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THE THEORY OF OPTIMAL DELEGATION WITH AN APPLICATION TO TARIFF CAPS

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THE THEORY OF OPTIMAL DELEGATION WITH AN APPLICATION TO TARIFF CAPS

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We consider a general representation of the delegation problem, with and without money burning, and provide sufficient and necessary conditions under which an interval allocation is optimal. We also apply our results to the theory of trade agreements among privately informed governments. For both perfect and monopolistic competition settings, we provide conditions under which tariff caps are optimal.

KEYWORDS: Optimal delegation, tariff caps, money burning, trade agreements.

1. INTRODUCTION

IN MANY IMPORTANT SETTINGS, a principal faces an informed but biased agent, and contingent transfers between the principal and agent are infeasible. The principal then chooses a permissible set of actions and "delegates" the agent to select any action from this set. The optimal form of delegation reflects an interesting tradeoff. The principal may wish to grant flexibility to the agent in order to utilize the agent's superior information as to the state of nature; however, the principal may also seek to restrict the agent's selection so as to limit the expression of the agent's bias.

The "delegation problem" contrasts with most of the mechanism-design literature, which assumes that contingent transfers are feasible. Contingent transfers may be infeasible, or at least severely restricted, in several settings of economic and political interest. For example, in a setting in which a regulator selects permissible prices or outputs for a monopolist with private information, legal rules may preclude contingent transfers between a regulator and a regulated firm.[2] Legal rules also limit contingent transfers in a variety of political settings. In other settings, contingent transfers may be discouraged due to social or ethical considerations.

The delegation problem was first defined and analyzed by Holmstrom (1977). He provided conditions for the existence of an optimal solution to the delegation problem. He also characterized optimal delegation sets in a series of examples, under the restriction that the delegation set takes the form of a

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[2]For further discussion, see Alonso and Matouschek (2008) and the references cited therein.

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single interval.[3] As Holmstrom (1977) argued, interval delegation is commonly observed. It is thus of special importance to understand when interval delegation is an optimal solution to the delegation problem.[4]

In this paper, we consider a general representation of the delegation problem and provide conditions under which interval delegation is an optimal solution to this problem. We also apply our results to the theory of trade agreements among privately informed governments, and establish conditions under which an optimal agreement takes the form of a tariff cap.

To develop a general representation of the delegation problem, we posit that the agent's action is taken from an interval on the real line and that the state has a continuous distribution over a bounded interval on the real line. While most of the delegation literature has focused on quadratic preferences, we consider a more general set of preferences. The principal's welfare function is continuous in the action and state and is twice differentiable and concave in the action. The state enters into the agent's welfare function in a multiplicative fashion, as is standard, and the agent's welfare function is twice differentiable and strictly concave in the action. We assume that the agent's preferred action is interior and strictly increasing in the state. We do not impose any conditions on the direction of the bias of the agent. We also analyze a modified delegation problem with a two-dimensional delegation set, where an action may be permitted only when an associated level of money is burned.

The possibility of money burning can be interpreted in many ways; as one example, our analysis includes situations in which "exceptional" actions are permitted only if wasteful administrative costs are incurred.[5] When money burning is allowed, we assume that it entails equal losses in the welfares of the agent and principal. To motivate this assumption, we note that, in some settings, such as the trade-agreement application that we discuss below, two players may seek to maximize the expected value of their joint welfare, with the understanding that one of the players will subsequently observe the state and choose an action from the permissible set to maximize his welfare. In this context, the "principal's" welfare corresponds to the players' joint welfare, and the agent's welfare is the welfare of the player who is subsequently informed

[3] Holmstrom (1977, p. 44) also established, for a specific example with quadratic preferences, that a single interval is optimal over all compact delegation sets.

[4] There is a large literature that followed Holmstrom's original work. See, for example, Martimort and Semenov (2006), Melumad and Shibano (1991), and Mylovanov (2008) and, more recently, Armstrong and Vickers (2010) and Frankel (2010).

