Reputation and Partial Default*

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Abstract

This paper presents a continuous-time reputation model of sovereign debt allowing for both varying levels of partial default and full default. In it, a government can be a non-strategic commitment type, or a strategic opportunistic type, and a government's reputation is its equilibrium Bayesian posterior of being the commitment type. Our equilibrium has that for bond levels reachable by both types without defaulting, bigger partial defaults (or bigger haircuts for bond holders) imply higher interest rates for subsequent bond issuances, as in the data.

1 Introduction

Countries which when partially defaulting impose bigger haircuts on their bond holders face bigger market consequences. In particular, the interest rates they face for future bond issuances are higher.[1] But it is far from clear that this should be the case. After all, the bigger the haircut, the better debt position the defaulting country is in.

Here, we propose a possible explanation: Bigger adverse consequences for bigger haircuts act as an *endogenous* equilibrium incentive to make sovereign governments willing to mix between differing haircut levels.

In our model, governments can have possible private "types" (in particular a strategic "opportunistic" type and a non-strategic "commitment" type). We assume the commitment type, which by assumption never fully defaults, is sometimes stochastically forced to partially default at varying haircut levels, while the opportunistic type can default at any level at any time. Our

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[1] Cruces and Trebesch (2013) were the first to point out this fact after compiling a comprehensive dataset on haircuts and bond prices for sovereign defaults. See also the subsequent work of Asonuma (2016).

equilibrium has the property that for bond levels reachable by the opportunistic type without defaulting, bond prices are higher the higher the equilibrium probability a country's government is the commitment type. Further, our equilibrium involves the opportunistic type mixing — for any positive length of time, it chooses a positive finite probability of defaulting at every possible haircut level.

For such mixing to be a best response for an opportunistic type, the equilibrium mixing probabilities must imply that when a country imposes a bigger haircut on its lenders, its reputation (the Bayesian posterior that it is the commitment type) falls further than if it had imposed a smaller haircut. The intuition is that the *benefit* of a larger haircut is, of course, the wiping out of more debt. But for the opportunistic type to then ever choose a smaller haircut, there has to be a corresponding, and equal, *cost* to imposing a larger haircut. The cost imposed by equilibrium is then this greater loss in reputation, and the corresponding greater increase in future interest rates.

Our model is similar to and based on the continuous time model of Amador and Phelan (forthcoming), referred to now as A&P21. In that model, however, countries can only fully default (that is, fully repudiate the debt ensuring no payments would ever be made to current bond holders). Here, we also allow countries to partially default.

There are several papers that model reputational considerations in sovereign debt markets, such as Cole, Dow and English (1995), Alfaro and Kanczuk (2005), D'Erasmo (2011), Egorov and Fabinger (2016), and Dovis (2019).[2] The initial quantitative models of sovereign debt effectively assumed that there are no partial defaults (all defaults are full), and the sovereign can subsequently re-enter financial markets with a clean slate.[3] This assumption has been relaxed by subsequent research that has modelled the process of renegotiation between lenders and the sovereign.[4] Aguiar, Amador, Hopenhayn and Werning (2016), Dvorkin, Sánchez, Sapriza and Yurdagul (2021), and Mihalache (2020) study the role of the maturity structure plays in restructurings. The latter two provide a quantitative analysis, but do not focus on the behavior of spreads after default and re-entering financial markets. In her study of cyclical renegotiation outcomes, Sunder-Plassmann (2018) is able to generate spreads that are higher after higher haircuts in an Eaton and Gersovitz

[2] See Amador and Phelan (forthcoming) for a discussion of these papers. Chatterjee, Corbae, Dempsey and Ríos-Rull (2020) explore the implications of reputational concerns in unsecured consumer credit markets.

[3] See for example Arellano (2008) and Aguiar and Gopinath (2006), which are based on the incomplete markets framework of Eaton and Gersovitz (1981). After re-entering financial markets following a default, there are two opposing forces at play in these models: first, default costs are low and expected to remain low in the future, raising the probability of future defaults. But debt is also low (as the country has fully defaulted), which lowers that probability. This second force dominates and generates counter-factually low spreads after a country re-enters financial markets following a default.

[4] See, for instance Yue (2010), who introduced bargaining over restructuring terms in the Eaton-Gersovitz framework. For an earlier model on renegotiation, see Bulow and Rogoff (1989). See also Salomao (2017), and Benjamin and Wright (2008).

(1981) style model by making the bargaining power of the government during a renegotiation counter-cyclical. This can lead to higher haircuts in defaults that occur during recessions (and those lower output states are associated with lower default costs in the future – potentially increasing spreads). Note that in our paper, output (which is constant) and debt are not sufficient state variables to capture the behavior of spreads, a fact emphasized by Cruces and Trebesch (2013). Arellano, Mateos-Planas and Ríos-Rull (2019) provides an alternative model where partial defaults lead to further defaults and increases in debt (because of arrears).

