

Trading on a Simplex: An Eaton-Kortum Diagram*

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Abstract

We present a deterministic re-interpretation of the [Eaton and Kortum \(2002\)](#) Ricardian model of international trade. The representation can be interpreted as a Dornbusch–Fischer–Samuelson generalization, amenable to diagrammatic analysis, to an N -country world under specific technological restrictions that nest Eaton-Kortum.

1 Introduction

The seminal paper of [Eaton and Kortum \(2002\)](#) (henceforth EK) builds on an earlier – also seminal – paper by [Dornbusch, Fischer, and Samuelson \(1977\)](#) (henceforth DFS). The DFS model is a Ricardian model with a continuum of goods, but only two countries. With only two countries, DFS are able to index goods by comparative advantage, rendering the equilibrium amenable to a transparent diagrammatic analysis. However, with more than two countries comparative advantage is a multi-dimensional object, and the diagrammatic analysis of DFS is lost (see [Eaton and Kortum, 2002](#), footnote 13). EK elegantly circumvent this obstacle by taking a *probabilistic* approach to characterize an N -country Ricardian model with a continuum of goods. Specifically, by assuming that productivity is drawn independently across goods and countries from a Fréchet distribution with common shape parameter, and exploiting the properties of the Fréchet and a continuum of goods, EK obtain a gravity representation of bilateral trade and other compact general equilibrium conditions. This allows for the analysis of a rich multi-country model that can

*The views expressed here are entirely those of the authors. They do not necessarily represent the views of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

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be taken to the data. However, the notion of “goods” becomes lost in the process, and the useful graphical intuition one obtains from DFS appears out of reach. This paper closes the loop by introducing a diagrammatic approach to analyzing EK that is reminiscent of the original DFS framework. Rather than using a probabilistic structure for technology, we specify technology directly as a measure of goods on a simplex. Trade shares are then areas of regions in the simplex, with no appeal to a law of large numbers over a continuum of goods. We can then perform comparative static exercises to graphically explore how the pattern of trade changes with trade costs or productivity, as in the original DFS.

The Armington model with CES preferences delivers the same gravity equation and aggregate predictions as EK. While the desire for tractability is shared, our approach has an advantage relative to the Armington model: its focus on technology. Preferences play a very limited role in characterizing the equilibrium and comparative statics; indeed, the elasticity of substitution plays no role whatsoever in the analysis below. This keeps the focus on the comparative advantage parameter that governs the dispersion of relative productivity across goods, a defining feature of both EK and DFS.

We proceed by first introducing a simple Ricardian model with a continuum of goods that is amenable to diagrammatic analysis. We then show that the core equilibrium conditions of this model coincide with those of the original EK model. Given that [Eaton and Kortum \(2002\)](#) already proposed a 2-country version (see their footnote 15), our benchmark below is an $N = 3$ country world. We show how it is straightforward to extend it to any N .

2 Environment

We consider a static environment with $N = 3$ countries, indexed by $i \in \{1, 2, 3\}$. It is straightforward to generalize to $N > 3$ countries, which we do after discussing the benchmark case. Labor is the only factor of production, and country i is endowed with L_i units of labor.

2.1 Technology and Goods

As in DFS, there is a unit-measure continuum of goods. With two countries, DFS are able to sort goods by comparative advantage in a line (that is, relative productivity of the good in the two countries). With three countries, comparative advantage becomes multi-dimensional. We let goods be indexed by their vector of labor productivities in each country, $(z_1, z_2, z_3) \in \mathbb{R}_{++}^3$, and the unit mass of goods has a distribution over such vectors.

The key step in our approach is to map the space of productivities $\mathbb{R}_{++}^3$ into a space more amenable to analysis. For a given $\theta > 0$ and $T_i > 0$, let $y_i = T_i z_i^{-\theta}$ and let us define the following

transformation of coordinates that maps $\mathbb{R}_{++}^3$ into a *comparative advantage vector* $\omega = (a_1, a_2, a_3)$ in the simplex $\Omega \equiv \{a \in \mathbb{R}_+^3 : a_1 + a_2 + a_3 = 1\}$ and a *world productivity* $s \in \mathbb{R}_{++}$:

$$a_i = \frac{y_i}{\sum_k y_k}, \quad s = \left(\sum_k y_k \right)^{-1/\theta}.$$

In this transformation, the value of a_i tracks the comparative advantage of producing the good in country i : a low value of a_i means country i is relatively productive in it. The world productivity s tracks how productive all countries are in producing the good: it scales the good's productivity in every country. To see this, we can invert the transformation. Let $A_i \equiv T_i^{1/\theta}$, then country i 's productivity in a good with comparative advantage ω and world productivity s is

$$z_i = A_i s a_i^{-1/\theta},$$

which shows that s scales the productivity of all countries in producing this good.

Note that we can always do such a change of coordinates for any θ and any $\{T_i\}$, and this change of coordinates generates a measure of goods μ on the set $\mathcal{N} \equiv \Omega \times \mathbb{R}_{++}$. We therefore index a good by $v = (\omega, s) \in \mathcal{N}$ and work directly in this space.

