Reputation and Partial Default[†]

By MANUEL AMADOR AND CHRISTOPHER PHELAN*

This paper presents a continuous-time reputation model of sovereign debt allowing for both varying levels of partial default and full default. In it, a government can be a nonstrategic commitment type or a strategic opportunistic type, and a government's reputation is its equilibrium Bayesian posterior of being the commitment type. Our equilibrium has that for bond levels reachable by both types without defaulting, bigger partial defaults (or bigger haircuts for bond holders) imply higher interest rates for subsequent bond issuances, as in the data. (JEL D83, E32, E43, G12, H63)

Countries which when partially defaulting impose bigger haircuts on their bond holders face bigger market consequences. In particular, the interest rates they face for future bond issuances are higher.¹ But it is far from clear that this should be the case. After all, the bigger the haircut, the better debt position the defaulting country is in.

Here, we propose a possible explanation: bigger adverse consequences for bigger haircuts act as an *endogenous* equilibrium incentive to make sovereign governments willing to mix between differing haircut levels.

In our model, governments can have possible private "types" (in particular a strategic "opportunistic" type and a nonstrategic "commitment" type). While we assume that the country, regardless of which type of government is in power, is sometimes forced to partially default at varying haircut levels, the opportunistic type can *voluntarily* default at any level at any time, while the commitment type can never voluntarily default. Our equilibrium has the property that for bond levels reachable by the opportunistic type without defaulting, bond prices are higher the higher the equilibrium probability a country's government is the commitment type. Further, our equilibrium involves the opportunistic type mixing—for any positive length of time, it chooses a positive finite probability of defaulting at every possible haircut level.

For such mixing to be a best response for an opportunistic type, the equilibrium mixing probabilities must imply that when a country imposes a bigger haircut on its lenders, its reputation (the Bayesian posterior that it is the commitment type) falls

^{*}Amador: University of Minnesota, Federal Reserve Bank of Minneapolis, and NBER (email: amador@umn. edu); Phelan: University of Minnesota, Federal Reserve Bank of Minneapolis, and NBER (email: cphelan@umn. edu). Peter Klenow was coeditor for this article. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

 $^{^{\}dagger}$ Go to https://doi.org/10.1257/aeri.20210739 to visit the article page for additional materials and author disclosure statement(s).

¹Cruces and Trebesch (2013) were the first to point out this fact after compiling a comprehensive dataset on haircuts and bond prices for sovereign defaults. See also the subsequent work of Asonuma (2016).

further than if it had imposed a smaller haircut. The intuition is that the *benefit* of a larger haircut is, of course, the wiping out of more debt. But for the opportunistic type to then ever choose a smaller haircut, there has to be a corresponding, and equal, *cost* to imposing a larger haircut. The cost imposed by equilibrium is then this greater loss in reputation and the corresponding greater increase in future interest rates.

Our model is similar to and based on the continuous time model of Amador and Phelan (2021)—hereafter, AP. In that model, however, countries can only fully default (i.e., fully repudiate the debt ensuring no payments would ever be made to current bond holders). Here, we also allow countries to partially default.

There are several papers that have modeled reputational considerations in sovereign debt markets. In their analysis of sovereign debt sustainability, Cole and Kehoe (1995, 1998) introduced two government types similar to ours: an "honest" (commitment) type and a "normal" (opportunistic) type. Cole, Dow, and English (1995); Alfaro and Kanczuk (2005); D'Erasmo (2011); Egorov and Fabinger (2016); and Dovis (2019) also featured alternating government types in a sovereign debt context.² In a related contribution, Chatterjee, Corbae, and Ríos-Rull (2008) developed a finite horizon model of unsecured consumer debt that features private information about the consumer's impatience rate (or the distribution of shocks). They are motivated by similar facts that arise in consumer finance: default by consumers lowers credit scores, and consumers with better scores borrow at lower rates. Chatterjee et al. (2020) explore the quantitative implications of these concerns for consumer unsecured debt in a richer model.

The initial quantitative models of sovereign debt effectively assumed that there are no partial defaults (all defaults are full), and the sovereign can subsequently reenter financial markets with a clean slate.³ This assumption has been relaxed by subsequent research that has modeled the process of renegotiation between lenders and the sovereign.⁴ Aguiar et al. (2019); Dvorkin et al. (2021); and Mihalache (2020) study the role the maturity structure plays in restructurings. The latter two provide a quantitative analysis but do not focus on the behavior of spreads after default and reentering financial markets. In her study of cyclical renegotiation outcomes, Sunder-Plassmann (2018) is able to generate spreads that are higher after higher haircuts in an Eaton and Gersovitz (1981) style model by making the bargaining power of the government during a renegotiation countercyclical. This can lead to higher haircuts in defaults that occur during recessions (and those lower output states are associated with lower default costs in the future—potentially increasing spreads). Arellano, Mateos-Planas, and Ríos-Rull (2019) provide an alternative model where partial defaults lead to further defaults and increases in debt (because of arrears). Recent work by Fourakis

⁴See, for instance, Yue (2010), who introduced bargaining over restructuring terms in the Eaton-Gersovitz framework. For an earlier model on renegotiation, see Bulow and Rogoff (1989). See also Salomao (2017) and Benjamin and Wright (2008).

²See AP for a discussion of these papers.

