Take the Short Route

Sovereign Default and Debt Maturity

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Crisis and Maturity

- In crisis times, countries do not issue long-term bonds
  - even though yield curve flattens or inverts
- Why are governments doing this?
  - Term premia
  - Hedging
  - Incentives
- Issuances vs Stocks

What do we do

- Sovereign debt model with outside option shocks
  - Welfare theorem (of sorts)
  - Long-term bond markets are not used
    - Strict losses otherwise – budget constraint shrinks
  - Only issue short-term debt – stocks are not determined

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  - The way default probabilities evolve over time
    - level/slope of yield curve does not matter
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- Key: *Marginal price* $\neq$ *Average price*
  - Bond auction data is promising

Environment

- Discrete time, infinite horizon
- Risk neutral foreign lenders, discount R^{-1}
- Government of a small open economy
    - Endowment $\{y_t\}_{t=0}^{\infty}$, deterministic
    - Preferences: felicity u and discount β

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- Assumption: $\beta R \leq 1$
- Eaton-Gersovitz timing / Markov equilibrium

Asset markets

Assets

- One-period bond, b_t
- Arbitrary portfolio of long-term bonds:

$$\boldsymbol{l}^t = \{l_0, l_1, \ldots\}$$

$\boldsymbol{l}^t_k = l_k$: payment due in k periods from now, $t + k$

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Notation

- Given inherited $(b, \boldsymbol{l})$: $b + \boldsymbol{l}_0$ is amount of debt maturing today
- $\boldsymbol{l}^t_{\geq k} = \{l_k, l_{k+1}, \dots\}$

Timing, choices and default

- Governments inherits previous claims: b and $\boldsymbol{l} = \{l_0, l_1, \dots\}$
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If no default

- Issues one-period bonds b'
- New stock of long-term claims: $\boldsymbol{l}' = \{l'_0, l'_1, \dots\}$
- Budget constraint

$$b \leq y_t - c - \boldsymbol{l}_0 + q(b', \boldsymbol{l}', t)b' + Q(\boldsymbol{l}, \boldsymbol{l}', b', t) \qquad \text{(BC)}$$

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Markov Equilibrium

- $q(b', \boldsymbol{l}', t)$: price of one-period bonds
- $Q(\boldsymbol{l}, \boldsymbol{l}', b', t)$: cost of moving stock of legacy debt from $\boldsymbol{l}$ to $\boldsymbol{l}'$

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Incomplete markets

- $\boldsymbol{l}$ is contingent only on time
- $\boldsymbol{l}' \in \Gamma(\boldsymbol{l}, t) \leftarrow$ restrictions on claims that can be traded
  - "Not trading" is allowed: $\boldsymbol{l}_{\geq 1} \in \Gamma(\boldsymbol{l}, t)$.

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If default

- Government's payoff is v_t^D
- Lenders get 0

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Assumptions on v_t^D

- Drawn from continuous CDF F_t
- Support $\left[\underline{v}_t^D, \overline{v}_t^D\right] \subset \left[\underline{V}, \overline{V}\right]$
- Independent across time
- $\underline{u} + \beta \overline{V} < \underline{v}_t^D$ and $\overline{u} + \beta \int v dF(v) > \overline{v}_t^D$

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Drop the time subscripts from now on when clear

Government's optimization

- $V(b, \boldsymbol{I})$: equilibrium value function
- Government defaults if $V(b, \boldsymbol{I}) < v^D$
- Bellman:

$$V(b, \boldsymbol{I}) = \max_{c, b', \boldsymbol{I}'} \left\{ u(c) + \beta \int \max \left\{ V(b', \boldsymbol{I}'), v^D \right\} dF(v^D) \right\}$$

$$\text{subject to (BC) and } \boldsymbol{I}' \in \Gamma(\boldsymbol{I}).$$

Lenders' optimality: One period bond

- Consider $b', \boldsymbol{I}$' state tomorrow
- Default probability is $F(V(b', \boldsymbol{I}'))$