[5] Other interpretations are also available. In a linear-quadratic setup, allowing for money burning is equivalent to allowing for stochastic allocations. For related work, see Goltsman, Hörner, Pavlov, and Squintani (2009) and Kovac and Mylovanov (2009). Within the context of a repeated game with privately observed and i.i.d. shocks, money burning can be interpreted as symmetric punishments. See Athey, Bagwell, and Sanchirico (2004) and Athey, Atkeson, and Kehoe (2005) for related themes. In the context of a consumption-savings problem, Amador, Werning, and Angeletos (2006) interpreted money burning as the possibility of selecting a consumption-savings bundle that lies in the interior of the consumer's budget set.

and chooses an action. A money burning expense incurred by the agent then also lowers the principal's welfare.[6]

To establish our findings, we utilize and extend the Lagrangian methods developed by Amador, Werning, and Angeletos (2006) in their analysis of a consumption-savings model. The Lagrangian method that they proposed, however, is not directly applicable to the general setting that we consider. First, in the setting without money burning, our constraint set features a continuum of equality constraints. Second, and more generally, our setup allows that the Lagrangian may fail to be concave with respect to the action.[7] The key to our approach is to construct valid Lagrange multipliers such that the Lagrangian is concave in the action when evaluated at those multipliers. We can then check first-order conditions for the maximization of the Lagrangian and thereby identify sufficient conditions for the optimality of interval delegation. Finally, we use standard Lagrangian techniques as well as simple perturbations to determine necessary conditions.

Our first proposition establishes sufficient conditions for an optimal solution to the delegation problem to take the form of interval delegation, where the sufficient conditions are expressed in terms of the welfare functions and the distribution of the state of nature. We provide conditions for both the setting without money burning as well as the setting with money burning. When the principal's welfare function is at most as concave as the agent's, the sufficient conditions for the two settings coincide. However, when the principal's welfare function is more concave than the agent's, the sufficient conditions for the optimality of interval delegation become tighter when money burning is feasible.

In our second proposition, we consider necessary conditions for the optimality of interval delegation. If the principal's welfare function is at least as concave as the agent's, then we use Lagrangian techniques to show, for the delegation problem with money burning, that our sufficient conditions are also necessary for the optimality of interval delegation. For other circumstances, we consider specific perturbations that enable us to identify necessary conditions. These perturbations are enough to identify a family of welfare functions for which the sufficient conditions of our first proposition are also necessary for

[6] Ambrus and Egorov (2009) identified an additional scenario in which money burning generates equal losses for the principal and agent; namely, if the agent has an ex ante participation constraint and ex ante (noncontingent) transfers are feasible, then the principal must compensate the agent for the agent's expected money burning expenses. As we discuss below, we also consider an extended model with imperfect transfers, which can be understood as relaxing the equal-loss assumption.

[7] In Amador, Werning, and Angeletos (2006), concavity of the Lagrangian obtains directly from the structure of the problem, and the constraint set does not feature a continuum of equality constraints. This allowed Amador, Werning, and Angeletos (2006) to obtain both necessary and sufficient conditions from Lagrangian methods.

the optimality of interval delegation. As we discuss below, this preference family includes, as special cases, the preferences commonly used in the literature.[8]

Our third proposition considers a special case in which the difference between the principal's welfare and the agent's welfare can be expressed as a function that depends on the action chosen but not on the state. For this case, the sufficient conditions for interval delegation take a particularly simple form. In particular, if the function that captures the welfare difference is decreasing when evaluated at the agent's preferred action (i.e., if the agent is biased to take actions that are higher than the principal would prefer), then the optimality of interval delegation holds for a broad range of settings if the density is nondecreasing and the agent's welfare is never more than twice as concave as the principal's welfare.

Using our findings, we also develop a new application of delegation theory to the theory of trade agreements among governments with privately observed political pressures. We consider trade between two countries. For a given good, the importing government sets a tariff, and governments negotiate a trade agreement to maximize their expected joint welfare. A trade agreement defines a set of permissible import tariffs and associated money burning levels, where we may think of money burning in this context as any wasteful bureaucratic procedures that a government must follow in the course of selecting certain tariffs. After the trade agreement is formed and the delegation set is selected, the importing government privately observes the level of political pressure from its import-competing industry and then selects its preferred tariff from the set of permissible tariffs. We can capture this scenario as a delegation problem with money burning, in which the "principal's" objective is to maximize expected joint government welfare, the agent's objective is to maximize the welfare of the importing government for any given level of political pressure, and the state variable is the level of political pressure.

