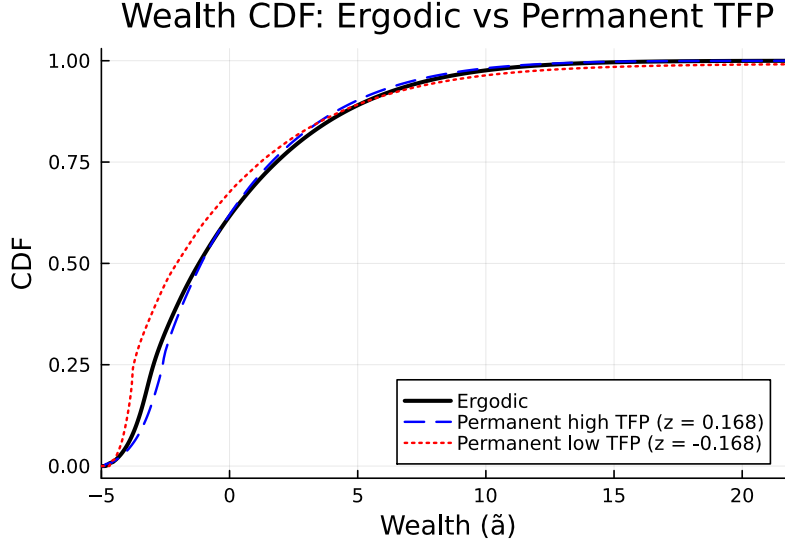


Figure 6: Wealth CDF in the HANK model. Solid black: ergodic distribution under aggregate shocks. Dashed blue and dotted red: stationary distributions when TFP is permanently fixed at  $\pm 2\sigma$  from its mean, with the monetary policy shock held at its unconditional mean.

One interesting feature is the close alignment between the RANK and HANK responses, suggesting that, at least for this calibration, incomplete markets do not lead to quantitatively different responses compared to the RANK benchmark.

### 3.4 Robustness: Inflation Costs and a Low-Debt Calibration

In Appendices B and C I present two robustness exercises. First, I include the inflation costs in the dividend policy, which makes dividends and cash-on-hand dependent on the inflation realization. Second, I simulate a low-debt calibration with  $B = 0.5$  instead of  $B = 5$ . In both cases, the results are very similar to the baseline calibration, although the accuracy of the forecast rules for the low-debt calibration is slightly lower. Appendix D performs additional accuracy tests for the forecast rules.

## 4 Conclusion

In all the three models above, endogenous market clearing objects (interest rates, wages, and/or output) are the key state variables for the forecasting rules and deliver a near-perfect fit. Impulse responses confirm that the forecast rules are conditionally accurate and also reveal no significant differences between the particular HANK calibration used here and the responses generated by a corresponding RANK model.