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- Lenders' optimality $\Rightarrow$ One-period bond price:

$$q(b', \boldsymbol{I}') = R^{-1} F(V(b', \boldsymbol{I}'))$$

Lenders' optimality: Long-term trades

- Given $b', \boldsymbol{I}'$, $\{V_{t+k}\}_{k=1}^{\infty}$
  - V_{t+k}: value in $t+k$ conditional on non-defaulting
  - Example: $V_{t+1} = V(b', \boldsymbol{I}')$

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- Discounting survival probability:

$$\rho_k(b', \boldsymbol{I}') = R^{-k} \Pi_{i=1}^{k} F(V_{t+i})$$

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- Lenders' optimality $\Rightarrow$ Long-term trades:

$$Q(\boldsymbol{I}, \boldsymbol{I}', b') = \sum_{k=1}^{\infty} \rho_k(b', \boldsymbol{I}')(\boldsymbol{I}'_{k-1} - \boldsymbol{I}_k)$$

Note: $Q(\boldsymbol{I}, \boldsymbol{I}_{\geq 1}, b') = 0$

Equilibrium definition

Markov Competitive Equilibrium

V, ρ, Q, q such that

- Government maximizes given Q, q.
- Q, q are prices consistent with V and ρ
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OK, this seems like a mess. How do we solve this?

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A *welfare theorem (of sorts)*

Towards a welfare theorem

- Incentive compatible allocation $\{c_k, V_k\}_{k=0}^{\infty}$:

$$V_k = u(c_k) + \beta \int \max\left\{V_{k+1}, v^D\right\} dF(v^D)$$

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- Planning problem
  - Sharing between government and *new* lenders

- If no default, have to pay legacy claims

  $\Rightarrow$ Resources left to be shared: $\{y_k - l_k\}$

Towards a welfare theorem: The planning problem

Planning problem

$$B^{\star}(v, \boldsymbol{I}) = \sup_{\{c_k, V_k\}} \left\{ \sum_{k=0} \underbrace{\left(\prod_{i=1}^{k} R^{-1} F(V_i) \right)}_{p_k} (y_k - I_k - c_k) \right\}$$

subject to:

$$V_0 \geq v$$

$\{c_k, V_k\}$ is an incentive compatible allocation

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- **Preliminaries:**
  - For $v \geq \underline{v}^D$, then $V_0 = v$ and $V_k \leq \max\{v, \overline{V}^D\}$

Going back to equilibrium ..

Equilibrium value functions

Equilibrium problem

$$V(b, \boldsymbol{I}) = \max_{c, b', \boldsymbol{I}'} \left\{ u(c) + \beta \int \max \left\{ V(b', \boldsymbol{I}'), v^D \right\} dF(v^D) \right\}$$

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Dual

$$B(v, \boldsymbol{I}) \equiv \max_{c, b', \boldsymbol{I}', v'} \left\{ y_t - \boldsymbol{I}_0 - c + q(b', \boldsymbol{I}')b' + Q(\boldsymbol{I}, \boldsymbol{I}', b) \right\}$$

$$v = u(c) + \beta \int \max\{v', v^D\} dF(v^D)$$

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Towards a welfare theorem

- Want to show that $B = B^\star$ together with implementation
- Complication: The presence of Q in the dual problem
  - In C.E., choosing $(b', \boldsymbol{I}')$ determines $v' = V(b', \boldsymbol{I}')$ and hence q
  - But Q is affected by all future decisions – not just today's

One inequality

Lemma 1

$B(v, \boldsymbol{I}) \leq B^{\star}(v, \boldsymbol{I})$

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- but (c, v') arbitrary

  $\Rightarrow$ choose any $\{c_k, V_k\}$ feasible in planning problem with $V_0 = v$

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Proof (cont.)

$$B(v, \boldsymbol{I}) \geq y_t - \boldsymbol{I}_0 + c_0 + R^{-1}F(V_1)B(V_1, \boldsymbol{I}_{\geq 1})$$

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- Last term $\to 0$

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- Last term $\to 0$
- As $\{c_k, V_k\}$ was arbitrary and delivers $v \Rightarrow B(v, \boldsymbol{I}) \geq B^\star(v, \boldsymbol{I})$