³See, for example, Arellano (2008) and Aguiar and Gopinath (2006), which are based on the incomplete markets framework of Eaton and Gersovitz (1981). After reentering financial markets following a default, there are two opposing forces at play in these models: first, default costs are low and expected to remain low in the future, raising the probability of future defaults. But debt is also low (as the country has fully defaulted), which lowers that probability. This second force dominates and generates counterfactually low spreads after a country reenters financial markets following a default.

(2021) shares our interest on the behavior of spreads after default. His paper performs a quantitative analysis of reputation in a sovereign debt model where creditors learn from all government actions and not just from default decisions (as we do here). In his baseline calibration, by the time default occurs, the government has already revealed its type by accumulating large amounts of debt.

Unlike most of the sovereign default literature, we impose no exogenous costs on governments to defaulting. While empirically GDP indeed drops after default events, it is difficult to attribute this *directly* to a government breaking a promise: no real resources are destroyed when a debt payment isn't made. Instead, such GDP drops may be equilibrium reactions by economic agents—the very process we are attempting to understand in this study. While by assumption GDP in our model is constant, Cole and Kehoe (1998) show how such GDP drops can occur in a reputation model with similarities to ours.

I. The Environment

The continuous and infinite time environment/game we consider is similar to and based on the environment of AP. As in AP, here we consider a small open economy endowed with a constant flow y of a consumption good whose government can borrow from risk-neutral price-taking outside lenders who discount the future at rate i > 0. The terms of such borrowing are that the country can issue long-term bonds at every date s which pay a coupon at date t of $(i + \lambda)e^{-\lambda(t-s)}$. This coupon schedule ensures that the price of a bond is one at date s if default cannot occur. Assuming exponentially decaying coupon payments is equivalent to the government paying an instantaneous coupon of $i + \lambda$ per unit of current debt, b(t), with such debt decaying at rate λ . We assume that the initial level of debt is zero and there exists an exogenous maximum level of debt, $\overline{B} < y/(i + \lambda)$ (which ensures that paying the required coupon is always feasible).

A. Players

As in AP, there is a countable list of potential *governments* where at any date $t \ge 0$, only one of the potential governments is active. We assume that the first government on the list is an *opportunistic type*, the second a *commitment type*, with the list then alternating between types. At all times, with Poisson arrival rate $\epsilon > 0$, an opportunistic type government is replaced by the next government on the list (a commitment type). With arrival rate $\delta > 0$, a commitment type government is replaced by an opportunistic type. Such switches are private.

B. Strategies

We assume that the commitment type is nonstrategic and continuously makes coupon payments $(i + \lambda)$ per unit of debt. Unlike AP, here we assume that both types are sometimes forced to *partially default*. To this end, let $\eta = {\eta_1, \ldots, \eta_N}$ denote an increasing grid of fractions representing the severity of a partial default, where a larger value of *n* represents a smaller haircut, or a larger level of remaining debt after the partial default. Let $\theta_n > 0$ denote the Poisson arrival rate of a shock that forces a government with debt b(t) to partially default and reset its debt to $\eta_n b(t)$. In particular, partial default resets the promised stream of payments of each existing bond proportionally to η_n of its previous value. Assume that these θ shocks are not publicly observed and that a commitment type never fully defaults.

While the commitment type can only partially default, and does so only when forced, an opportunistic type can voluntarily both fully and partially default. If a full default occurs, current bond holders get no additional payments and the stock of outstanding debt is set to zero. If an opportunistic type voluntarily partially defaults, coupon payments are adjusted exactly as in the case of a forced default at that level. Since both types can partially default, and whether a partial default is forced or voluntary is not publicly observed, a partial default at any level does not mechanistically reveal the government's type.

As in AP, we assume that strategies are *Markov*. The payoff relevant state variables here are the level of debt b(t) and the government's reputation $\rho(t)$. In AP, where only full default or no default was possible at any given moment, these two state variables could be reduced, without loss, to a single state variable that implied them both, time since last default, denoted τ . Here, this is no longer true. With partial default, time since last full default no longer mechanistically implies debt and reputation. Nevertheless, here we again look for strategies as a function of τ , but consider τ to be a more abstract object, best thought of as the time on a stopwatch that can be reset back either to zero (in the case of full default) or to *endogenous* earlier time points depending on the level of partial default, η . In particular, let $b(\tau)$ be the level of debt if there are no defaults for τ periods starting from no debt. We search for equilibria where upon a partial default from $b(\tau)$ to debt level $\eta b(\tau)$, the stopwatch is reset to the endogenous amount of time it takes a country to go from zero debt to debt level $\eta b(\tau)$ conditional on not defaulting at any level during that time, denoted $\tau^*(\eta b(\tau))$ (i.e., $\tau^*(b)$ is the inverse function of the equilibrium $b(\tau)$).

For the commitment type, we assume that as long as it is in control, it follows a prespecified expenditure rule determined by the expectations of international financial markets of how a commitment type should act. That is, as long as the commitment type is in control, the stock of debt evolves according to

(1)
$$b'(\tau) = H(b(\tau), q(\tau))$$

for some exogenous function H, where $q(\tau)$ represents the price of a bond when the stopwatch displays τ (from now on referred to as "period τ ") after the realization of the period τ default event. The main purpose of this assumption is that games where informed players (in our case, a government that knows its type) have rich action spaces are notoriously difficult to characterize. Thus, assuming the commitment type follows an exogenous debt rule eliminates the choice of the level of debt as a signaling device and allows us to focus solely on the implications of needing the opportunistic type to be willing to follow the equilibrium default strategy.⁵ In addition, the debt sustainability literature uses widely similar fiscal rules.⁶

⁵For an alternative, see Fourakis (2021).