2 The Environment

The continuous and infinite time environment/game we consider is similar to and based on the environment of A&P21. As in A&P21, here we consider a small open economy endowed with a constant flow y of a consumption good whose government can borrow from risk-neutral price-taking outside lenders who discount the future at rate $i > 0$. The terms of such borrowing is that the country can issue long term bonds at every date s which pay a coupon at date t of $(i+\lambda)e^{-\lambda(t-s)}$. This coupon schedule ensures that the price of a bond is one at date s if default cannot occur. Assuming exponentially decaying coupon payments is equivalent to the government paying an instantaneous coupon of $i + \lambda$ per unit of current debt, $b(t)$, with such debt decaying at rate λ. We assume the initial level of debt is zero, and there exists an exogenous maximum level of debt, $\overline{B} < y/(i + \lambda)$ (which ensures paying the required coupon is always feasible).

2.1 Players

As in A&P21, there is a countable list of potential *governments* where at any date $t \geq 0$, only one of the potential governments is active and makes decisions. We assume the first government on the list is an *opportunistic type*, the second a *commitment type*, with the list then alternating between types. At all times, with Poisson arrival rate $\epsilon > 0$, an opportunistic type government is replaced by the next government on the list (a commitment type). With arrival rate $\delta > 0$, a commitment type government is replaced by an opportunistic type. Such switches are private.

2.2 Strategies

We assume the commitment type is non-strategic and continuously makes coupon payments $(i + \lambda)$ per unit of debt. Unlike A&P21, here we assume the commitment type is sometimes forced to *partially default*. To this end, let $\eta = \{\eta_1, \ldots, \eta_N\}$ denote an increasing grid of fractions representing the severity of a partial default, where a larger value of n represents a smaller haircut, or a larger level of remaining debt after the partial default. Let $\theta_n > 0$ denote the Poisson arrival rate of a shock which forces a commitment government with debt $b(t)$ to partially default and reset its debt to $\eta_n b(t)$. In particular, partial default resets the promised stream of payments of each existing bond proportionally to η_n of its previous value. Assume these θ shocks are not publicly observed, and that a commitment type never fully defaults.

An opportunistic type, however, can both fully and partially default. It can always fully default, and, like the commitment type, can partially default at level n, resetting debt to $\eta_n b(t)$. If a full default occurs, current bond holders get no additional payments, and the stock of outstanding debt is set to zero. If an opportunistic type partially defaults, coupon payments are adjusted exactly as if the commitment type partially defaults. Since both types can partially default, a partial default at any level does not mechanistically reveal the government's type.

As in A&P21, we assume that strategies are *Markov*. The payoff relevant state variables here are the level of debt $b(t)$ and the government's reputation $\rho(t)$. In A&P21, where only full default or no default was possible at any given moment, these two state variables could be reduced, without loss, to a single state variable that implied them both, time since last default, denoted τ. Here, this is no longer true. With partial default, time since last full default no longer mechanistically implies debt and reputation. Nevertheless, here we again look for strategies as a function of τ, but consider τ to be a more abstract object, best thought of as the time on a stopwatch which can be reset back to either zero (in the case of full default) or to *endogenous* earlier time-points depending on the level of partial default, n. In particular, let $b(\tau)$ be the level of debt if there are no defaults for τ periods starting from no debt. We search for equilibria where upon a partial default from $b(\tau)$ to debt level $\eta_n b(\tau)$, the stopwatch is reset to the endogenous amount of time it takes a country to go from zero debt to debt level $\eta_n b(\tau)$ conditional on not defaulting at any level during that time, denoted $\tau^*(\eta_n b(\tau))$. (That is, $\tau^*(b)$ is the inverse function of $b(\tau)$.)

For the commitment type, we assume that as long as it is in control, it follows a pre-specified expenditure rule determined by the expectations of international financial markets of how a commitment type should act. That is, as long as the commitment type is in control, the stock of debt evolves according to

$$b'(\tau) = H(b(\tau), q(\tau)) \tag{1}$$

for some exogenous function H, where $q(\tau)$ represents the price of a bond when the stopwatch displays τ (from now on referred to as *period* τ) after the realization of the period τ default event.