TFP Shock: HANK vs RANK

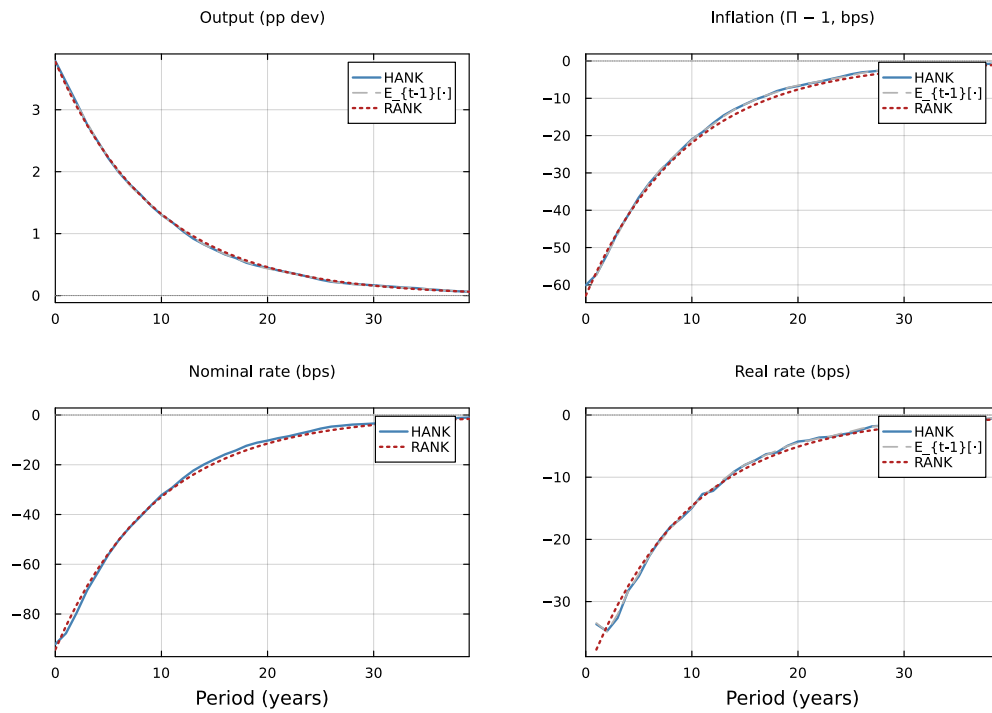


Figure 7: GIRF to a one-standard-deviation TFP shock. Solid blue: HANK model. Dashed gray: one-step-ahead forecast  $\mathbb{E}_{t-1}[\cdot]$ . Dotted red: RANK analytical IRF. The plot for the real rate does not include the impact on the realized return on impact.

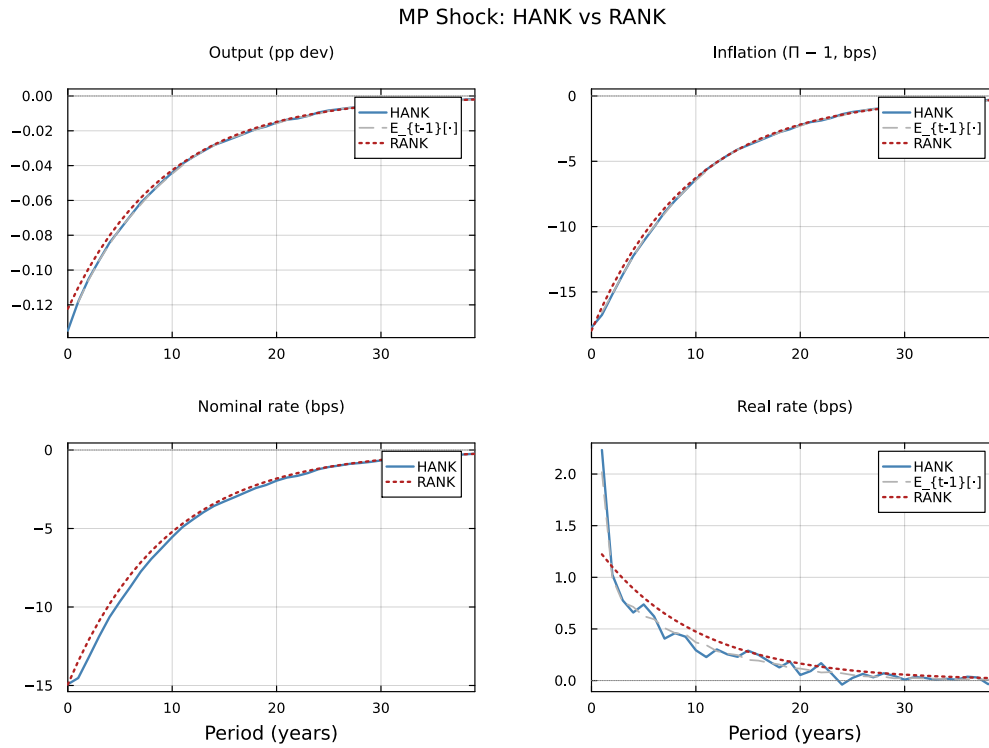


Figure 8: GIRF to a one-standard-deviation monetary policy shock. Solid blue: HANK model. Dashed gray: one-step-ahead forecast  $\mathbb{E}_{t-1}[\cdot]$ . Dotted red: RANK analytical IRF. The plot for the real rate does not include the impact on the realized return on impact.

It is worth noting that in the models above, the wealth distribution is concentrated in regions where policy functions are non-linear. Approximate aggregation therefore does not follow simply from linearity of individual policies. Werning (2015) constructed a real business cycle model where the aggregate behavior is equivalent to that of a representative agent model, despite potentially large heterogeneity in the wealth distribution and non-linear policy functions. This linearity with respect to aggregates, in spite of non-linearities at the individual level, may be the mechanism behind the approximate aggregation I have obtained here.