⁶For recent examples, see Lorenzoni and Werning (2019); Gourinchas, Philippon, and Vayanos (2017); and Martin and Philippon (2017).

It follows from the sequential budget constraint that $c(\tau) = y - (i + \lambda)b(\tau) + q(\tau)[b'(\tau) + \lambda b(\tau)]$, and thus consumption for the commitment type is determined by $c(\tau) = C(b(\tau), q(\tau))$, where the function *C* is given by

$$C(b,q) \equiv y - (i + \lambda)b + q(H(b,q) + \lambda b).$$

We impose the following further conditions on H(b,q) (and thus, implicitly, C(b,q)):

ASSUMPTION 1: Let $\mathbb{X} \equiv [0, \overline{B}] \times [0, 1]$. The function $H: \mathbb{X} \to \mathbb{R}$ satisfies the following:

- (*i*) *H* is Lipschitz continuous.
- (*ii*) *H* is weakly decreasing in b.
- *(iii) H* is weakly increasing in *q*.
- (iv) There exists $\underline{q} \in \left(0, \frac{i+\lambda}{i+\lambda+\delta+\epsilon}\right)$ such that H(0,q) = 0 for all $q \in [0,\underline{q}]$, and H(0,q) > 0 for all $q \in (q,1]$.
- $(v) \ H(\bar{B},1) \leq 0.$
- (vi) *H* is differentiable in the set of $(b,q) \in \mathbb{X}$ such that H(b,q) > 0.

Restrictions (ii) and (iii) guarantee that the commitment type increases its debt by more the higher the price of its bonds and the lower the inherited debt stock.⁷

For the *opportunistic* type, in addition to the Markov restriction, we impose a restriction that it always chooses a level of borrowing (and thus consumption) that is identical to that which would have been chosen by a commitment government facing the same debt and price. (In AP, we show that this restriction is without loss.) With this restriction, the only decision left under the control of the opportunistic government is whether and how much to default. Let *default level* n = 0 denote full default and default level $n \in \{1, \ldots, N\}$ denote the partial default, resetting debt *b* to $\eta_n b$ by adjusting future coupon payments proportionally.

We assume that a strategy for an opportunistic government consists of two vectors, $\alpha(\tau)$ and $\gamma(\tau)$, describing the government behavior before and after a date *T*. Here, *T* is the (equilibrium) amount of time necessary for a government starting with $\rho = 0$ to achieve, by not defaulting, its maximum reputation, $\rho = 1$. The first vector, $\alpha(\tau) = \{\alpha_0(\tau), \ldots, \alpha_N(\tau)\}$, denotes for all $\tau < T$, the Poisson arrival rate of the opportunistic government *voluntarily* defaulting at each level *n*. The second vector, $\gamma(\tau) = \{\gamma_0(\tau), \ldots, \gamma_N(\tau)\}$, denotes for all $\tau \ge T$, the *probability* of the opportunistic government defaulting at each level *n* at precisely period τ . We further restrict attention to strategies where $\sum_{n=0}^{N} \gamma_n(\tau) = 1$ for all $\tau \ge T$, or that an

⁷See AP for justification of the other restrictions.

opportunistic type certainly and immediately defaults at some level for all $\tau \geq T$. Such a strategy restriction is definitely *not* without loss. We nevertheless attempt to construct equilibria in this class since due to similarities between this model and AP, we believe (but do not prove) that all Markov equilibria will have these features.⁸ We note that even though we restrict attention to strategies in this class, in an equilibrium, the opportunistic government must not want to deviate when it is allowed to use any default strategy (and not just those in this class).

C. Payoffs

If the government does *not* default at period τ , it issues additional bonds $H(b(\tau), q(\tau)) + \lambda b(\tau)$ at endogenous price $q(\tau)$ and its consumption is $C(b(\tau), q(\tau))$. If the government fully defaults (i.e., defaults at level n = 0), then τ is reset to zero with debt b(0) = 0. If the government partially defaults at level $n \ge 1$, debt is reset from $b(\tau)$ to $\eta_n b(\tau)$, and τ is reset to $\tau^*(\eta_n b(\tau))$.

There are no direct costs of choosing to fully or partially default and no restrictions on government borrowing from then on. In particular, in the case of full default, the government immediately issues new additional bonds H(0,q(0)) at endogenous price q(0) and its consumption is C(0,q(0)). In the case of partial default, the government immediately issues new additional bonds $H(\eta_n b(\tau), q(\tau^*(\eta_n b(\tau)))) + \lambda \eta_n b(\tau)$ at endogenous price $q(\tau^*(\eta_n b(\tau)))$ and its consumption is $C(\eta_n b(\tau), q(\tau^*(\eta_n b(\tau))))$.

The opportunistic type receives a flow payoff equal to $u(c(\tau))$ as long as it is continuously in power and discounts future payoffs at rate r > 0. We assume that $u: \mathbb{R}_+ \to [\underline{u}, \overline{u}]$ for some finite values \underline{u} and \overline{u} and that u is strictly increasing. We make no other assumptions on the preferences of the opportunistic type. (As in AP, a preview of our results is that our constructed Markov equilibrium is essentially *independent* of u and r. Other than more is preferred to less, and now is preferred to later, the preferences of the opportunistic type will not matter at all.)⁹

D. Beliefs

Recall that $\rho(\tau)$ represents the international market's beliefs, or Bayesian posterior, that the government is the commitment type when the stopwatch is at τ . If the government *fully* defaults at any period $\tau \ge 0$, Bayesian updating implies that ρ immediately jumps to zero since only the opportunistic type can fully default.