It follows from the sequential budget constraint that $c(\tau) = y-(i+\lambda)b(\tau)+q(\tau)(b'(\tau)+\lambda b(\tau))$, and thus consumption for the commitment type is determined by $c(\tau) = C(b(\tau), q(\tau))$ where the function C is given by

$$C(b, q) \equiv y - (i + \lambda)b + q(H(b, q) + \lambda b)$$

We further impose the following further conditions on $H(b,q)$ (and thus, implicitly, $C(b,q)$):

Assumption 1. *Let $\mathbb{X} \equiv [0,\overline{B}] \times [0,1]$. The function $H : \mathbb{X} \to \mathbb{R}$ satisfies the following: (i) H is Lipschitz continuous; (ii) H is weakly decreasing in b; (iii) H is weakly increasing in q; (iv) There exists $\underline{q} \in (0, \frac{i+\lambda}{i+\lambda+\delta+\epsilon})$ such that $H(0,q) = 0$ for all $q \in [0,\underline{q}]$, and $H(0,q) > 0$ for all $q \in (\underline{q},1]$; (v) $H(\overline{B},1) \leq 0$; (vi) H is differentiable in the set of $(b,q) \in \mathbb{X}$ such that $H(b,q) > 0$.*

Restrictions (ii) and (iii) guarantee that the commitment type increases its debt by more the higher the price of its bonds and the lower the inherited debt stock.[5]

For the *opportunistic* type, in addition to the Markov restriction, we impose a restriction that it always chooses a level of borrowing (and thus consumption) that is identical to that which would have been chosen by a commitment government facing the same debt and price. (In A&P21, we show this restriction is without loss.) With this restriction, the only decision left under the control of the opportunistic government is whether and how much to default. Let *default level* $n = 0$ denote full default and default level $n \in \{1, \dots, N\}$ denote the partial default resetting debt b to $\eta_n b$ by adjusting future coupon payments proportionally.

Given these restrictions, we assume that a strategy for an opportunistic government consists of a scalar T, and two vectors, $\alpha(\tau)$ and $\gamma(\tau)$. The first vector, $\alpha(\tau) = \{\alpha_0(\tau), \dots, \alpha_N(\tau)\}$, denotes for all $\tau < T$, the Poisson arrival rate of the opportunistic government defaulting at each level n. The second vector, $\gamma(\tau) = \{\gamma_0(\tau), \dots, \gamma_N(\tau)\}$, denotes for all $\tau \geq T$, the *probability* of the opportunistic government defaulting at each level n at precisely period τ. We further assume $\sum_{n=0}^{N} \gamma_n(\tau) = 1$ for all $\tau \geq T$, or that an opportunistic type certainly and immediately defaults at some level for all $\tau \geq T$. Such a strategy specification is definitely **not** without loss. We nevertheless attempt to construct equilibria in this class.

2.3 Payoffs

If the government does *not* default at period τ, it issues additional bonds $H(b(\tau), q(\tau)) + \lambda b(\tau)$ at endogenous price $q(\tau)$ and its consumption is $C(b(\tau), q(\tau))$. If the government fully defaults (that is, defaults at level $n = 0$), then τ is reset to zero with $b_0 = 0$. If the government partially defaults at level $n \geq 1$, debt is reset from $b(\tau)$ to $\eta_n b(\tau)$, and τ is reset to $\tau^*(\eta_n b(\tau))$.

There are no direct costs of choosing to fully or partially default and no restrictions on government borrowing from then on. In particular, in the case of full default, the government immediately issues new additional bonds $H(0, q(0))$ at endogenous price $q(0)$ and its consumption is $C(0, q(0))$. In the case of partial default, the government immediately issues new additional bonds $H(\eta_n b(\tau), q(\tau^*(\eta_n b(\tau)))) + \lambda \eta_n b(\tau)$ at endogenous price $q(\tau^*(\eta_n b(\tau)))$ and its consumption is $C(\eta_n b(\tau), q(\tau^*(\eta_n b(\tau))))$.

[5] See A&P (2021) for justification of the other restrictions.

The opportunistic type receives a flow payoff equal to $u(c(\tau))$ as long as it is continuously in power, and discounts future payoffs at rate $r > 0$. We assume that $u : \mathbb{R}_+ \to [\underline{u}, \overline{u}]$ for some finite values $\underline{u}$ and $\overline{u}$, and that u is strictly increasing. We make no other assumptions on the preferences of the opportunistic type. (As in A&P21, a preview of our results is that our constructed Markov equilibrium is essentially *independent* of u and r. Other than more is preferred to less, and now is preferred to later, the preferences of the opportunistic type will not matter at all.)

2.4 Beliefs

Recall $\rho(\tau)$ represents the international market's beliefs, or Bayesian posterior, that the government is the commitment type when the stopwatch is at τ. If the government *fully* defaults at any period $\tau \geq 0$, Bayesian updating implies ρ immediately jumps to zero, since only the opportunistic type can fully default.