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## A The RANK Model

The RANK comparison in Section 3 uses the standard three-equation New Keynesian model with the same preference and firm parameters as the HANK model.

The three equations are:

$$\begin{aligned}\hat{y}_t &= \mathbb{E}_t[\hat{y}_{t+1}] - \frac{1}{\gamma} (\hat{i}_t - \mathbb{E}_t[\hat{\pi}_{t+1}]), \\ \hat{\pi}_t &= \beta \mathbb{E}_t[\hat{\pi}_{t+1}] + \tilde{\kappa} \hat{y}_t, \\ \hat{i}_t &= \phi \hat{\pi}_t + \varepsilon_t,\end{aligned}$$

where  $\hat{y}_t$  is the output gap,  $\hat{\pi}_t$  is inflation, and  $\hat{i}_t$  is the nominal interest rate. The natural rate of output responds to TFP shocks by  $\psi_a = (1 + \varphi)/(\gamma + \varphi)$ , so the output gap is given by  $\hat{y}_t = y_t - \psi_a z_t$ . The slope of the NKPC is  $\tilde{\kappa} = \frac{\epsilon - 1}{\kappa}(\gamma + \varphi)$ , where  $\epsilon$  is the elasticity of substitution and  $\kappa$  is the Rotemberg adjustment cost.

TFP and the monetary policy shock follow AR(1) processes:

$$z_t = \rho_z z_{t-1} + \nu_t^z, \quad \varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \nu_t^\varepsilon.$$

Conjecture that  $\hat{y}_t = c_x x_t$ ,  $\hat{\pi}_t = p_x x_t$ , and  $\hat{i}_t = i_x x_t$  for each shock  $x \in \{z, \varepsilon\}$ , where  $x_t$  follows  $x_t = \rho_x x_{t-1} + \nu_t^x$ . Substituting into the three equations and solving:

For a monetary policy shock ( $x = \varepsilon$ ):

$$\begin{aligned} D_\varepsilon &= (1 - \rho_\varepsilon)(1 - \beta\rho_\varepsilon) + \frac{\tilde{\kappa}}{\gamma}(\phi - \rho_\varepsilon), \\ c_\varepsilon &= -(1 - \beta\rho_\varepsilon)/(\gamma D_\varepsilon), \\ p_\varepsilon &= -\tilde{\kappa}/(\gamma D_\varepsilon), \\ i_\varepsilon &= \phi p_\varepsilon + 1. \end{aligned}$$

For a TFP shock ( $x = z$ ), the natural rate of output responds by  $\psi_a = (1 + \varphi)/(\gamma + \varphi)$ :

$$\begin{aligned} D_z &= (1 - \rho_z)(1 - \beta\rho_z) + \frac{\tilde{\kappa}}{\gamma}(\phi - \rho_z), \\ c_z &= \tilde{\kappa} \psi_a (\phi - \rho_z)/(\gamma D_z), \\ p_z &= -\tilde{\kappa} \psi_a (1 - \rho_z)/D_z, \\ i_z &= \phi p_z. \end{aligned}$$

The impulse responses are then  $\hat{y}_t = c_x \rho_x^t x_0$ ,  $\hat{\pi}_t = p_x \rho_x^t x_0$ , and  $\hat{i}_t = i_x \rho_x^t x_0$ , where  $x_0$  is the initial shock size, set equal to the period-1 GIRF shock in the HANK model so that the two are directly comparable.

The realized real rate follows from the Fisher equation,  $\hat{r}_t = \hat{i}_{t-1} - \hat{\pi}_t$ , with the convention that the nominal rate is predetermined (set one period in advance). At  $t = 1$ , the economy starts at steady state so  $\hat{i}_0 = 0$  and the initial real rate response is purely driven by surprise inflation:

$$\hat{r}_1 = -\hat{\pi}_1, \quad \hat{r}_t = \hat{i}_{t-1} - \hat{\pi}_t \quad \text{for } t \geq 2.$$

## B Inflation Costs in the Dividend Policy

In the baseline HANK model, the Rotemberg price adjustment costs do not reduce real profits. This appendix considers the alternative where they do. The dividend policy becomes

$$d(s, w, Y) = \left[ 1 - \frac{w}{z(s)} - \frac{\kappa}{2} (\Pi(s, w, Y) - 1)^2 \right] Y,$$

instead of  $d = (1 - w/z)Y$ . All other parameters are identical to the baseline (Table 3).