⁸ For instance, there can't be a Markov equilibrium where the opportunistic type does not eventually default with probability one since indifference requires consumption to be constant and greater than the endowment, while outside lenders breaking even requires consumption to eventually be lower than the endowment. But once consumption starts falling, optimization by the opportunistic type implies immediate default at some level.

⁹ Similarly to Cole, Dow, and English (1995), the opportunistic government only values consumption while in power. We made this assumption for simplicity: it allows us to narrow attention to equilibria where consumption is constant when the opportunistic government is in power. If the opportunistic government enjoys utility from consumption after it is out of power, we need to keep its utility while in power constant instead. Such a model is significantly more complex. Nevertheless, for this case we can show that constant utility implies that $c(\tau)$ is increasing with τ for $\tau < T$. This in turn implies that our main result—that larger haircuts imply larger increases in risk premia—should still hold. For a version of this environment where governments care about outcomes after they are out of power, without private information, see Aguiar and Amador (2011).

AER: INSIGHTS

Next, consider $\tau < T$. If the government *partially* defaults at level *n* at τ , Bayesian updating depends on the opportunistic type's strategy. Specifically, if a partial default of level *n* occurs, Bayesian updating implies that $\rho(\tau)$ jumps to

$$\frac{\rho(\tau)\theta_n}{\rho(\tau)\theta_n + [1 - \rho(\tau)] [\theta_n + \alpha_n(\tau)]}.$$

This is the arrival rate of an *n*-level default by the commitment type divided by the arrival rate of an *n*-level default by either type.

If the government doesn't default at any level at date $\tau < T$, Bayesian updating again depends on the opportunistic type's strategy. Bayesian updating in this case implies that¹⁰

(2)
$$\rho'(\tau) = \underbrace{\left[1 - \rho(\tau)\right]\epsilon - \rho(\tau)\delta}_{\text{drift toward long-run reputation}} + \underbrace{\rho(\tau)\left[1 - \rho(\tau)\right]\left[\alpha_0(\tau) + \sum_{n=1}^N \alpha_n(\tau)\right]}_{\text{reputation gain from no default}}$$

Note the higher the arrival of rate of complete or partial default, $\alpha_0(\tau) + \sum_{n=1}^N \alpha_n(\tau)$, the greater reputation gain from not defaulting.

For $\tau \ge T$, $\rho(\tau) = 1$, since by assumption an opportunistic government immediately defaults at some level for all $\tau \ge T$. Further, Bayesian updating requires ρ to remain at one if no default occurs. As when $\tau < T$, if a government fully defaults, Bayesian updating requires ρ to jump to zero. If a government partially defaults at level *n*, Bayesian updating requires ρ to jump to

$$\frac{\theta_n}{\theta_n + \delta \gamma_n(\tau)}.$$

This is again the arrival rate of an *n*-level default by the commitment type divided by the arrival rate of an *n*-level default by either type, where the arrival rate of an *n*-level default by an opportunistic type is the arrival rate of a type switch, δ , multiplied by the probability of an immediate *n*-level default, $\gamma_n(\tau)$.

¹⁰This formula is the derivative of the following with respect to Δ evaluated at $\Delta = 0$:

$$\rho(\tau + \Delta) = (1 - \delta\Delta) \frac{\rho(\tau)(1 - \sum_{n=1}^{N} \theta_n \Delta)}{\rho(\tau)(1 - \sum_{n=1}^{N} \theta_n \Delta) + [1 - \rho(\tau)]\{1 - \sum_{n=0}^{N} [\theta_n + \alpha_n(\tau)]\Delta\}} + \epsilon\Delta \left\{ 1 - \frac{\rho(\tau)(1 - \sum_{n=1}^{N} \theta_n \Delta)}{\rho(\tau)(1 - \sum_{n=1}^{N} \theta_n \Delta) + [1 - \rho(\tau)]\{1 - \sum_{n=0}^{N} [\theta_n + \alpha_n(\tau)]\Delta\}} \right\}$$

E. Prices

Our construction of a Markov equilibrium will take a candidate initial price q(0) as given. Prices for all periods $0 < \tau \leq T$ are then determined by the delay differential equation¹¹

$$(3) \underbrace{(i+\lambda)}_{\text{discount rate}} q(\tau) = \underbrace{(i+\lambda)}_{\text{coupon}} + \underbrace{q'(\tau)}_{\text{capital gain}} - \underbrace{[1-\rho(\tau)]\alpha_0(\tau)}_{\text{arrival rate of full default}} \times \underbrace{q(\tau)}_{\text{capital loss if full default}} - \sum_{n=1}^{N} \underbrace{\{\rho(\tau)\theta_n + [1-\rho(\tau)][\theta_n + \alpha_n(\tau)]\}}_{\text{arrival rate of partial default}} \underbrace{[q(\tau) - q(\tau^*(\eta_n b(\tau)))\eta_n]}_{\text{capital loss if partial default}}.$$

For $\tau \ge T$,

(4)
$$\underbrace{(i+\lambda)}_{\text{discount rate}} q(\tau) = \underbrace{(i+\lambda)}_{\text{coupon}} + \underbrace{q'(\tau)}_{\text{capital gain}} - \underbrace{\delta\gamma_0(\tau)}_{\text{arrival rate of full default}} \times \underbrace{q(\tau)}_{\text{capital loss if full default}} - \sum_{n=1}^{N} \underbrace{\left[\theta_n + \delta\gamma_n(\tau)\right]}_{\text{arrival rate}} \underbrace{\left[q(\tau) - q\left(\tau^*(\eta_n b(\tau))\right)\eta_n\right]}_{\text{capital loss if partial default}}.$$