Next, consider $\tau < T$. If the government *partially* defaults at level n at τ, Bayesian updating depends on the opportunistic type's strategy. Specifically, if a partial default of level n occurs, Bayesian updating implies $\rho(\tau)$ jumps to

$$\frac{\rho(\tau)\theta_n}{\rho(\tau)\theta_n + (1-\rho(\tau))\alpha_n(\tau)}.$$

This is the arrival rate of an n-level default by the commitment type divided by the arrival rate of an n-level default by either type.

If the government doesn't default at any level at date $\tau < T$, Bayesian updating again depends on the opportunistic type's strategy. Bayesian updating in this case implies[6]

$$\rho'(\tau) = (1-\rho(\tau))\epsilon + \rho(\tau)\left((1-\rho(\tau))\left(\alpha_0(\tau) + \sum_{n=1}^{N}(\alpha_n(\tau)-\theta_n)\right) - \delta\right). \tag{2}$$

For $\tau \geq T$, $\rho(\tau) = 1$, since by assumption an opportunistic government immediately defaults at some level for all $\tau \geq T$. Further, Bayesian updating requires ρ to remain at one if no default occurs. As when $\tau < T$, if government fully defaults, Bayesian updating requires ρ to jump to

[6]This formula is the derivative of the following with respect to Δ evaluated at $\Delta = 0$

$$\begin{aligned}\rho(\tau+\Delta) = (1-\delta\Delta)&\frac{\rho(\tau)(1-\sum_{n=1}^{N}\theta_n\Delta)}{\rho(\tau)(1-\sum_{n=1}^{N}\theta_n\Delta) + (1-\rho(\tau))(1-\sum_{n=0}^{N}\alpha_n(\tau)\Delta)} \\ +\epsilon\Delta(1 - &\frac{\rho(\tau)(1-\sum_{n=1}^{N}\theta_n\Delta)}{\rho(\tau)(1-\sum_{n=1}^{N}\theta_n\Delta) + (1-\rho(\tau))(1-\sum_{n=0}^{N}\alpha_n(\tau)\Delta)}).\end{aligned}$$

zero. If a government partially defaults at level n, Bayesian updating requires ρ to jump to

$$\frac{\theta_n}{\theta_n + \delta\gamma_n(\tau)}.$$

This is again the arrival rate of an n-level default by the commitment type divided by the arrival rate of an n-level default by either type, where the arrival rate of an n-level default by an opportunistic type is the arrival rate of a type switch, δ, multiplied the probability of an immediate n-level default, $\gamma_n(\tau)$.

2.5 Prices

Our construction of a Markov equilibrium will take candidate initial prices $q(0)$ as given. Prices for all periods $0 < \tau \leq T$ are then determined by the delay differential equation[7]

$$\begin{aligned} q'(\tau) = -(i+\lambda) + q(\tau)(i+\lambda+(1-\rho(\tau))\alpha_0(\tau)) + \\ \sum_{n=1}^{N}(q(\tau) - q(\tau^*(\eta_n b(\tau)))\eta_n)(\rho(\tau)\theta_n + (1-\rho(\tau))\alpha_n(\tau)). \end{aligned} \tag{3}$$

For $\tau \geq T$,

$$\begin{aligned} q'(\tau) = -(i+\lambda) + q(\tau)(i+\lambda+\delta\gamma_0(\tau)) + \\ \sum_{n=1}^{m}(q(\tau) - q(\tau^*(\eta_n b(\tau)))\eta_n)(\theta_n + \delta\gamma_n(\tau)). \end{aligned} \tag{4}$$

3 Markov Equilibria

We consider a collection $(b(\tau), q(\tau), \rho(\tau), T, \{\alpha_n(\tau)\}_{n=0}^N, \{\gamma_n(\tau)\}_{n=0}^N)$ to be a Markov equilibrium if

1. (Foreign investors break even in equilibrium.) For all τ, $q(\tau)$ is the expected discounted (by i) value of the stream of coupon payments for a bond issued at period τ.

[7] This formula is the limit as $\Delta \to 0$ of $\frac{q(\tau+\Delta)-q(\tau)}{\Delta}$ where

$$\begin{aligned} q(\tau) = (i+\lambda)\Delta + e^{-(i+\lambda)\Delta} q(\tau+\Delta)\left(\rho(\tau)(1-\sum_{n=1}^{N}\theta_n\Delta) + (1-\rho(\tau))(1-\sum_{n=0}^{N}\alpha_n(\tau)\Delta)\right) \\ + e^{-(i+\lambda)\Delta}\sum_{n=1}^{N} q(\tau^*(\eta_n b(\tau)))\eta_n\left(\rho(\tau)\theta_n\Delta + (1-\rho(\tau))\alpha_n(\tau)\Delta\right). \end{aligned}$$

2. (Market beliefs are rational.) $\rho : \mathbb{R}^+ \to [0,1]$; satisfies Bayes rule.