Our first restriction in the commodity space is that there exists a transformation that induces a uniform measure on the simplex:¹

Assumption 1 (Uniformity on the Simplex). *There exists a transformation with $\theta > 0$ and $T_1, T_2, T_3 > 0$ such that the fraction of goods whose comparative-advantage vectors lie in any region of Ω equals that region's share of the simplex's area.*

When Assumption 1 holds, it does so for a unique θ . The values of T_i are also unique up to a common factor. That is, a technology can be compatible with at most one θ .

We show below that this assumption implies that the fraction of goods sourced from different locations takes the EK gravity form. Our next assumption strengthens Assumption 1 and is the one that will guarantee that expenditure shares take the same gravity form as the fraction of

¹More technically, for a region $E \subseteq \Omega$, let $|E|$ denote its area. Assumption 1 states that

$$\mu(E \times \mathbb{R}_{++}) = \frac{|E|}{|\Omega|} \quad \text{for every region } E \subseteq \Omega.$$

goods sourced (given the preferences we specify below):²

Assumption 2 (Uniformity on the Simplex Given World Productivity). *There exists a transformation with $\theta > 0$ and $T_1, T_2, T_3 > 0$ such that for any s , the fraction of goods among those with world productivity s whose comparative-advantage vectors lie in any region of Ω equals that region's share of the simplex's area.*

The final technological assumption is the standard iceberg transport cost: delivering one unit of a good from i to j requires shipping $\tau_{ij} \geq 1$ units.

2.2 The Firm's Problem

Given the technology description above, firms are competitive and hire labor at a wage w_i in country $i = 1, 2, 3$. The cost of producing good $v = (\omega, s) \in \mathcal{N}$ in country i for delivery in country j is:

$$\frac{\tau_{ij} a_i^{1/\theta} w_i}{A_i s}.$$

Competition implies that the price of good v in country j is the marginal cost of the minimum-cost supplier:

$$p_j(v) = \frac{1}{s} \min_i \left\{ \tau_{ij} a_i^{1/\theta} w_i / A_i \right\}. \quad (1)$$

Given that s is the same for all countries for this good, which country is the cheapest supplier of a good $v = (\omega, s)$ depends only on ω , the comparative advantage vector.

2.3 The Household's Problem

The representative household in country i has preferences characterized by the utility function $U_i(\{c(v)\}_{v \in \mathcal{N}})$. We assume that U_i is quasi-concave. To determine the pattern of production we do not need to say more about preferences at this stage. Later on, we impose a type of symmetry restriction on preferences across goods.

The household's problem in country i is:

$$\max_{\{c_i(v)\}} U_i(\{c_i(v)\}) \text{ subject to } X_i \equiv \int_{\mathcal{N}} p_i(v) c_i(v) d\mu(v) \leq w_i L_i,$$

²Assumption 2 strengthens Assumption 1. With $|E|$ as in footnote 1, Assumption 2 states that

$$\mu(E \times B) = \frac{|E|}{|\Omega|} \mu(\Omega \times B) \quad \text{for every region } E \subseteq \Omega \text{ and set of values of world productivity } B \subseteq \mathbb{R}_{++}.$$

Setting $B = \mathbb{R}_{++}$ gives Assumption 1.

where we define X_i to be the total expenditure of country i .

Let $\Omega_{ji} \subseteq \Omega$ denote the set of comparative advantage vectors for goods that country i purchases from country j (as we discussed above, this choice is only a function of ω). That is, $\omega \in \Omega_{ji}$ if $j = \arg \min_k \left\{ \tau_{ki} a_k^{1/\theta} w_k / A_k \right\}$, where we ignore the measure-zero set of ties. Let X_{ji} denote the expenditure of country i on goods from country j :

$$X_{ji} = \int_{\Omega_{ji} \times \mathbb{R}_{++}} p_i(v) c_i(v) d\mu(v).$$

2.4 Equilibrium

The definition of equilibrium is standard:

Definition 1. An equilibrium is a set of prices and consumption in each country $\{p_i(v), c_i(v)\}$ for $v \in \mathcal{N}$, and a vector of wages $\{w_i\}$, such that households optimize given prices and wages, prices satisfy (1), and global expenditure on goods sourced in i equals labor income in i : $\sum_j X_{ij} = w_i L_i$.

3 A Graphical Representation in the Simplex

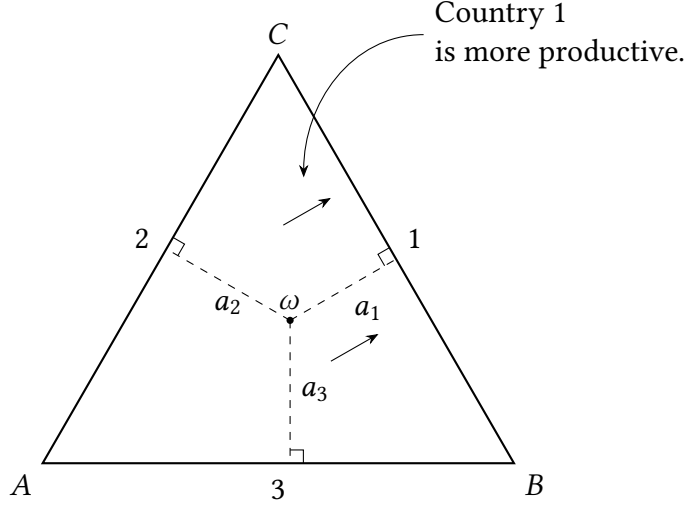
We now show how the pattern of production can be represented graphically using a simplex. In what follows we highlight the roles of Assumptions 1 and 2 in generating the results.