II. Markov Equilibria

We consider a collection $(b(\tau), q(\tau), \rho(\tau), T, \{\alpha_n(\tau)\}_{n=0}^N, \{\gamma_n(\tau)\}_{n=0}^N)$ to be a Markov equilibrium if:

- (Foreign investors break even in equilibrium.) For all τ , $q(\tau)$ is the expected discounted value of the stream of coupon payments for a bond issued at period τ .
- (Market beliefs are rational.) $\rho: \mathbb{R}^+ \to [0,1]$; satisfies Bayes rule.
- (Debt evolution and budget constraint.) The level of debt, as a function of τ, evolves according to the prespecified expenditure rule H.
- (Opportunistic type optimizes.) For all τ , no other default strategy improves the continuation expected lifetime payoff of the opportunistic type.

¹¹This formula is the limit as $\Delta \to 0$ of $\frac{q(\tau + \Delta) - q(\tau)}{\Delta}$, where $q(\tau) = (i + \lambda)\Delta + e^{-(i+\lambda)\Delta}q(\tau + \Delta) \left\{ \rho(\tau) \left(1 - \sum_{n=1}^{N} \theta_n \Delta\right) + \left[1 - \rho(\tau)\right] \left[1 - \left(\alpha_0 + \sum_{n=1}^{N} (\theta_n + \alpha_n(\tau))\right)\Delta\right] \right\} + e^{-(i+\lambda)\Delta} \sum_{n=1}^{N} q(\tau^*(\eta_n b(\tau)))\eta_n \left\{ \rho(\tau)\theta_n\Delta + [1 - \rho(\tau)] [\theta_n + \alpha_n(\tau)]\Delta \right\}.$

III. Constructing a Markov Equilibrium

In this section, we construct a Markov equilibrium.

The main idea for the equilibrium construction, as in AP, is to first ensure that consumption for the opportunistic type is always at a constant $c^* > y$ for all periods $\tau < T$ and that consumption of the opportunistic type is weakly less than c^* for all $\tau \ge T$. The construction next ensures that reputation ρ is reset after a partial default so that after a default of level *n* in period τ , ρ jumps to $\rho(\tau^*(\eta_n b(\tau)))$. Such a construction ensures that an opportunistic government is indifferent between defaulting at any level or not defaulting for $\tau < T$ and is indifferent between default levels (and is willing to certainly and immediately default at some level) for $\tau \ge T$.

To this end, let Q(b,c) denote the price that causes a commitment type with debt *b* to set its consumption to *c*. That is, Q(b,c) is such that C(b,Q(b,c)) = c. Assumption 1 guarantees that Q(b,c) is strictly increasing in *b* and *c*, reflecting the fact that to maintain a level of consumption higher than *y*, the bond price must be higher at a higher debt level (as the government must be generating positive revenue from new issuances to sustain c > y) and that a higher consumption requires a higher bond price, given a debt level.

Consider then a solution to the following autonomous first-order differential equation:

(5)
$$b'(\tau) = H(b(\tau), Q(b(\tau), c^*))$$

with initial condition b(0) = 0. Its solution, along with $q(\tau) = Q(b(\tau), c^*)$, will define the candidate $(b(\tau), q(\tau))$ before period *T* as the debt level and bond price paths that keep consumption at c^* . Once these candidate paths for debt and bond prices are determined, they will be used to determine the candidate paths of default arrival rates, $\alpha_n(\tau)$, and reputation, $\rho(\tau)$. We then define our candidate *T* as the earliest period τ such that $\rho(\tau) = 1$.

To define the candidate $\rho(\tau)$ implied by our candidate $b(\tau)$ and $q(\tau)$ for $\tau \leq T$, we derive a delay differential equation. First, to ensure that ρ jumps to $\rho(\tau^*(\eta_n b(\tau)))$ after an *n* type partial default, one needs

(6)
$$\alpha_n(\tau) = \frac{\rho(\tau) - \rho(\tau^*(\eta_n b(\tau)))}{[1 - \rho(\tau)]\rho(\tau^*(\eta_n b(\tau)))}\theta_n$$

for all $n \ge 1$. Given this and solving equation (3) for $\alpha_0(\tau)$ and substituting into equation (2), one derives the delay differential equation

(7)
$$\rho'(\tau) = \epsilon + \rho(\tau) \frac{q'(\tau) + i + \lambda}{q(\tau)} - \rho(\tau)(i + \lambda + \epsilon + \delta) + \rho(\tau) \sum_{n=1}^{N} \left[\frac{q(\tau^*(\eta_n b(\tau)))}{q(\tau)} \frac{\rho(\tau)}{\rho(\tau^*(\eta_n b(\tau)))} \eta_n - 1 \right] \theta_n$$

Since our candidate $b(\tau)$ and $q(\tau)$ (and thus $q'(\tau)$) have been previously determined, this delay differential equation, with initial condition $\rho(0) = 0$, solves for our candidate $\rho(\tau)$. Define T as the smallest τ such that $\rho(\tau) = 1$.

This defines our candidate $(b(\tau), q(\tau), \rho(\tau), \{\alpha_n(\tau)\}_{n=0}^N)$ up to *T*, all as a function of the assumed q(0).