A Welfare theorem and implementation

- Lemma 1: $B(v, \boldsymbol{I}) \leq B^{\star}(v, \boldsymbol{I})$
  - Uses the equilibrium pricing restrictions on Q
- Lemma 2: $B(v, \boldsymbol{I}) \geq B^{\star}(v, \boldsymbol{I})$
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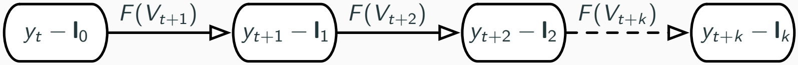
Welfare Theorem

$B(v, \boldsymbol{I}) = B^{\star}(v, \boldsymbol{I})$, and this can be attained by trading only one-period bonds

The planning solution: The costs of default



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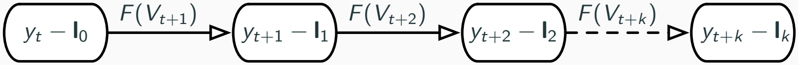
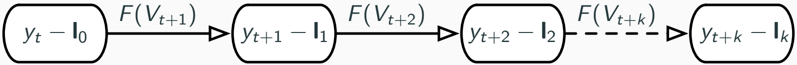


- Lower c_t and increase V_{t+k}
    - Raises likelihood of "reaching" $t + k$
    - Raises cost of delivering initial v (consumption not smoothed)

The planning solution: The role of legacy claims



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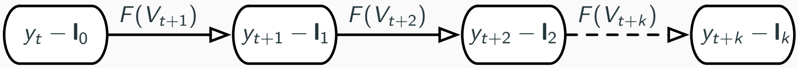
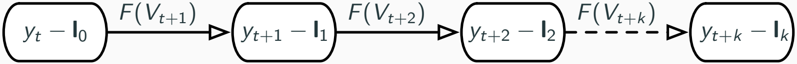


- $\boldsymbol{l}_k \uparrow$:
    - Less surplus to split in $t + k$
    - Less incentives to tilt consumption and reach that period
    - Default more likely

The planning solution: The role of legacy claims



- $\boldsymbol{l}_k \uparrow$:
  - Less surplus to split in $t + k$
  - Less incentives to tilt consumption and reach that period
  - Default more likely

- Higher $\boldsymbol{l}_k$ seems inefficient
  - Implementation: no need to play with $\boldsymbol{l}$
  - What if you do? Can that be optimal too? **No**

Convexity of B

- Let $\{c_k, V_k\}$ be an optimal allocation given $(v, \boldsymbol{I})$

$$B^\star(v, \boldsymbol{I}) = \sum_{k=0}^{\infty} p_k \left(y_{t+k} - c_k - \boldsymbol{I}_k\right)$$

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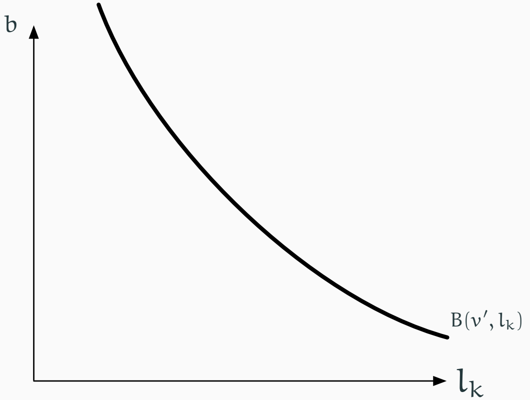
- Value function is convex in $\boldsymbol{I}$
  - Tangent is given by prices: $\nabla B^\star = \{-p_k\}_0^\infty$
- Strictly convex in $\boldsymbol{I}$ if default probability interior
  - $\{p_k\}$ changes when $\boldsymbol{I}$ changes

Convexity ⇒ Trading long-term bonds is costly



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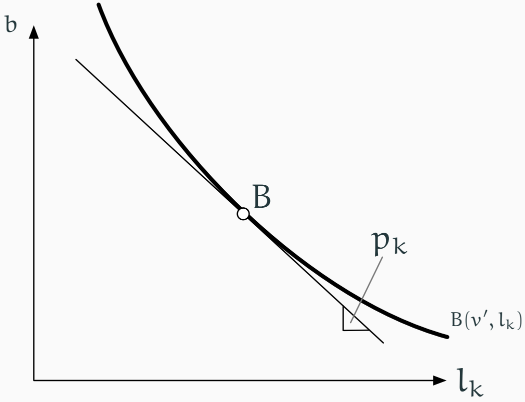