In this general context, we consider two settings in which a role for a trade agreement arises. The first setting is a standard model with perfect competition, in which the familiar "terms-of-trade" externality provides the rationale for a trade agreement between governments. The second setting follows the "new-trade" theory of intra-industry trade and features monopolistic competition. In the monopolistic competition model that we present, an import tariff switches expenditures away from foreign varieties and toward domestic varieties, generating a negative international ("profit-shifting") externality even though the import tariff does not alter the terms of trade. In both settings, we assume an additively separable utility function and the existence of

[8]Other papers obtain the optimality of interval (or pooling) allocations in different settings. For example, see Athey, Atkeson, and Kehoe (2005) in the context of a monetary policy game and Athey, Bagwell, and Sanchirico (2004), Athey and Bagwell (2008), and McAfee and McMillan (1992) in the context of collusion. Since we do not consider additional expectational constraints or allow for multiple agents, our results do not directly apply to these papers.

an "outside" or numeraire good that is produced in both countries and freely traded. Both settings have received significant attention in the literature on trade agreements.[9]

For both settings, we establish conditions under which an optimal trade agreement does not employ money burning and takes the form of a tariff cap. Our findings thus provide an interpretation of a fundamental design feature of the GATT/WTO trade agreement, whereby governments negotiate "tariff bindings" or "bound tariff levels" rather than precise tariffs. As the World Trade Report 2009 (World Trade Organization (2009, p. 105)) states, the "*concept of a tariff binding—i.e., committing not to increase a duty beyond an agreed level—is at the heart of the multilateral trading system.*"[10] A tariff binding is simply a tariff cap. Our analysis also provides an interpretation of "binding overhang," whereby a WTO member government applies a tariff that falls below its negotiated bound level. While the pattern of binding overhang varies across products and countries, the World Trade Organization (2009, p. xix) notes that "*binding overhangs are a prominent feature of the WTO commitments of most members.*"[11] In our model, a government that incurs high political pressure applies a tariff that equals the cap, but a government applies a tariff below the cap when its political pressure is sufficiently low. Our analysis thus indicates conditions under which binding overhang occurs with positive probability in an optimal trade agreement. Finally, we note that our assumption that contingent transfers are unavailable can be motivated in the trade agreement setting, since explicit monetary transfers between governments are not required by WTO rules and are rarely used in WTO dispute resolutions.

In particular, using our third proposition, we consider both perfect and monopolistic competition settings, and provide simple conditions under which an optimal trade agreement takes the form of a tariff cap. For the setting with perfect competition, we consider further two particular specifications. The first

[9]For related models with perfect competition, see, for example, Bagwell and Staiger (2005), Feenstra and Lewis (1991), Grossman and Helpman (1995), Helpman and Krugman (1989, Chap. 2), and Horn, Maggi, and Staiger (2010). Related models with monopolistic competition were analyzed by Chang (2005), DeRemer (2012), Flam and Helpman (1987), Helpman and Krugman (1989, Chap. 7), and Ossa (2012), for example. See Bagwell and Staiger (2012) for discussion of the rationale for a trade agreement in imperfectly competitive markets when import and export policies are available.

[10]In GATT and now the WTO, market access commitments are achieved through tariff bindings. GATT Article II.1(a) states "*each contracting party shall accord to the commerce of the other contracting parties treatment no less favorable than that provided for in the appropriate Part of the appropriate Schedule annexed to this Agreement.*" In GATT parlance, a contracting party is a country and the treatment provided for in the schedule of concessions is the bound tariff.

[11]Work by Bouet and Laborde (2010) suggests that binding overhang is also quantitatively significant. Using a CGE model, they concluded that world trade would fall by 9.9% in a scenario where the applied tariffs of major economies were raised to bound rates. For recent empirical analyses of the pattern of binding overhang, see Bacchetta and Piermartini (2011) and Beshkar, Bond, and Rho (2011).

specification is a linear-quadratic model of trade. For a broad range of parameter values, we show that a tariff cap is optimal under this specification if the density function that determines political pressure is nondecreasing and also under a condition that allows for decreasing densities. The second specification is an endowment model with log utility. We establish related conditions under which a tariff cap is optimal for this specification. We also confirm with this specification that our approach can handle nonquadratic preferences. This latter point is confirmed with even greater force in the setting with monopolistic competition, where consumer demand exhibits constant elasticity of substitution across different varieties. We identify here a range of values for the elasticity of substitution such that the optimality of the tariff cap can again be easily confirmed if the density is nondecreasing.

An important feature of both settings is that the Lagrangian may be concave in the action only when evaluated at appropriate multipliers. In addition to providing new insights about the optimality of tariff caps and the phenomenon of binding overhang, our trade application thus also serves to illustrate the enhanced generality that our approach affords.