For $\tau > T$, the candidate $b(\tau)$ and $q(\tau)$ are no longer determined by the paths necessary to hold consumption constant at c^* . Candidates for $b(\tau)$ and $q(\tau)$ are constructed for $\tau > T$ by using b(T) and q(T) as initial conditions and using the differential equations (1) and (4), substituting $\gamma_n(\tau) = \frac{\theta_n}{\delta} \left[\frac{1}{\rho(\tau^*(\eta_n b(\tau)))} - 1 \right]$ for $n \ge 1$

and $\gamma_0(\tau) = 1 - \sum_{n=1}^N \gamma_n(\tau)$ into (4), yielding the delay differential equation

(8)
$$q'(\tau) = -(i+\lambda) + q(\tau)(i+\lambda+\delta)$$

$$- q(au) \sum_{n=1}^{N} \left[rac{qig(au^*ig(\eta_n b(au)ig)ig)}{q(au)
hoig(au^*ig(\eta_n b(au)ig)ig)} \eta_n - 1
ight] heta_n$$

Setting $\gamma_n(\tau)$ to this value ensures that reputation ρ jumps to $\rho(\tau^*(\eta_n b(\tau)))$ after a partial default of level *n*, and setting $\gamma_0(\tau)$ to this value ensures that after a type switch, the newly born opportunistic type immediately defaults at some level.

Is this candidate equilibrium an equilibrium? For this, we need to check four conditions. First, we need our constructed $\rho(\tau) \in [0,1]$ for any $\tau \in (0,T]$. Second, we need our constructed $\sum_{n=1}^{N} \gamma_n(\tau) \in [0,1]$ for all $\tau \ge T$. Third, we need $C(b(\tau), q(\tau))$ $\le c^*$ for all $\tau \ge T$. This and consumption equal to c^* for all $\tau \in [0,T]$ ensures that optimization by the opportunistic government is satisfied. Finally, we need prices $q(\tau)$ to actually represent the expected present discounted value of the coupon payments of a bond issued at period τ . This is ensured if and only if $q(\tau)$ converges to a finite limit as $\tau \to \infty$. In particular, $q(\tau)$ must converge to

(9)
$$\frac{i+\lambda+\sum_{n=1}^{N}\frac{q(\tau^{*}(\eta_{n}\bar{b}))}{\rho(\tau^{*}(\eta_{n}\bar{b}))}\eta_{n}\theta_{n}}{i+\lambda+\delta+\sum_{n=1}^{N}\theta_{n}},$$

where $\overline{b} \equiv \lim_{\tau \to \infty} b(\tau)$. Note that for any arbitrary c^* , such convergence will in general not occur. For low c^* , the corresponding $q(0) = Q(0, c^*)$ and q(T) will also be low and the pricing equation (4) will cause $q(\tau)$ to diverge downward. Intuitively, the differential equation (4) is "justifying" a too low q(T) through ever increasing capital losses. Likewise, for high c^* , the corresponding $q(0) = Q(0, c^*)$ and q(T) will also be high, and the pricing equation (4) will cause $q(\tau)$ to diverge upward. Here, the differential equation (4) justifies a too-high q(T) through ever-increasing capital gains.

IV. Main Result

PROPOSITION 1: Let a collection $(b(\tau), q(\tau), \rho(\tau), \{\alpha_n(\tau)\}_{n=0}^N, \{\gamma_n(\tau)\}_{n=0}^N\}$ be an equilibrium constructed as in the previous section. Then for all $b(\tau) \leq b(T)$ and $m > n, q(\tau^*(\eta_m b(\tau))) > q(\tau^*(\eta_n b(\tau))).$

PROOF:

That $c^* > y$ implies that $b(\tau)$ is strictly increasing in τ for $\tau < T$. This implies that $\tau^*(\eta_m b(\tau)) > \tau^*(\eta_n b(\tau))$ as $\eta_n b(\tau) < \eta_m b(\tau) < b(\tau) \leq b(T)$. That H(b,q) is decreasing in b and increasing in q, along with $q \in (0,1)$, implies that C(b,q) is also decreasing in b and increasing in q. Thus for consumption, $c(\tau)$, to be constant at c^* for $\tau < T$, $q(\tau)$ must also be strictly increasing in τ for $\tau < T$. Given that we have just shown that $\tau^*(\eta_m b(\tau)) > \tau^*(\eta_n b(\tau))$, the result of the proposition follows.

This result implies that for partial defaults possibly done in equilibrium by the opportunistic type, the fall in bond prices following a partial default is greater the greater the bond holder haircut. (In our construction, partial defaults to debt values greater than b(T) are only ever done by the commitment type in equilibrium. If a type switch from commitment to opportunistic occurs when $b(\tau) \ge b(T)$, the opportunistic type immediately either fully defaults or partially defaults at a level *n* such that $\eta_n b(\tau) < b(T)$.) Note that our result that bigger partial defaults imply bigger drops in bond prices holds independent of the values of θ_n (the arrival rates of forced partial defaults) or any other parameters of the model such as H, i, λ , $\{\eta_1, \ldots, \eta_N\}$, \overline{B} , ϵ , or δ .

The intuition for this result can be seen in the proof: since equilibrium consumption of the opportunistic type is always greater than the country's endowment, debt must be increasing with the time-on-the-stopwatch τ as long as the opportunistic type is possibly in power. This then implies that bond prices must be increasing to keep consumption constant (a necessity for indifference). Further, reputation is increasing since no default is informative. This means that debt and reputation are increasing with the time-on-the-stopwatch τ . A larger haircut implies a larger reduction in the debt value. In our equilibrium construction, that lower debt value is thus necessarily associated with a lower reputation and a lower price.