The converged forecast rule coefficients are:

$$\begin{aligned} \log w_{t+1} &= -0.027 + 0.780 \log z_{t+1} - 2.061 \log \varepsilon_{t+1} - 0.567 \log z_t + 1.532 \log \varepsilon_t \\ &\quad + 0.757 \log w_t - 0.021 \log Y_t, \quad R_w^2 = 0.9993, \\ \log Y_{t+1} &= -0.017 + 0.891 \log z_{t+1} - 1.005 \log \varepsilon_{t+1} - 0.613 \log z_t + 0.553 \log \varepsilon_t \\ &\quad - 0.098 \log w_t + 0.778 \log Y_t, \quad R_Y^2 = 0.9975. \end{aligned}$$

Figure 9 shows the policy functions and Figures 10–11 display the GIRFs. The results are very similar to the baseline, confirming that the inclusion of inflation adjustment costs in dividends has a negligible quantitative effect in this calibration.

## C Low-Debt Calibration

This appendix considers a calibration with substantially lower government bond supply,  $B = 0.5$  instead of  $B = 5$ . This tightens the effective borrowing constraint (since  $\tilde{a} \geq -B$ ) and reduces the liquidity available to households.

Because the lower bond supply changes the steady-state real rate, the Taylor rule target  $\bar{R}$  is adjusted accordingly, as shown in Table 5. All other parameters are identical to the baseline (Table 3).

Table 5: Parameters that differ from the baseline HANK model.

Parameter	Description	Baseline	Low-debt
$B$	Government bond supply	5.0	0.5
$\bar{R}$	Taylor rule target gross rate	1.023	1.002

The converged forecast rule coefficients are:<sup>6</sup>

<sup>6</sup>In this specification, the OLS forecast rules are estimated with a small ridge penalty ( $10^{-4}$ ) following Maliar and Maliar (2014) because of the multicollinearity among  $\log z_t$ ,  $\log w_t$ , and  $\log Y_t$ .