3.1 Assumption 1: Comparative Advantage and Sourcing

Under Assumption 1, the commodity space maps into Ω with a uniform measure. As we discuss above, given wages, ω is the only component of a good that determines its sourcing choice.

Figure 1 shows the corresponding simplex. The edges of the depicted equilateral triangle with height 1 correspond to the three countries $i \in \{1, 2, 3\}$. Each point inside the triangle represents goods with comparative advantage vector $\omega = (a_1, a_2, a_3)$, where a_i is the perpendicular distance from the point to the edge i . Points closer to edge i represent goods for which country i is relatively productive (recall that a_i represents inverse relative productivity). For example, as ω gets closer to the edge 1, the productivity index a_1 approaches 0, implying that country 1's relative productivity goes to ∞ . This means that goods with an ω close to the edge 1 are those goods where country 1 has a comparative advantage. This is a graphical depiction of comparative advantage and is our multi-dimensional analogue to the DFS schedule $A(z)$.

Figure 1: The Comparative Advantage Simplex



DFS define a “marginal good” where the cost of supplying a good to a given country is the same when sourced from the two countries. In our case, we define instead a locus of comparative advantage vectors. Specifically, let $\{w_i\}$ be a vector of equilibrium wages. Consider a consumer in country 1. Let $M^1(i, j)$ be the set of comparative advantage vectors for which the cost is the same whether the good is produced in i or j :

$$M^1(i, j) = \left\{ \omega = (a_1, a_2, a_3) \in \Omega \text{ such that } \frac{\tau_{i1} w_i a_i^{1/\theta}}{A_i} = \frac{\tau_{j1} w_j a_j^{1/\theta}}{A_j} \right\}.$$

This set is a locus in our simplex along which a_i/a_j is constant:

$$\frac{a_i}{a_j} = \left(\frac{\tau_{j1} w_j / A_j}{\tau_{i1} w_i / A_i} \right)^\theta.$$

For example, $M^1(2, 3)$ is a ray from the vertex A where the edges 2 and 3 meet, as depicted in Figure 2. Similar rays from the other vertices correspond to $M^1(1, 2)$ and $M^1(1, 3)$. In Figure 2 these loci intersect at the point labeled O . For a good with comparative advantage O , country 1 faces the same cost regardless of source country. On the interior of the triangle AOB whose base is country 3 and has edges AO and BO , country 1 will source from country 3. Similarly for the other two partitions. Keep in mind that point O and the associated threshold rays are drawn from the perspective of country 1 consumers; with trade costs, these objects will vary across alternative destination countries.

In anticipation, we flag the importance of the point O . As will become clear, key equilibrium objects such as expenditure shares and the price distribution will be indexed by this point. Similarly, many comparative statics will follow from understanding how O moves in response.

The expression for the coordinates of point O has a familiar form. At point $O = (a_1^O, a_2^O, a_3^O)$, we have

$$a_2^O = \left(\frac{\tau_{11}w_1/A_1}{\tau_{21}w_2/A_2} \right)^\theta a_1^O, \quad a_3^O = \left(\frac{\tau_{11}w_1/A_1}{\tau_{31}w_3/A_3} \right)^\theta a_1^O.$$

The fact that $a_1 + a_2 + a_3 = 1$ implies

$$a_1^O = \frac{(\tau_{11}w_1/A_1)^{-\theta}}{\sum_{j=1}^3 (\tau_{j1}w_j/A_j)^{-\theta}}.$$

Similar expressions hold for a_2^O and a_3^O . In particular, for $i = 1, 2, 3$, we have:

$$a_i^O = \frac{(\tau_{i1}w_i/A_i)^{-\theta}}{\sum_{j=1}^3 (\tau_{j1}w_j/A_j)^{-\theta}}. \quad (2)$$

The expression (2) resembles the standard CES expenditure share formula, but importantly we have not imposed any such restriction on preferences in the derivation or linked it to any notion of expenditure. Nevertheless, it does reflect the fraction of goods sourced from each country. Consider the goods sourced from country 3; that is, triangle AOB . The area of this triangle is $(1/2)a_3^O \times \overline{AB}$, where $\overline{AB}$ is the length of the base of AOB . As all sides of ABC have the same length, the total area is $1/2\overline{AB} \times \sum_i a_i^O = 1/2\overline{AB}$. By Assumption 1, a_i^O represents the fraction of goods that country 1 sources from country i .