V. An Example

In this section, we present an example that computationally meets our equilibrium criteria.¹² The example parameters, where possible, are the same as in AP. In particular, for the commitment type's borrowing function H(b,q), we choose

(10)
$$H(b,q) = \max\left\{r^{\star} - \left(\frac{i+\lambda}{q} - \lambda\right), 0\right\}(y-b).$$

¹²The code is available at https://github.com/manuelamador/partial_default.



Notes: Equilibrium paths for q, b, ρ , and c starting from $\rho = 0$ and b = 0. H is as in equation (10). The rest of the parameters are y = 1, $\epsilon = 0.01$, $\delta = 0.02$, i = 0.01, $\lambda = 0.2$, $r^* = 0.15$, $\eta = \{0.25, 0.75\}$, $\theta_1 = \theta_2 = 0.005$. The value of T = 30.9 is represented by the vertical line.



Figure 2. Equilibrium Paths for α_n and γ_n

We normalize y = 1 and choose other parameters consistent with a unit of time being one year. Thus, our choice of $\epsilon = 0.01$ and $\delta = 0.02$ implies a 1 percent chance per year that an opportunistic government dies in the next year to be replaced by a commitment government and a 2 percent chance per year that a commitment government dies to be replaced by an opportunistic government. We set the outside world discount rate i = 0.01 and $\lambda = 0.2$, corresponding to a yearly principal payoff of 20 percent or roughly five-year debt.

For the parameters associated with partial default, we set the grid of partial default levels $\eta = \{0.25, 0.75\}$ and the Poisson arrival rates that a government is forced to partially default at these levels at $\theta = \{0.005, 0.005\}$.¹³

¹³Computationally, we can handle a much finer grid on partial default levels as well as θ_n values being nonconstant. The coarse grid is chosen only to make visualization easier.





FIGURE 3. INCREASE IN INTEREST RATES FOR NEW BOND ISSUANCES AFTER PARTIAL DEFAULT

Figure 1 presents the computed equilibrium functions $b(\tau)$, $q(\tau)$, and $\rho(\tau)$, along with the implied function $c(\tau)$. Here, the graduation date T = 30.9. Figure 2 presents the computed equilibrium functions $\alpha_n(\tau)$ and $\gamma_n(\tau)$. Note that $\alpha_n(\tau) \to \infty$ for $n \in \{0, 1, 2\}$ as $\tau \to T$. That is, as $\tau \to T$, voluntary default at some level becomes certain. For $\tau \ge T$, by construction, a newly born opportunistic type immediately defaults at some level but mixes between default levels. Finally, Figure 3 illustrates our main result (Proposition 1) for these parameters: for all $\tau > 0$, if a government wipes out 25 percent of its debt (or $\eta = 0.75$), bond yields associated with new issuances rise by less than if it wipes out 75 percent of its debt (or $\eta = 0.25$).

VI. Other Equilibria

Are there other Markov equilibria with positive borrowing? While not proved here, we think not. Mixing by the opportunistic type imposes substantial discipline on equilibria (specifically indifference), which our construction exploits.

The intuition for why all equilibria, for $\tau < T$, should involve mixing over haircut levels is as follows: suppose that at some date and history, a proposed strategy called for the opportunistic type to set the Poisson arrival rate of voluntarily choosing that haircut to zero. Bayesian updating then implies that a partial default at this level has no information and thus causes no updating, implying that a deviating opportunistic type could partially default at this level with no cost to its reputation (all benefits and no costs). So instead suppose that at that date and history, a proposed strategy called for the opportunistic type to, with some positive probability, partially default at that level at exactly that date. Given that the probability of a commitment type partially defaulting at any exact date is zero (given the Poisson arrival rate assumption on forced partial defaults), Bayesian updating implies that if such a partial default occurs, the government's reputation jumps to zero. But then why should an

171

opportunistic government only partially default when it can fully default and suffer no worse a consequence? This logic suggests that voluntary partial defaults by the opportunistic type have to happen as strictly positive Poisson arrival events as well, implying indifference as an equilibrium condition, as in our construction.

VII. Conclusion

In papers in the tradition of Eaton and Gersovitz (1981), bigger partial defaults have ambiguous or counterfactual effects on bond prices since larger partial defaults put a country in a better debt position going forward. In this paper, we have demonstrated that a reputational model can be augmented to incorporate partial defaults. We have shown the ingredients that are necessary for this and provided a possible reputational explanation for why larger haircuts imply larger effects on bond prices or interest rates for future bond issuances.

Compared to our reputation model without partial defaults, this model also delivers more realistic time series. Without partial default, reputation either gradually improves or is completely destroyed. Likewise, without partial default, bond prices continue to gradually get higher, unless they suddenly revert to the worst they can possibly ever be. In this paper, bond prices can and do fall to intermediate levels.

An important feature in the empirical literature this model does *not* match is that default history matters in determining credit spreads even when comparing countries with similar debt levels. (See Cruces and Trebesch 2013.) In particular, for a given debt level, in our model a country that has this debt level because debt gradually increased to this level will have the same credit spread as a country that achieved this debt level by partially defaulting to it from a higher level. This occurs in our simple model since with no real shocks and with countries starting with zero debt, reputation and debt levels move perfectly together *on the equilibrium path*. However, *off the equilibrium path*, a lower debt price (as shown in AP, Section VI). Enhancing the current model with endowment shocks is a promising way to reconcile this fact and is the subject of future work.