### Household and Firm Policy Functions in HANK

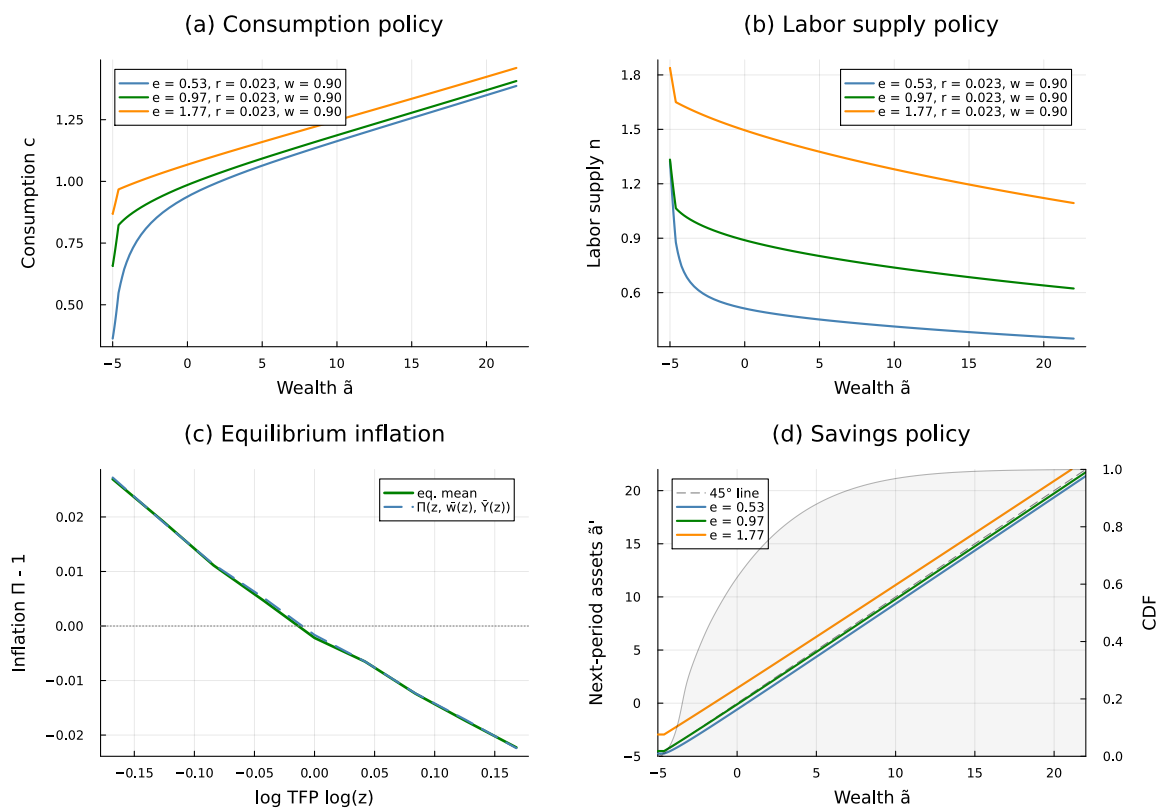


Figure 9: Policy functions for the HANK model with Rotemberg costs in dividends.

TFP Shock: HANK vs RANK

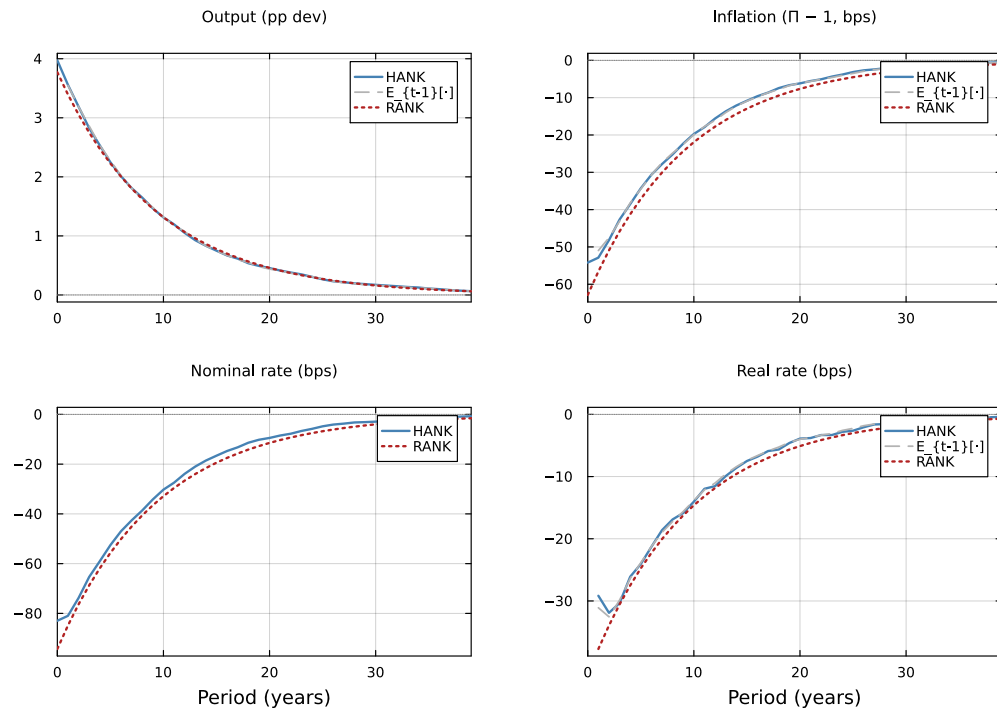


Figure 10: GIRF to a TFP shock (inflation costs in dividends). Solid blue: HANK. Dashed gray:  $\mathbb{E}_{t-1}[\cdot]$ . Dotted red: RANK.