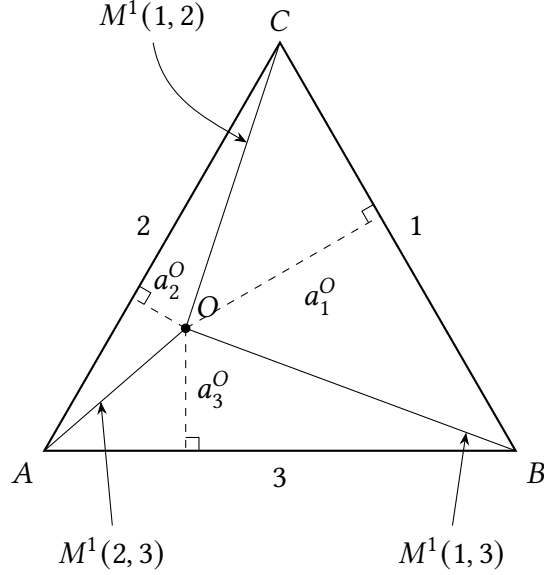
A similar analysis can be done for countries 2 and 3. The respective lines $M^2(i, j)$ and $M^3(i, j)$ will determine the intersection points O for each destination country.

3.2 Assumption 2: Prices and Sources

To characterize prices, we now restrict attention to the case in which all goods $v = (\omega, s)$ have the same world productivity, which we normalize to $s = 1$ (which can be done by rescaling T_i). In this case, Assumption 2 holds automatically under Assumption 1.

Under the assumption that $s = 1$ for all goods, every point in the simplex represents just one good. Point O is then the good for which all potential source countries have the same cost of supplying country 1, and it is the good with the highest price in country 1, which we denote $\bar{p}_1$. To see this, note that every good ω has at least one source country i with $a_i \leq a_i^O$; which implies

Figure 2: The Threshold Goods



that that country can supply the good at a cost weakly lower than at O . Hence

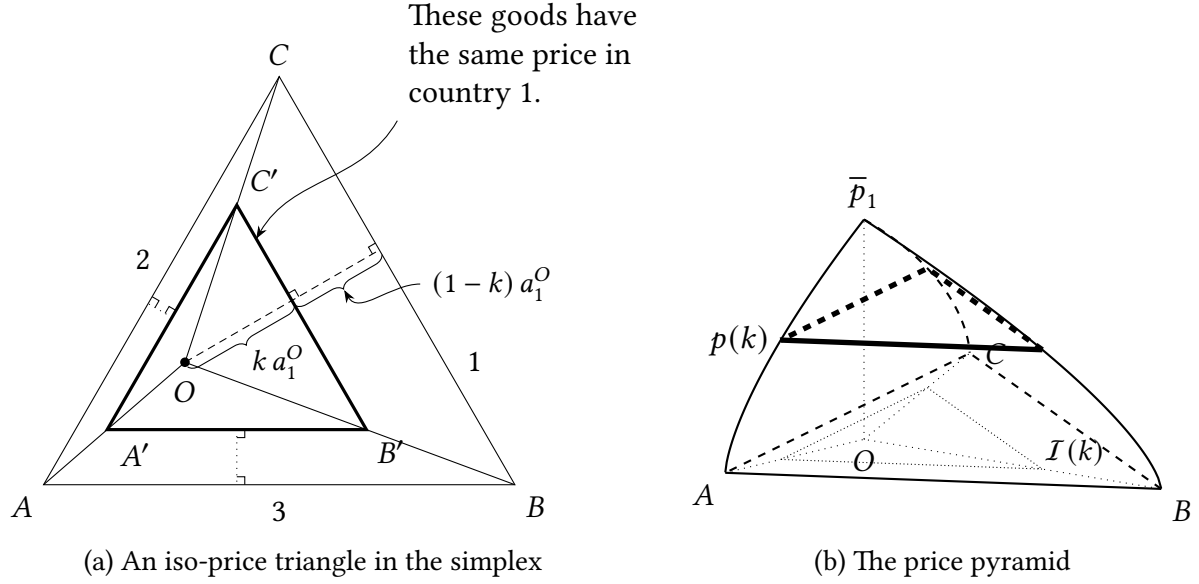
$$p_1(\omega, 1) = \min_k \frac{\tau_{k1} a_k^{1/\theta} w_k}{A_k} \leq \frac{\tau_{i1} a_i^{1/\theta} w_i}{A_i} \leq \frac{\tau_{i1} (a_i^O)^{1/\theta} w_i}{A_i} = \bar{p}_1 \quad \text{for all } \omega \in \Omega.$$

Consider the triangle $A'B'C'$ depicted in Panel (a) of Figure 3. The edge $A'B'$ is parallel to AB ; hence, goods on this edge are produced by country 3 and all have the same cost (recall that the distance to the edge is inversely related to productivity). Similarly for the other edges of the triangle. At the vertices, costs are the same for the respective pair of countries. Consumers in country 1 therefore face the same price for all goods along the edge of this triangle. We call this triangle an *iso-price* triangle. On the interior of this triangle, goods are more expensive (there is less comparative advantage), and outside the triangle goods are cheaper.

In Panel (b) we show a three-dimensional representation of the “price pyramid” with the simplex of goods as the base and the height at each point the price of the good. The maximum is $\bar{p}_1$, the price at point O , and prices are zero along the edges of the simplex as productivity goes to infinity. The vertical edges of the price pyramid are depicted for $\theta = 1.5$ and hence are slightly concave relative to the base.