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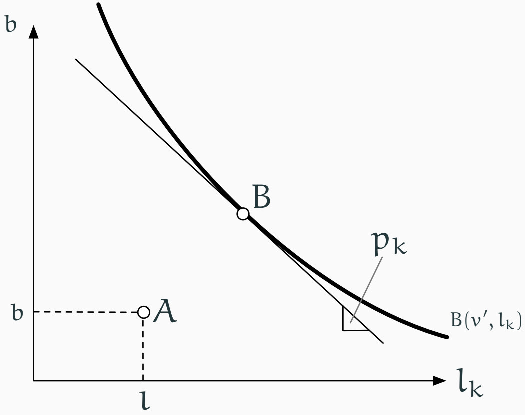

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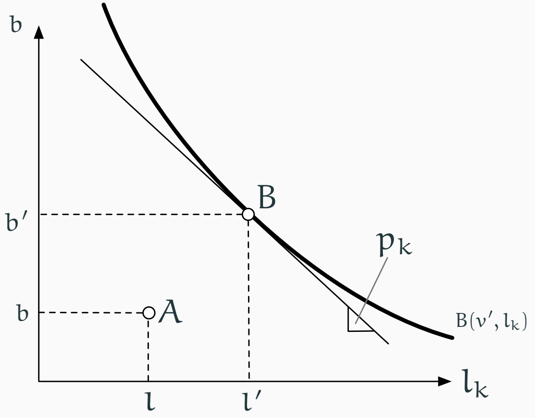

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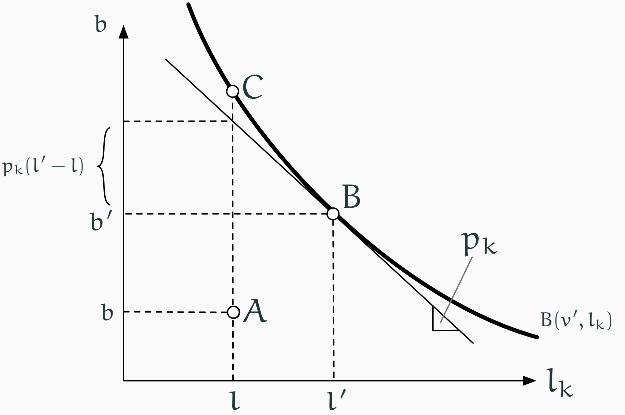

Taking stock

- Even though prices are always actuarially fair in equilibrium the price *schedule* deters the government from trading
    - Buy backs: greater incentives to save in the future
        - Increases the price of long-term bonds
    - Same for issuances in reverse: deter savings in the future
        - Lower the price of long-term bonds
    - All of these paths are already feasible with short-term trades
        - And cheaper

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  - All of these paths are already feasible with short-term trades
    - And cheaper
- Prices moving with trades is key
  - If prices were constant, maturity is irrelevant.

A Contract Theory Intuition

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  - Only one “principal” dealing with the “agent” at all times

A Contract Theory Intuition

- One-period debt is optimal because it is an exclusive contract
    - Only one "principal" dealing with the "agent" at all times
- The following contract would also optimal:
    - Issue debt for N periods with time-varying coupons
    - Commit not to re-issue debt until the last coupon is paid
- But not time consistent
- Legacy bondholders at the mercy of future fiscal policy

A C.E. Intuition

- Short-term debt is like a variable cost
    - The default cost is paid in interest rate when rolled over incentivizing the government to do something about it
- Long-term debt is like a sunk cost
    - Default premium is paid at the time of issuance – but after that ...