MP Shock: HANK vs RANK

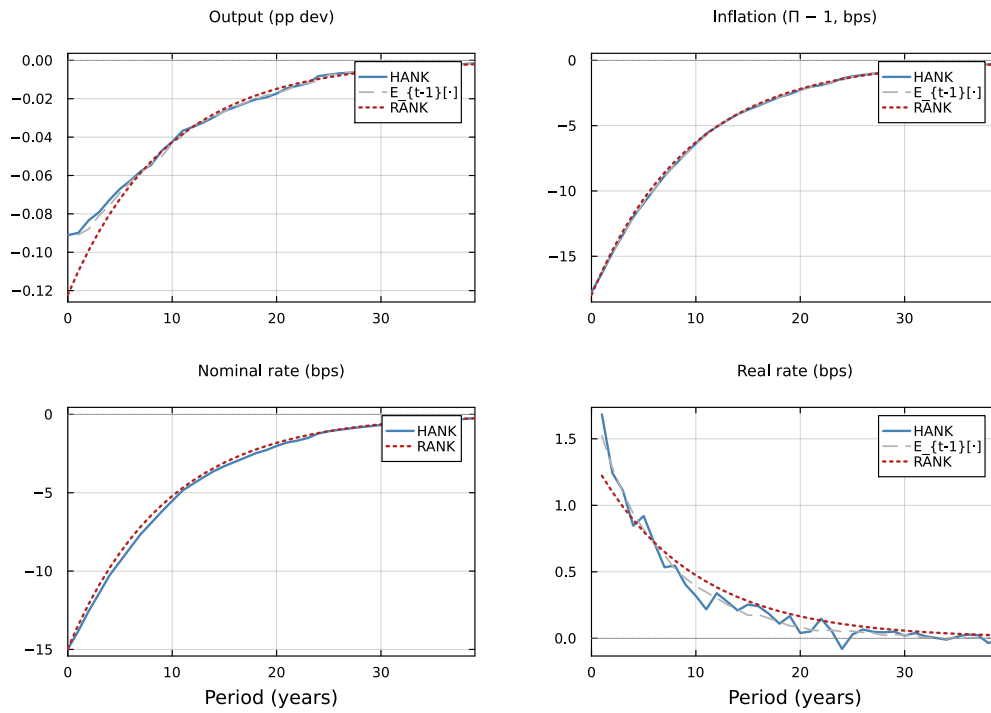


Figure 11: GIRF to a monetary policy shock (inflation costs in dividends). Solid blue: HANK. Dashed gray:  $\mathbb{E}_{t-1}[\cdot]$ . Dotted red: RANK.

$$\begin{aligned} \log w_{t+1} &= -0.098 + 0.737 \log z_{t+1} - 2.217 \log \varepsilon_{t+1} + 0.265 \log z_t + 0.299 \log \varepsilon_t \\ &\quad + 0.319 \log w_t - 0.535 \log Y_t, \quad R_w^2 = 0.9857, \\ \log Y_{t+1} &= -0.043 + 0.868 \log z_{t+1} - 1.094 \log \varepsilon_{t+1} + 0.112 \log z_t + 0.146 \log \varepsilon_t \\ &\quad + 0.137 \log w_t - 0.228 \log Y_t, \quad R_Y^2 = 0.9973. \end{aligned}$$

The  $R^2$ s are lower than in the baseline, especially so for the wage forecast rule. Figure 12 shows the policy functions and Figures 13–14 display the GIRFs. One significant difference is that the wealth distribution is significantly more compressed in the low-debt calibration as expected (see the shaded CDF in panel (d) of Figure 12).