3. (Debt evolution and budget constraint.) The level of debt, as a function of τ, evolves according to the pre-specified expenditure rule H.

4. (opportunistic type optimizes.) For all τ, no other default strategy improves the continuation expected lifetime payoff of the opportunistic type.

4 Constructing a Markov Equilibrium

In this section, we construct a Markov equilibrium.

The main idea for the equilibrium construction, as in A&P21, is to first ensure that consumption for the opportunistic type is always at a constant $c^\star > y$ for all periods $\tau < T$, and that consumption of the opportunistic type is weakly less than $c^\star$ for all $\tau \geq T$. The construction next ensures that reputation ρ is reset after a partial default so that after a default of level n in period τ, ρ jumps to $\rho(\tau^*(\eta_n b(\tau)))$. Such a construction ensures that an opportunistic government is indifferent between defaulting at any level or not defaulting for $\tau < T$, and is indifferent between default levels (and is willing to certainly and immediately default at some level) for $\tau \geq T$.

To this end, let $Q(b,c)$ denote the price which causes a commitment type with debt b to set its consumption to c. That is, $Q(b,c)$ is such that $C(b, Q(b,c)) = c$. Assumption 1 guarantees that $Q(b,c)$ is strictly increasing in b and c, reflecting the fact that to maintain a level of consumption higher than y, the bond price must be higher at a higher debt level (as the government must be generating positive revenue from new issuances to sustain $c > y$); and that a higher consumption requires a higher bond price, given a debt level.

Consider then a solution to the following autonomous first order differential equation:

$$b'(\tau) = H(b(\tau), Q(b(\tau), c^\star)) \tag{5}$$

with initial condition $b(0) = 0$. Its solution, along with $q(\tau) = Q(b(\tau), c^\star)$ will define the candidate $(b(\tau), q(\tau))$ before period T, as the debt level and bond price paths that keep consumption at $c^\star$. Once these candidate paths for debt and bond prices are determined, they will be used to determine the candidate paths of default arrival rates, $\alpha_n(\tau)$, and reputation, $\rho(\tau)$. We then define our candidate T as the earliest period τ such that $\rho(\tau) = 1$.

To define the candidate $\rho(\tau)$ implied by our candidate $b(\tau)$ and $q(\tau)$ for $\tau \leq T$, we derive a delay differential equation. First, to ensure that ρ jumps to $\rho(\tau^*(\eta_n b(\tau)))$ after an n type partial default, one needs

$$\alpha_n(\tau) = \frac{\rho(\tau)}{1-\rho(\tau)} \frac{1-\rho(\tau^*(\eta_n b(\tau)))}{\rho(\tau^*(\eta_n b(\tau)))} \theta_n, \tag{6}$$

for all $n \geq 1$. Given this and solving equation (3) for $\alpha_0(\tau)$ and substituting into equation (2), one derives the delay differential equation

$$\begin{aligned}\rho'(\tau) &= \epsilon + \rho(\tau)\frac{q'(\tau)+i+\lambda}{q(\tau)} - \rho(\tau)(i+\lambda+\epsilon+\delta)\\ &\rho(\tau)\sum_{n=1}^{N}\left(\frac{q(\tau^*(\eta_n b(\tau)))}{q(\tau)}\frac{\rho(\tau)}{\rho(\tau^*(\eta_n b(\tau)))}\eta_n - 1\right)\theta_n.\end{aligned} \tag{7}$$

Since our candidate $b(\tau)$ and $q(\tau)$ (and thus $q'(\tau)$) have been previously determined, this delay differential equation, with initial condition $\rho(0) = 0$, solves for our candidate $\rho(\tau)$. Define T as the smallest τ such that $\rho(\tau) = 1$.

This defines our candidate $(b(\tau), q(\tau), \rho(\tau), \{\alpha_n(\tau)\}_{n=0}^{N})$ up to T, all as a function of the assumed $q(0)$.

For $\tau > T$, the candidate $b(\tau)$ and $q(\tau)$ are no longer determined by the paths necessary to hold consumption constant at $c^\star$. Candidates for $b(\tau)$ and $q(\tau)$ are constructed for $\tau > T$ by using $b(T)$ and $q(T)$ as initial conditions and using the differential equations (1) and (4), substituting $\gamma_n(\tau) = \frac{\theta_n}{\delta}(\frac{1}{\rho(\tau^*(\eta_n b(\tau)))} - 1)$ for $n \geq 1$ and $\gamma_0(\tau) = 1 - \sum_{n=1}^{N}\gamma_n(\tau)$ into (4), yielding the delay differential equation

$$q'(\tau) = -(i+\lambda) + q(\tau)(i+\lambda+\delta) - q(\tau)\sum_{n=1}^{N}\left(\frac{q(\tau^*(\eta_n b(\tau)))}{q(\tau)\rho(\tau^*(\eta_n b(\tau)))}\eta_n - 1\right)\theta_n. \tag{8}$$

Setting $\gamma_n(\tau)$ to this value ensures that reputation ρ jumps to $\rho(\tau^*(\eta_n b(\tau)))$ after a partial default of level n, and setting $\gamma_0(\tau)$ to this value ensures that after a type switch, the newly born opportunistic type immediately defaults at some level.