We can index the iso-price triangles by a scaling factor $k \in [0, 1]$. Suppose the triangle has distance to the edges of the full simplex given by $(1 - k) \times (a_1^O, a_2^O, a_3^O)$; that is, in Figure 3, the perpendicular distance between $A'B'$ and AB is $(1 - k)a_3^O$, and similarly for the other edges. The

Figure 3: Iso-Price Loci when $s = 1$



highest price, $\bar{p}_1$, can be expressed as³

$$\bar{p}_1 = \left(\sum_i (\tau_{i1} w_i / A_i)^{-\theta} \right)^{-\frac{1}{\theta}}. \quad (3)$$

Then goods along the boundary of $A'B'C'$ have price $p = (1-k)^{1/\theta} \bar{p}_1$. The fraction of goods with price higher than p is the area of $A'B'C'$ relative to that of the simplex, and the relative areas of the three sub-triangles with vertex O represent the fraction of goods with price higher than p sourced from each country.

Let $I(k)$ denote the boundary of the iso-price triangle that is obtained by shrinking the simplex by a factor of k around the point O . All goods on $I(k)$ have the same price; specifically for $k \in [0, 1]$

$$I(k) \equiv \{ \omega \in \Omega : p_1(\omega, 1) = (1-k)^{1/\theta} \bar{p}_1 \equiv p(k) \}. \quad (4)$$

Since we take the perspective of country 1 throughout, we drop the country subscript on $p(k)$ and on the price distribution below. This iso-price triangle is depicted by the thick contour $p(k)$ in Panel (b) of Figure 3. Its projection onto the simplex denoted by the dotted lines in panel (b) represents the set $I(k)$.

As we vary k , we shrink or expand the iso-price triangle around point O , tracing out the distribution of prices for consumer 1. The areas sourced in each country shrink or expand with

³To see this, note that $\bar{p}_1 = (a_i^O)^{1/\theta} \tau_{i1} w_i / A_i$, for any i as the costs are equal across sources at point O . Inverting, we have $a_i^O = \bar{p}_1^\theta (\tau_{i1} w_i / A_i)^{-\theta}$. Summing across i and using $\sum_i a_i^O = 1$, we can solve for $\bar{p}_1$ given by equation (3).

k , as well. Given the proportionality, the fraction sourced from each country is stable for each price. More generally, for any iso-price triangle, the fraction of goods with higher price from each country is the same as in the full price distribution. Thus, we obtain the well known result from Eaton and Kortum (2002): the distribution of prices is independent of the source country.

We have chosen to illustrate the result under the restriction $s = 1$, but let us briefly point out that it holds more generally under Assumption 2. To see this, note that a good with comparative advantage vector ω and world productivity s has price $p_1(\omega, s) = p_1(\omega, 1)/s$. Then, for any world productivity s , by Assumption 2, among goods with world productivity s and price above any p , a fraction a_i^O is sourced from country i . Integrating across different values of world productivity maintains this fraction and preserves the result that the distribution of prices is independent of the source country. This also explains why Assumption 1 is not enough: even if the total projection onto Ω is uniform, regions of the simplex supplied by different countries may contain goods with different world productivity, and hence may feature different price distributions.

4 Demand and Expenditure Shares

So far we have not used any knowledge of consumer demand. In what follows, we will assume *that households spend the same amount on goods that have the same price and world productivity*. More formally:

Assumption 3 (Symmetry of Preferences). *For each $i \in \{1, 2, 3\}$, at any solution to country i 's household problem, goods with the same price and world productivity s receive the same consumption.*

Together with Assumption 2, this assumption is sufficient to recover the expenditure share formulas of Eaton and Kortum (2002). Note that no restriction is placed on the elasticity of substitution across goods. In our benchmark with all goods sharing the same world productivity $s = 1$, this assumption is the same as requiring that households spend the same amount on goods with equal prices. When world productivity varies across goods, this assumption allows households to treat goods with different world productivity differently, as long as they do not distinguish goods by their comparative advantage vector ω .

Recall that for the pattern of production, we only need to know the comparative advantage vectors. We can then consider expenditure patterns one value of world productivity at a time. In Section 3.2 we show that for a given world productivity s and a price p , a fraction a_i^O of the goods with a price above p is sourced from country i . This means that for any price interval $[p, p + \Delta)$,

the share of goods sourced from country i is also a_i^O . By Assumption 3, all goods with the same price and world productivity receive the same expenditure. Hence, since this holds for every sub-interval of prices, a fraction a_i^O of country 1's spending on goods with prices in $[p, p + \Delta)$ and world productivity s goes to country i . As this is true for every price interval and every value of world productivity, adding up we get

$$X_{i1} = a_i^O X_1.$$

Using the expression for a_i^O from (2), we have

$$\frac{X_{i1}}{X_1} \equiv \pi_{i1} = a_i^O = \frac{(\tau_{i1} w_i / A_i)^{-\theta}}{\sum_{j=1}^3 (\tau_{j1} w_j / A_j)^{-\theta}}, \quad \text{for all } i = 1, 2, 3.$$