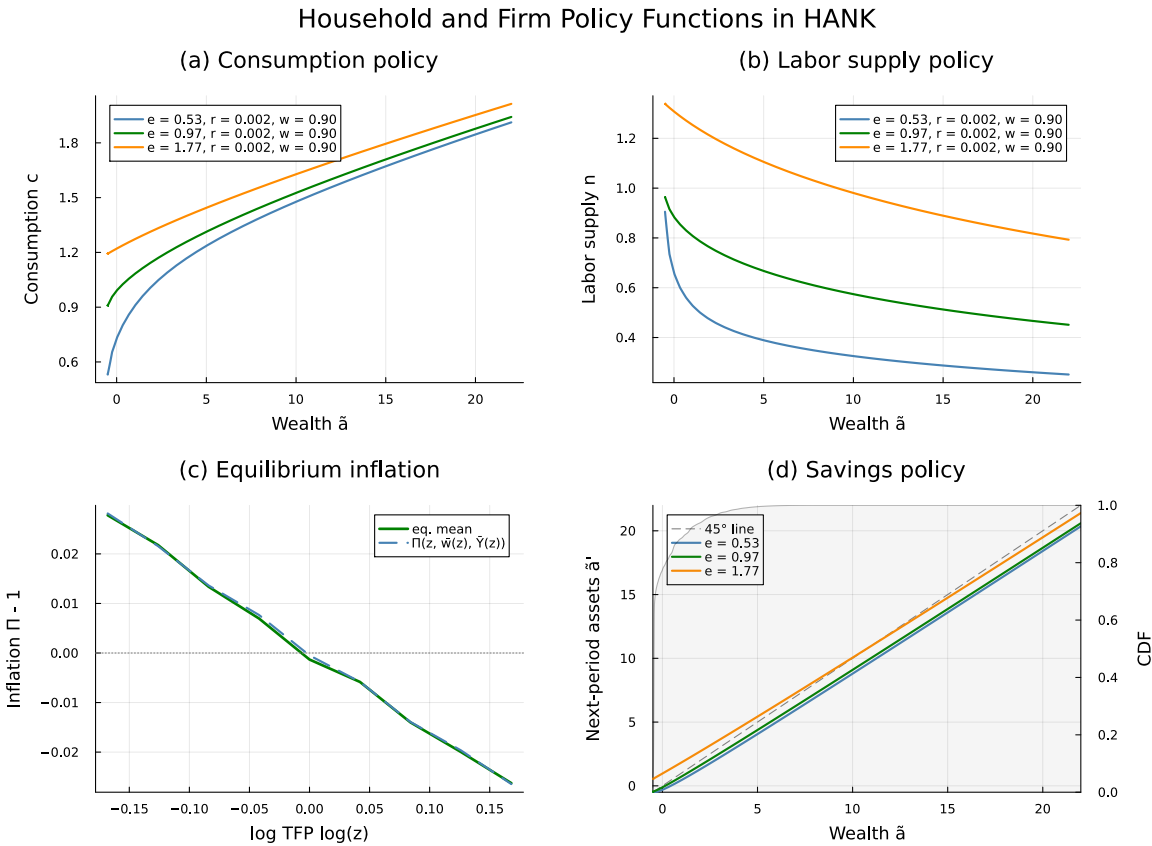


Figure 12: Policy functions for the HANK model with low bond supply ( $B = 0.5$ ).

The results on output and inflation are similar to the baseline, although there is a larger difference between the HANK and RANK response to the TFP shock early on. The differences are still quantitatively modest in this particular calibration.

TFP Shock: HANK vs RANK

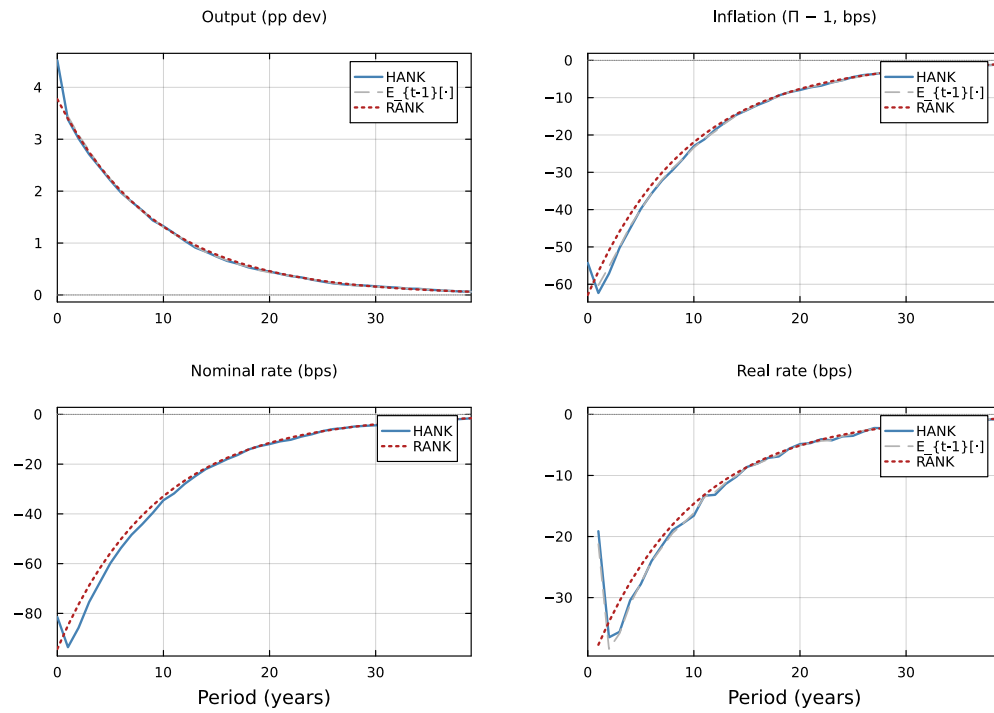


Figure 13: GIRF to a TFP shock (low-debt calibration). Solid blue: HANK. Dashed gray:  $\mathbb{E}_{t-1}[\cdot]$ . Dotted red: RANK.