A Stationary Economy

- Constant y
- Log utility
- $v^D = \frac{u((1-\tau)y)}{1-\beta}$ where τ is random
- Assets: One period bond, and a perpetuity
- Two cases $\beta R = 1$ and $\beta R < 1$

A Stationary Economy: Euler Equation

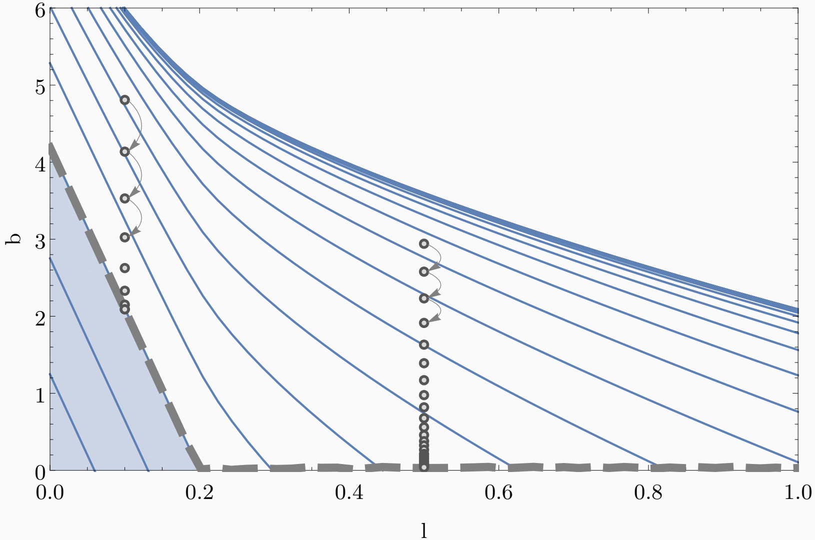
- If $v' > \overline{v}$:

$$\frac{1}{u'(c')} - \frac{\beta R}{u'(c)} = 0$$

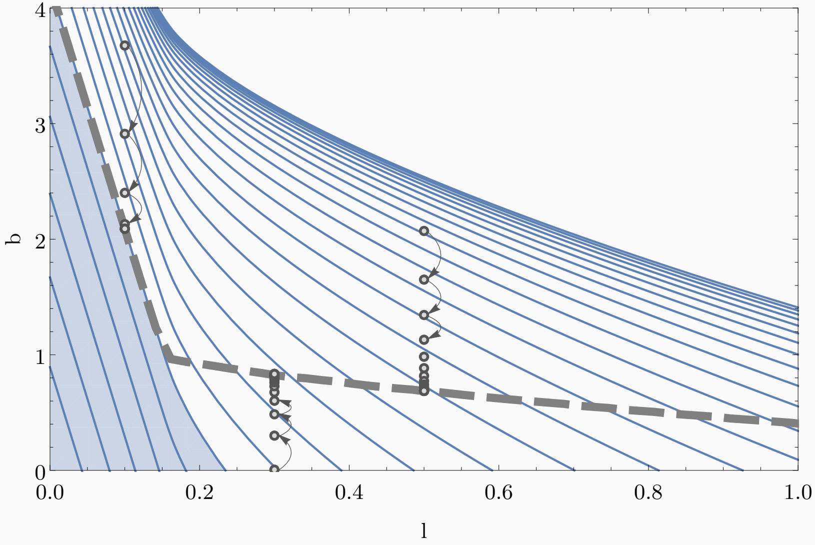
- If $v' \in (\underline{v}, \overline{v})$:

$$\frac{1}{u'(c')} - \frac{\beta R}{u'(c)} = \frac{f(v')}{F(v')} B^{\star}(v', \boldsymbol{I})$$

An Example Economy: $\beta R = 1$



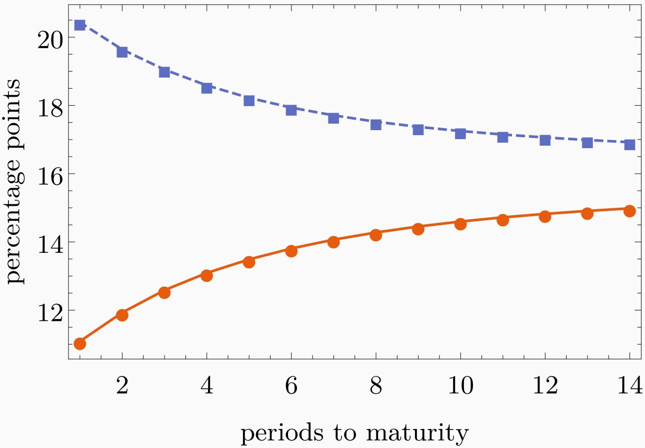
An Example Economy: $\beta R < 1$



Yield curve

- Example shows another thing
- Given that bond prices (levels) are actuarially fair
    - Yield curve reflects the sequence of default probabilities
- The costs of trading long-term bonds is independent of this