Is this candidate equilibrium an equilibrium? For this, we need to check four conditions. First, we need our constructed $\rho(\tau) \in [0,1]$ for any $\tau \in (0,T]$. Second, we need our constructed $\sum_{n=1}^{N}\gamma_n(\tau) \in [0,1]$ for all $\tau \geq T$. Third, we need $C(b(\tau), q(\tau)) \leq c^\star$ for all $\tau \geq T$. This and consumption equal to $c^\star$ for all $\tau \in [0,T]$ ensures optimization by the opportunistic government is satisfied. Finally, we need prices $q(\tau)$ to actually represent the expected present discounted value of the coupon payments of a bond issued at period τ. This is ensured if and only if $q(\tau)$ converges to a finite limit as $\tau \to \infty$. In particular, , $q(\tau)$ must converge to

$$\frac{i+\lambda+\sum_{n=1}^{N}\frac{q(\tau^*(\eta_n\bar{b}))}{\rho(\tau^*(\eta_n\bar{b}))}\eta_n\theta_n}{i+\lambda+\delta+\sum_{n=1}^{N}\theta_n}, \tag{9}$$

where $\bar{b} \equiv \lim_{\tau\to\infty} b(\tau)$. Note for any arbitrary $c^\star$, such convergence will in general **not** occur.

For low $c^\star$, the corresponding $q(0) = Q(0, c^\star)$ and $q(T)$ will also be low, and the pricing equation (4) will cause $q(\tau)$ to diverge downward. Intuitively, the differential equation (4) is "justifying" a too low $q(T)$ through ever increasing capital losses. Likewise, for high $c^\star$, the corresponding $q(0) = Q(0, c^\star)$ and $q(T)$ will also be high, and the pricing equation (4) will cause $q(\tau)$ to diverge upward. Here, the differential equation (4) justifies a too high $q(T)$ through ever increasing capital gains.

5 Main Result

Proposition 1. *Let a collection* $(b(\tau), q(\tau), \rho(\tau), \{\alpha_n(\tau)\}_{n=0}^N, \{\gamma_n(\tau)\}_{n=0}^N)$ *be an equilibrium constructed as in the previous section. Then for all* $b \leq b(T)$ *and* $m > n$, $q(\tau^*(\eta_m b(\tau))) > q(\tau^*(\eta_n b(\tau)))$.

Proof. That $c^\star > y$ implies $b(\tau)$ is increasing in τ for $\tau < T$. That $H(b, q)$ is decreasing in b and increasing in q, along with $q \in (0, 1)$ implies $C(b, q)$ is also decreasing in b and increasing in q. Thus for consumption, $c(\tau)$, to be constant at $c^\star$ for $\tau < T$, $q(\tau)$ must also be increasing in τ for $\tau < T$. □

This result implies that for partial defaults possibly done in equilibrium by the opportunistic type, the fall in bond prices following a partial default is greater the greater the bondholder haircut. (In our construction, partial defaults to debt values greater than $b(T)$ are only ever done by the commitment type in equilibrium. If a type switch from commitment to opportunistic occurs when $b(\tau) \geq b(T)$, the opportunistic type immediately either fully defaults or partially defaults at a level n such that $\eta_n b(\tau) < b(T)$.) Note, our result that bigger partial defaults imply bigger drops in bond prices holds independent of the values of θ_n (the arrival rates of partial default by the commitment type), or any other parameters of the model such as H, i, λ, $\{\eta_1, \ldots, \eta_N\}$, $\overline{B}$, ϵ, or δ.