REFERENCES

- Aguiar, Mark, and Gita Gopinath. 2006. "Defaultable Debt, Interest Rate and the Current Account." Journal of International Economics 69 (1): 64–83.
- Aguiar, Mark, and Manuel Amador. 2011. "Growth in the Shadow of Expropriation." Quarterly Journal of Economics 126 (2): 651–97.
- Aguiar, Mark, Manuel Amador, Hugo Hopenhayn, and Ivan Werning. 2019. "Take the Short Route: Equilibrium Default and Debt Maturity." *Econometrica* 87 (2): 423–62.
- Alfaro, Laura, and Fabio Kanczuk. 2005. "Sovereign Debt as a Contingent Claim: A Quantitative Approach." *Journal of International Economics* 65 (2): 297–314.
- Amador, Manuel, and Christopher Phelan. 2021. "Reputation and Sovereign Default." *Econometrica* 89 (4): 1979–2010.
- Amador, Manuel, and Christopher Phelan. 2023. "Replication data for: Reputation and Partial Default." American Economic Association [publisher], Inter-university Consortium for Political and Social Research [distributor]. https://doi.org/10.3886/E167961V1.
- Arellano, Cristina. 2008. "Default Risk and Income Fluctuations in Emerging Economics." American Economic Review 98 (3): 690–712.
- Arellano, Cristina, Xavier Mateos-Planas, and José-Víctor Ríos-Rull. 2019. "Partial Default." NBER Working Paper 26076.

- Asonuma, Tamon. 2016. "Serial Sovereign Defaults and Debt Restructurings." IMF Working Paper 2016/066.
- Benjamin, David, and Mark Wright. 2008. "Recovery Before Redemption: A Model of Delays in Sovereign Debt Renegotiations." Unpublished.
- Bulow, Jeremy, and Kenneth Rogoff. 1989. "A Constant Recontracting Model of Sovereign Debt." Journal of Political Economy 97 (1): 155–78.
- Chatterjee, Satyajit, Dean Corbae, and José-Víctor Ríos-Rull. 2008. "A Finite-Life Private-Information Theory of Unsecured Consumer Debt." Journal of Economic Theory 142 (1): 149–77.
- Chatterjee, Satyajit, Dean Corbae, Kyle P. Dempsey, and José-Víctor Ríos-Rull. 2020. "A Quantitative Theory of the Credit Score." NBER Working Paper 27671.
- Cole, Harold L., and Patrick J. Kehoe. 1995. "The Role of Institutions in Reputation Models of Sovereign Debt." *Journal of Monetary Economics* 35 (1): 45 – 64.
- Cole, Harold L., and Patrick J. Kehoe. 1998. "Models of Sovereign Debt: Partial Versus General Reputations." *International Economic Review* 39(1): 55–70.
- **Cole, Harold L., James Dow, and William B. English.** 1995. "Default, Settlement, and Signalling: Lending Resumption in a Reputational Model of Sovereign Debt." *International Economic Review* 36 (2): 365–85.
- Cruces, Juan J., and Christoph Trebesch. 2013. "Sovereign Defaults: The Price of Haircuts." American Economic Journal: Macroeconomics 5 (3): 85–117.
- **D'Erasmo, Pablo.** 2011. "Government Reputation and Debt Repayment in Emerging Economies." Unpublished.
- Dovis, Alessandro. 2019. "Efficient Sovereign Default." Review of Economic Studies 86 (1): 282–312.
- Dvorkin, Maximiliano, Juan M. Sánchez, Horacio Sapriza, and Emircan Yurdagul. 2021. "Sovereign Debt Restructurings." American Economic Journal: Macroeconomics 13 (2): 26–77.
- Eaton, Jonathan, and Mark Gersovitz. 1981. "Debt with Potential Repudiation: Theoretical and Empirical Analysis." *Review of Economic Studies* 48 (2): 289–309.
- Egorov, Konstantin, and Michal Fabinger. 2016. "Reputational Effects in Sovereign Default." Unpublished.
- Fourakis, Stelios. 2021. "Sovereign Default and Government Reputation." Unpublished.
- Gourinchas, Pierre-Olivier, Thomas Philippon, and Dimitri Vayanos. 2017. "The Analytics of the Greek Crisis." In NBER Macroeconomics Annual, Vol. 31, edited by Martin Eichenbaum and Jonathan A. Parker, 1–81. Chicago: University of Chicago Press.
- Lorenzoni, Guido, and Iván Werning. 2019. "Slow Moving Debt Crises." American Economic Review 109 (9): 3229–63.
- Martin, Philippe, and Thomas Philippon. 2017. "Inspecting the Mechanism: Leverage and the Great Recession in the Eurozone." *American Economic Review* 107 (7): 1904–37.
- Mihalache, Gabriel. 2020. "Sovereign Default Resolution Through Maturity Extension." Journal of International Economics 125C: 103326.
- Salomao, Juliana. 2017. "Sovereign Debt Renegotiation and Credit Default Swaps." Journal of Monetary Economics 90: 50–63.
- Sunder-Plassmann, Laura. 2018. "Writing Off Sovereign Debt: Default and Recovery Rates Over the Cycle." *Journal of International Money and Finance* 81: 221–41.
- Yue, Vivian Z. 2010. "Sovereign Default and Debt Renegotiations." Journal of International Economics 80 (2): 176–87.