Indeed now we have that the expenditure share on country i 's goods is **as if** consumers had a CES demand, with $\tau_{i1} w_i / A_i$ representing the cost of sourcing from country i , and $1 + \theta$, the elasticity of substitution.⁴ More generally, for every destination $d \in \{1, 2, 3\}$ we have:

$$\frac{X_{id}}{X_d} \equiv \pi_{id} = \frac{(\tau_{id} w_i / A_i)^{-\theta}}{\sum_{j=1}^3 (\tau_{jd} w_j / A_j)^{-\theta}}, \quad \text{for all } i = 1, 2, 3. \quad (5)$$

The fact that the share of expenditure on country i is the same as the share of the *number* of goods from country i is due to two sets of assumptions. On the technology side, Assumption 2 makes the set of goods that a country supplies to another a scaled version of the goods that any other country supplies to the same destination, at every price and world productivity s . This is what makes the distribution of prices independent of the source country for any world productivity. On the demand side, Assumption 3 then implies that for any world productivity, expenditure on goods with the same price is the same, and thus the fraction of expenditure on goods from country i equals the fraction of goods sourced from that country.

5 Closing the Model

The analysis so far has been predicated on a vector of wages (w_1, w_2, w_3) . To close the model, we need to ensure that the labor market clears and that trade is balanced:

$$X_{i1} + X_{i2} + X_{i3} = w_i L_i = X_i, \quad \text{for all } i = 1, 2, 3. \quad (6)$$

⁴This also shows the equivalence of the Eaton and Kortum (2002) model with an Armington model with an elasticity of substitution given by $1 + \theta$.

Equations (5) and (6) are sufficient to determine the equilibrium wages and expenditure shares (one of the equations is redundant given Walras' law, but recall that we also need to set a numeraire). On wages and prices we have one degree of freedom, and our numeraire will be country 1's wage: $w_1 = 1$.

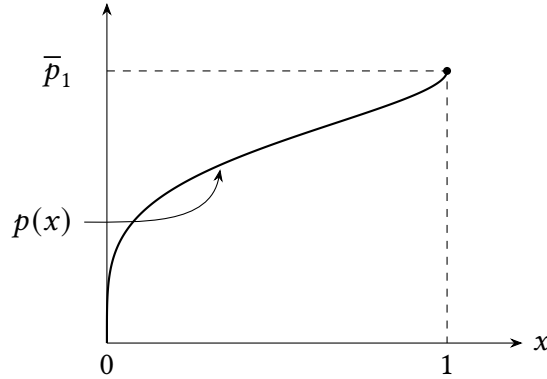
Note that the equilibrium conditions do not depend on world productivity s beyond Assumption 2, nor on preferences beyond Assumption 3.

6 The Price Distribution and Welfare

We have already seen that demand does not matter for the equilibrium determination of the wages or expenditure shares, beyond the restriction imposed in Assumption 3. In this section, we use the simplex to characterize the price distribution faced by consumers in a country for the case where $s = 1$. This shows why the pattern of substitution in demand does not matter for welfare either (beyond the symmetry restriction on preferences).

Taking the perspective of country 1, recall that $\mathcal{I}(k)$ defined in (4) defines the locus of goods with price $p(k) = (1 - k)^{1/\theta} \bar{p}_1$. This set is the boundary of triangle $A'B'C'$ in Figure 3. Inside this triangle, $p(\omega, 1) > p(k)$. As the iso-price triangle is similar to the simplex, but scaled down by a factor k , it has area k^2 times the area of the simplex.⁵ That is, the share of goods with prices above $p(k)$ is k^2 , and hence the share of goods with prices less than $p(k)$ is $1 - k^2$.

Figure 4: The Price Distribution



Following French (2015), it is convenient to invert the price distribution.⁶ Specifically, $x =$

⁵Recall that similar triangles have sides of equal ratios. By definition, the perpendicular distance from the edge of $A'B'C'$ to O is k times a_i^O , where i is the respective edge. This implies that, for example, $A'B'$ has a length k times that of AB . As both the base and height of $A'B'C'$ are scaled by k the area of the iso-price triangle is k^2 times the simplex.

⁶We thank Dave Donaldson for this reference.

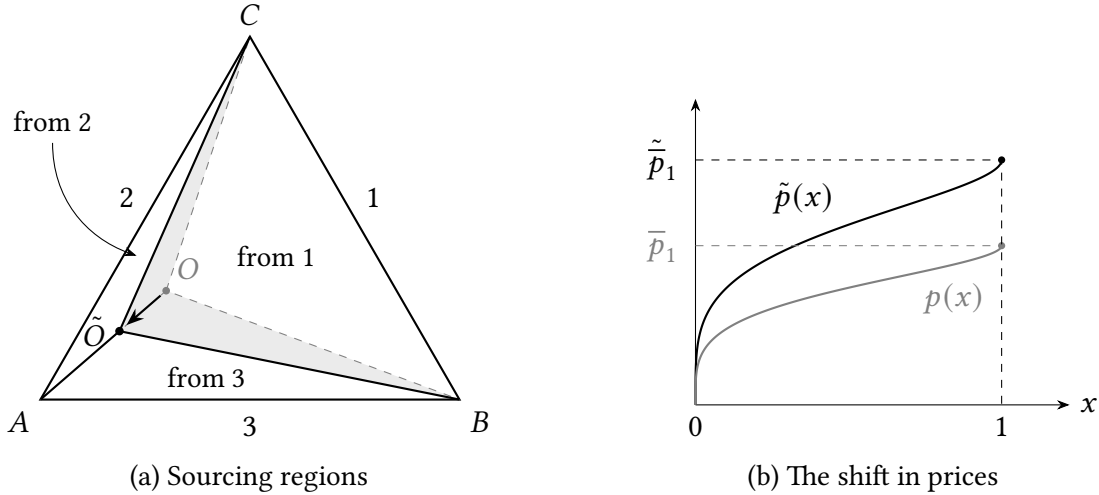
$1 - k^2$ is the share of prices less than $p(k)$. Inverting, $k = (1 - x)^{1/2}$. Substituting into $p(k) = (1 - k)^{1/\theta} \bar{p}_1$, we have