MP Shock: HANK vs RANK

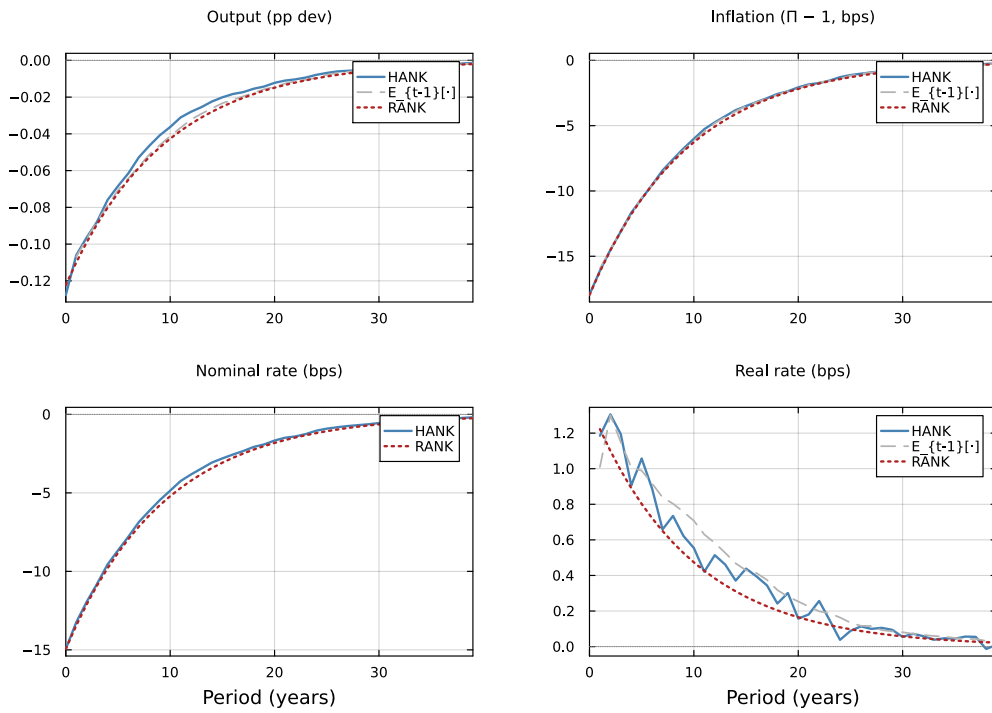


Figure 14: GIRF to a monetary policy shock (low-debt calibration). Solid blue: HANK. Dashed gray:  $\mathbb{E}_{t-1}[\cdot]$ . Dotted red: RANK.

## D Forecast Rule Accuracy

This appendix reports the Den Haan (2010b) accuracy diagnostic for all models. The test compares two paths of each forecasted aggregate variable: (i) the market-clearing series from the simulation, and (ii) a series generated by iterating the forecast rule forward from the same initial condition. The table reports

$$\text{err}_t = \frac{|\hat{x}_t^{\text{fc}} - x_t|}{\bar{x}},$$

where  $\hat{x}_t^{\text{fc}}$  is the forecast-only path and  $\bar{x}$  is the sample mean of  $x_t$  over post-burn-in periods. For the Huggett models the forecasted variable is the gross interest rate  $R_t$ ; for the HANK models the forecasted variables are the real wage  $w_t$  and aggregate output  $Y_t$ .

Table 6: Den Haan accuracy statistics for all models, computed along a single simulated shock path of 1,800 post-burn-in periods.

Model	Variable	Mean err. (%)	Max err. (%)	$R^2$
Huggett baseline	$R$	0.005	0.112	0.9997
Imrohoroglu	$R$	0.039	0.149	0.9973
HANK baseline	$w$	0.046	0.973	0.9993
	$Y$	0.022	0.468	0.9975
HANK infl. costs	$w$	0.140	1.579	0.9993
	$Y$	0.339	3.054	0.9975
HANK low debt	$w$	0.621	1.694	0.9857
	$Y$	0.308	0.860	0.9973

The Huggett models achieve the highest accuracy. The HANK models show larger errors.