6 An Example

In this section we present an example which computationally meets our equilibrium criteria. The example parameters, where possible, are the same as in A&P21. In particular, for the commitment type's borrowing function $H(b, q)$, we choose

$$H(b, q) = \max \left\{ r^\star - \left(\frac{i + \lambda}{q} - \lambda \right), 0 \right\} (y - b) . \tag{10}$$

We normalize $y = 1$, and choose other parameters consistent with a unit of time being one year. Thus our choice of $\epsilon = 0.01$ and $\delta = 0.02$ implies a 1% chance per year that an opportunistic

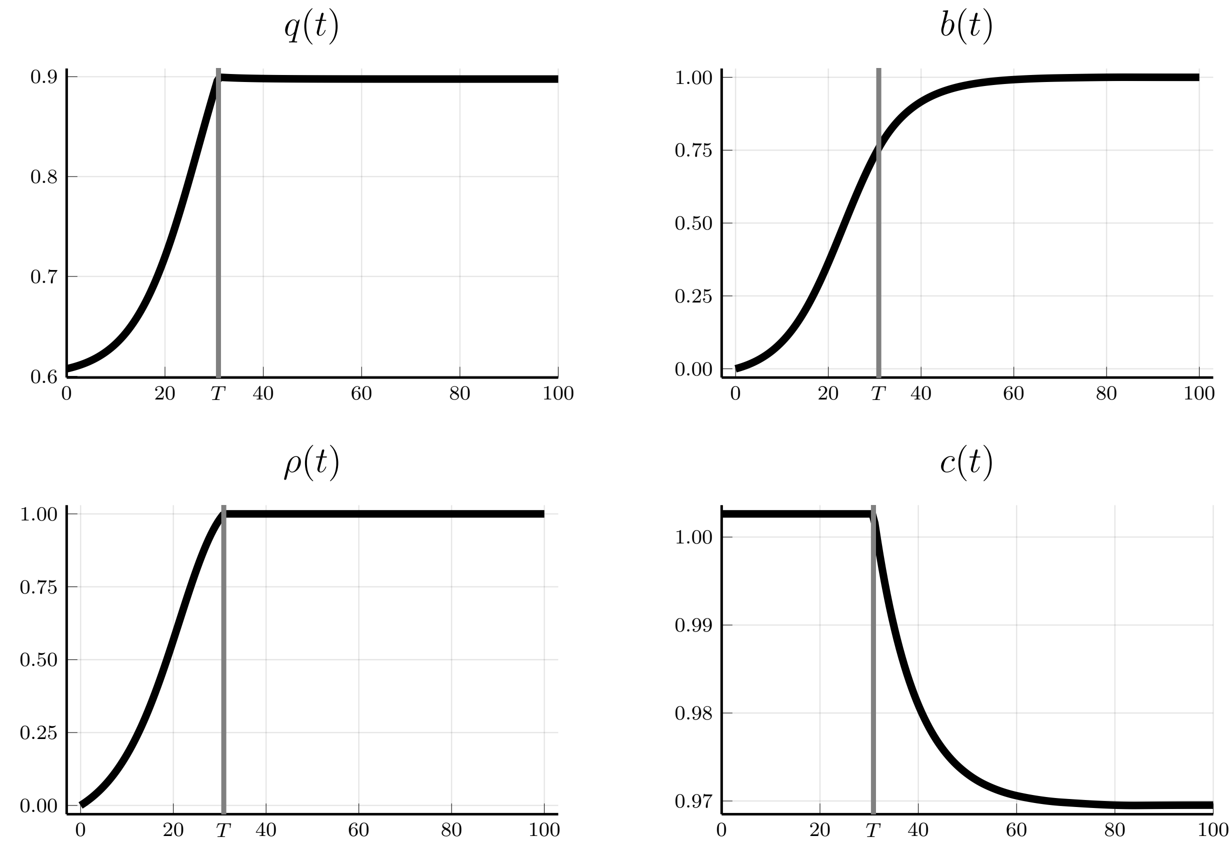

Figure 1: Equilibrium paths for q, b, ρ, and c starting from $\rho_0 = 0$ and $b_0 = 0$. H is as in equation (10). The rest of the parameters are $y = 1$, $\epsilon = 0.01$, $\delta = 0.02$, $i = 0.01$, $\lambda = 0.2$, $r^\star = 0.15$, $\gamma = \{.25, .75\}$, $\theta_1 = \theta_2 = .005$. The value of $T = 30.9$ is represented by the vertical line.