$$p(x) = \bar{p}_1 \left[1 - (1 - x)^{1/2} \right]^{1/\theta}. \quad (7)$$

Figure 4 depicts such a price distribution. Note that price distribution is indexed by $\bar{p}_1$, the price of good O . Just as for trade shares, the location of this point is all we need to know to evaluate the equilibrium from the perspective of a country 1 consumer, holding τ_{11} , w_1 and A_1 fixed.

In this spirit, we can evaluate comparative statics with respect to labor endowments L_i , productivity shifters A_i , or trade costs τ_{ij} by evaluating what happens to O . Given (7), such changes will shift the price distribution, but not alter the relative price between two goods at given ranks: in equilibrium, a change in labor endowments or trade costs always generates a proportional shift in the distribution of prices.

Figure 5: An Increase in Trade Costs



For example, consider an increase in trade costs from countries 2 and 3 into country 1; that is, τ_{i1} , $i = 2, 3$, increase. We depict the shift in Panel (a) of Figure 5. With fixed wages, an increase in trade costs shifts point O further away from country 1's edge. Country 1 spends more of its income on its own goods and imports less from 2 and 3. Moreover, $\bar{p}_1$ increases to $\tilde{\bar{p}}_1$.⁷ Note that this is illustrative, as in equilibrium, the wages will change. Panel (b) of Figure 5 shows the resulting effect on the price distribution: the entire price distribution shifts up and $\tilde{p}(x) =$

⁷The highest price for country 1 households is the cost of good O , which is from (3)

$$\bar{p}_1 = (a_1^O)^{1/\theta} \tau_{11} w_1 / A_1.$$

Given our numeraire $w_1 = 1$ and $\tau_{11} = 1$, this implies that $\bar{p}_1$ increases with the distance of O from country 1's edge.

$\kappa p(x) > p(x)$ at every percentile x with $\kappa = \tilde{\bar{p}}_1/\bar{p}_1$.

[French \(2015\)](#) obtained the welfare changes from analyzing the expenditure function and exploiting the proportionality of the price distribution. Following his approach, consider a change in $\{\tau_{ij}\}, \{A_i\}, \{w_i\}$ that moves the initial equilibrium of prices of country 1 from p_1 to a new $\tilde{p}_1$. Under Assumption 3, the expenditure function scales with $\bar{p}_1$.⁸ That is,

$$\frac{e(\tilde{p}_1, u)}{e(p_1, u)} = \frac{\tilde{\bar{p}}_1}{\bar{p}_1}.$$

Let u_1 and $\tilde{u}_1$ denote utility in the initial and in the new equilibrium. Define the welfare change as the proportional change in income necessary to attain the new utility at the initial prices. That is,

$$\frac{e(p_1, \tilde{u}_1)}{e(p_1, u_1)} = \frac{e(\tilde{p}_1, \tilde{u}_1) \frac{\bar{p}_1}{\tilde{\bar{p}}_1}}{e(p_1, u_1)} = \frac{\tilde{w}_1 L_1 / \tilde{\bar{p}}_1}{w_1 L_1 / \bar{p}_1},$$

since $e(p_1, u_1) = w_1 L_1$ and $e(\tilde{p}_1, \tilde{u}_1) = \tilde{w}_1 L_1$. Note that the welfare change is independent of preferences beyond the symmetry assumption.

Given that $\tau_{11} = 1$, equation (5) implies that the share of country 1's expenditure on its own goods is:

$$\pi_{11} \equiv \frac{X_{11}}{X_1} = \frac{(w_1/A_1)^{-\theta}}{\sum_{j=1}^3 (\tau_{j1} w_j/A_j)^{-\theta}} = \left(\frac{w_1}{A_1 \bar{p}_1} \right)^{-\theta},$$

so that $w_1/\bar{p}_1 = A_1 \pi_{11}^{-1/\theta}$. With L_1 and A_1 fixed, the proportional change in welfare between an