Example Economy $\beta R < 1$: Yield curves



Full Efficiency

- Promise to deliver B_{legacy} in value to legacy lenders:

$$B^{\star}(v, \boldsymbol{I}) = \sup_{\{c_k, V_k\}, \hat{\boldsymbol{I}}} \left\{ \sum_{k=0}^{\infty} \left(\prod_{i=1}^{k} R^{-1} F(V_i) \right) (y_k - \hat{\boldsymbol{I}}_k - c_k) \right\}$$

subject to:

$$V_0 \geq v$$

$\{c_k, V_k\}$ is an incentive compatible allocation

$$\sum_{k=0}^{\infty} \left(\prod_{i=1}^{k} R^{-1} F(V_i) \right) \hat{\boldsymbol{I}}_k \geq B_{\text{legacy}}$$

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- Last constraint binds and $\hat{\boldsymbol{I}}_k$ drops out from the objective
- Converting entire portfolio to one-period debt is efficient for all
- Cannot be achieved in a decentralized equilibrium: holdouts

Full efficiency: Holdouts

- Modified problem (+ holdout constraint):

$$B^{\star}(v, \boldsymbol{I}) = \sup_{\{c_k, V_k\}, \hat{\boldsymbol{I}}} \left\{ \sum_{k=0}^{\infty} \left(\prod_{i=1}^{k} R^{-1} F(V_i) \right) (y_k - \hat{\boldsymbol{I}}_k - c_k) \right\}$$

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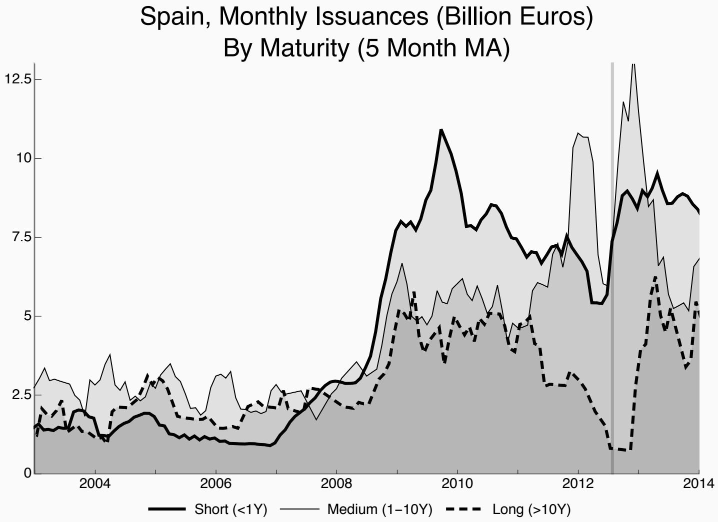
$$\sum_{k=0}^{\infty} \left(\prod_{i=1}^{k} R^{-1} F(V_i) \right) \hat{\boldsymbol{I}}_k \geq \sum_{k=0}^{\infty} \left(\prod_{i=1}^{k} R^{-1} F(V_i) \right) \boldsymbol{I}_k$$

- Problem collapses to our previous planning problem

Spain: Issuances



Conclusions

- Governments issue short-term when default risk is high. Why?
- This paper:
  - Sovereign debt model without insurance/hedging component
  - Sensitivity of long-term prices to issuances is larger than short
  - Marginal $\neq$ Average is key
- Zero issuances is focal
  - behavior of stocks depends
- Missing: Self-fulfilling debt crisis a la Cole-Kehoe
- Next: Separating hedging vs incentives