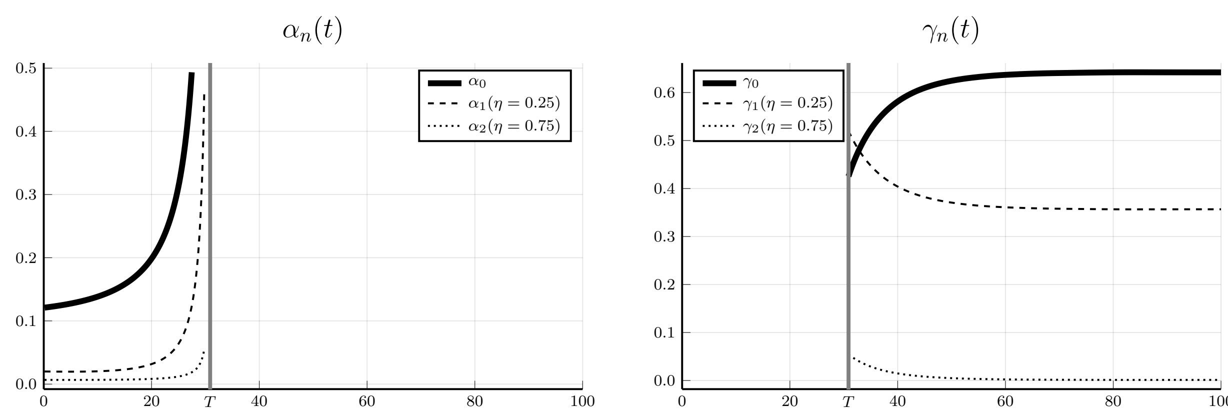

Figure 2: Equilibrium paths for α_n and γ_n.

government dies in the next year to be replaced by a commitment government, and a 2% chance per year that a commitment government dies to be replaced by an opportunistic government. We set the outside world discount rate $i = 0.01$ and $\lambda = 0.2$, corresponding to a yearly principal payoff of 20% or roughly five-year debt.

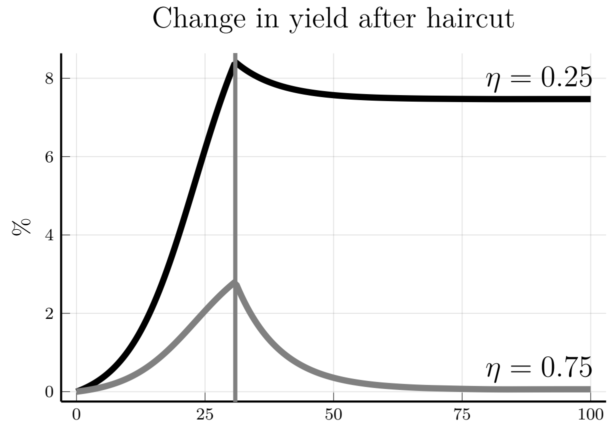

Figure 3: Increase in interest rates for new bond issuances after partial default.

For the parameters associated with partial default, we set the grid of partial default levels $\eta = \{.25, .75\}$, and the Poisson arrival rates that the commitment type is forced to partially default at these levels at $\{.005, .005\}$.[8]

Figure 1 presents the computed equilibrium functions $b(\tau)$, $q(\tau)$, and $\rho(\tau)$, along with the implied function $c(\tau)$. Here, the graduation date $T = 30.9$. Figure 2 presents the computed equilibrium functions $\alpha_n(\tau)$ and $\gamma_n(\tau)$. Note $\alpha_n(\tau) \to \infty$ for $n \in \{0, 1, 2\}$ as $\tau \to T$. That is, as $\tau \to T$, default at some level becomes certain. For $\tau \geq T$, by construction, a newly born opportunistic type immediately defaults at some level, but mixes between default levels. Finally, Figure 3 illustrates our main result (Proposition 1) for these parameters: For all $\tau > 0$, if a government wipes out 25% of its debt (or $\eta = .75$) bond yields associated with new issuances rise by less than if it wipes out 75% of its debt (or $\eta = .25$).

7 Other Equilibria

Are there other Markov equilibria with positive borrowing? While not proved here, we think not. Mixing by the opportunistic type imposes substantial discipline on equilibria, (specifically indifference), which our construction exploits.

The intuition for why all equilibria should involve mixing over haircut levels is as follows: Suppose at some date and history a proposed strategy called for the opportunistic type to set the Poisson arrival rate of that haircut to zero. Bayesian updating then implies a partial default at this level must have been done by the commitment type, implying a deviating opportunistic type

[8] Computationally, we can handle a much finer grid on partial default levels, as well as θ_n values being non-constant. The course grid is chosen only to make visualization easier.

could both partially default at this level and improve its reputation (all benefits, and no costs). So instead suppose at that date and history, a proposed strategy called for the opportunistic type to, with some positive probability, partially default at that level at exactly that date. Given that the probability of a commitment type partially defaulting at any exact date is zero (given the Poisson arrival rate assumption on forced partial defaults by the commitment type), Bayesian updating implies if such a partial default occurs, the government's reputation jumps to zero. But then why should an opportunistic government only partially default when it can fully default and suffer no worse a consequence? This logic suggests that partial defaults by the opportunistic type have to happen as strictly positive Poisson arrival events as well, implying indifference as an equilibrium condition, as in our construction.

8 Conclusion

In papers in the tradition of Eaton and Gersovitz (1981), bigger partial defaults have ambiguous or counterfactual effects on bond prices, since larger partial defaults put a country in a better debt position going forward. We provide here a possible reputational explanation for why larger haircuts imply larger effects on bond prices or interest rates for future bond issuances.

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