⁸Let $e(p, u)$ denote the expenditure function of country 1 and consider a path of model-generated prices $p_1(t)$, driven by costs $c_k(t) = \tau_{k1}(t)w_k(t)/A_k(t)$ that move smoothly, as t goes from 0 to 1, from their initial to their new equilibrium values. Abusing notation, we let $p_1(t, v)$ represent the price of good v at t . At every t , among goods with world productivity s and price q , a fraction $a_k^O(t)$ is sourced from country k , and the price of each such good sourced from k grows at rate $\dot{c}_k(t)/c_k(t)$, except at sourcing boundaries, which have measure zero. The average price change among these goods is therefore

$$q \sum_k a_k^O(t) \frac{\dot{c}_k(t)}{c_k(t)} = q \frac{\dot{\bar{p}}_1(t)}{\bar{p}_1(t)},$$

where the equality follows by differentiating $\bar{p}_1(t) = (\sum_k c_k(t)^{-\theta})^{-1/\theta}$. Assumption 3 implies that goods with the same world productivity and price receive the same Hicksian consumption. Thus, grouping goods by world productivity and price, Shephard's lemma gives

$$\frac{d}{dt} e(p_1(t), u) = \int c^h(t, v, u) \dot{p}_1(t, v) d\mu(v) = \frac{\dot{\bar{p}}_1(t)}{\bar{p}_1(t)} \int c^h(t, v, u) p_1(t, v) d\mu(v) = \frac{\dot{\bar{p}}_1(t)}{\bar{p}_1(t)} e(p_1(t), u).$$

Integrating from $t = 0$ to $t = 1$ yields $e(\tilde{p}_1, u)/e(p_1, u) = \tilde{\bar{p}}_1/\bar{p}_1$, where $p_1 = p_1(0)$ and $\tilde{p}_1 = p_1(1)$ are the initial and new equilibrium prices. This replicates [French's \(2015\)](#) result.

initial equilibrium and a counterfactual one (tildes) is

$$\frac{\tilde{w}_1 L_1 / \tilde{p}_1}{w_1 L_1 / \bar{p}_1} = \left(\frac{\tilde{\pi}_{11}}{\pi_{11}} \right)^{-1/\theta},$$

the well known [Arkolakis, Costinot, and Rodríguez-Clare \(2012\)](#) formula: the change in the own expenditure share and θ are sufficient for the welfare effects of any change in foreign labor endowments and trade costs.

One may still wonder why the substitution forces in demand do not matter at all for welfare comparisons in this Ricardian model. It is important to recall that the relative price between goods at any two given ranks is always unaffected, and hence substitution forces in demand do not enter into welfare comparisons.⁹

The analysis of this section restricted attention to $s = 1$. When world productivity varies across goods, the price of good (ω, s) is $p(\omega, s)/s$. So prices scale with $1/s$, and the welfare results above are unchanged.

7 Extension to N Countries

It is straightforward to extend the setup to include $N \neq 3$ countries. The case of $N = 2$ collapses to a version of the [Dornbusch, Fischer, and Samuelson \(1977\)](#) model. For the case of $N > 3$, besides the obvious adjustments to the formulas above, the only major change is to the price distribution, which becomes:

$$p(x) = \bar{p}_1 \left[1 - (1 - x)^{1/(N-1)} \right]^{1/\theta}.$$

8 The Mapping to Eaton-Kortum

In EK, the productivity of country i in each good is drawn independently across goods and countries from the Fréchet distribution

$$F_i(z) = e^{-T_i z^{-\theta}}, \quad z > 0,$$

with $T_i = A_i^\theta$. These draws, once transformed, do not lie on a unit simplex as world productivity varies across goods. However, the EK model is a special case of our commodity space. We obtain it by imposing [Assumption 2](#) and further restricting the measure of goods across values of world

⁹Given that the price at every rank is proportional to $\bar{p}_1$, and hence relative prices do not adjust, the consumption basket is a Hicksian composite good with a price proportional to $\bar{p}_1$. We thank Kei-Mu Yi for bringing this to our attention.

productivity s to have the density

$$h(s) = \frac{\theta}{2} s^{-3\theta-1} e^{-s^{-\theta}}, \quad s > 0.$$

It is straightforward to check that the resulting productivities in this case are those that are generated by the independent Fréchet productivity draws of EK.

Note that sourcing shares, expenditure shares, wages, and the welfare changes remain unchanged relative to the benchmark with $s = 1$, as variation in world productivity does not affect the equilibrium or the welfare results under Assumption 2. Only the distribution of prices changes. The mass of goods with price below q in country 1 becomes $1 - \exp(-(q/\bar{p}_1)^\theta)$, which is the distribution in Eaton and Kortum (2002).

The key to the EK results regarding expenditure shares, wages, and welfare changes is not the independent Fréchet distributions. What is crucial is that the fraction of goods whose comparative advantage lies in any region of the simplex equals that region's share of its area, and that the world productivity of a good does not depend on its comparative advantage. That is Assumption 2.

9 Conclusion

Eaton and Kortum (2002) write in their seminal paper that “Dornbusch, Fischer, and Samuelson work out the two-country case, but their approach does not generalize to more countries.” In footnote 15, they describe how their new model can be mapped into a DFS model when restricted to only two countries. But they write in footnote 13 that “With more than two countries there is no such natural ordering of commodities.” In this note, we have shown how this can be done by positing a simplex representation of technology.

In ongoing work we extend the model to incorporate the correlated productivity model of Lind and Ramondo (2023). This requires relaxing Assumptions 1 and 2.

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