Finally, we also show how our analysis can be extended to include the possibility of imperfect ("leaky bucket") contingent transfers. For the trade settings that we consider, we establish that the optimality of a tariff cap is robust to the possibility of transfers that are sufficiently inefficient; however, if a transfer instrument is available that is sufficiently efficient, governments can improve on a simple tariff cap.

Our work relates to two main literatures. The first is the literature on optimal delegation. We establish that important characterizations of optimal delegation in previous work can be captured as special cases of our findings. In particular, Alonso and Matouschek (2008) analyzed the optimal delegation problem when money burning is not allowed. They considered a setting with quadratic welfare functions and provided necessary and sufficient conditions for interval delegation to be optimal.[12] Their welfare functions are included in the family of welfare functions for which we provide necessary and sufficient conditions for the optimality of interval delegation. Likewise, we show that the minimum savings result of Amador, Werning, and Angeletos (2006) can be captured as a special case of our first proposition when money burning is allowed. Finally, our sufficient conditions for interval delegation when money burning is permitted include the sufficient conditions identified by Ambrus and Egorov (2009) for the special case of quadratic welfare functions and a uniform distribution for the state of nature.

The second literature addresses the economic theory of trade agreements.[13] An extensive set of research considers the purpose and design of the WTO,

[12]Alonso and Matouschek (2008) also characterized the value of delegation, provided associated comparative statics results, and obtained a characterization when interval delegation is not optimal.

[13]See Bagwell and Staiger (2010) for a recent survey of this literature.

but the trade-agreement literature has only recently addressed the economics of tariff caps (i.e., bindings) and the associated possibility of binding overhang. In the emerging theory literature that considers tariff bindings and binding overhang, our paper relates most closely to Bagwell and Staiger (2005). Relative to this paper, our paper contributes in three main respects. First, Bagwell and Staiger (2005) characterized the optimal tariff cap; however, they did not establish conditions under which an optimal trade agreement takes the form of a tariff cap. Second, Bagwell and Staiger (2005) analyzed a linear-quadratic model with perfect competition, whereas we consider this model as a special case in the setting with perfect competition and consider, as well, a setting with monopolistic competition.[14] Third, we consider a multidimensional policy space, in which a trade agreement can specify tariffs as well as money burning (i.e., wasteful bureaucratic procedures).[15]

The paper is organized as follows. The basic model is presented in Section 2. In Section 3, we present sufficient and also necessary conditions for interval delegation to solve the delegation problem without and with money burning, respectively. Our tariff-cap application is found in Section 4. In Section 5, we discuss in more detail the relationship between our findings and those of Alonso and Matouschek (2008), Amador, Werning, and Angeletos (2006), and Ambrus and Egorov (2009). Section 6 concludes. The Appendix contains several proofs. Additional details and proofs are found in the Supplemental Material (Amador and Bagwell (2013)).

2. BASIC SETUP

We consider a setting with a principal and an agent. The principal has a welfare function given by $w(\gamma, \pi) - t$, while the agent has a welfare function given by $\gamma\pi + b(\pi) - t$. The value of π represents an action or allocation, and the value of γ represents a state or shock that is private information to the agent. The value of t represents an action that reduces everyone's utility. We thus generalize the standard delegation problem to include the possibility of money burning.

[14]See also Bagwell (2009), who considered the linear-quadratic model when political pressures can take only two types. He found that optimal delegation then does not take the form of a tariff cap. Our work also provides a foundation for recent work by Amador and Bagwell (2012) and Beshkar, Bond, and Rho (2011), as we discuss further in Section 4.1.2.

[15]Tariff bindings and binding overhang have also received attention in other modeling frameworks. In a model with contracting costs, Horn, Maggi, and Staiger (2010) established that binding overhang can occur. Maggi and Rodríguez-Clare (2007) analyzed a model in which applied tariffs are set at bound levels in equilibrium, and yet the potential to apply a tariff below the bound level induces ex post lobbying that mitigates an ex ante problem of over-investment. Our work is also related to work by Feenstra and Lewis (1991). A key difference is that Feenstra and Lewis (1991) allowed for monetary transfers between governments, whereas we do not allow for (perfect) contingent transfers.

We assume that γ has a continuous distribution F with bounded support $\Gamma = [\underline{\gamma}, \overline{\gamma}]$ and with an associated continuous and strictly positive density f. The action π is chosen from a set Π which is an interval of the real line with nonempty interior. We assume, without loss of generality, that $\inf \Pi = 0$, and define $\overline{\pi}$ to be in the extended reals and such that $\overline{\pi} = \sup \Pi$. For the remainder of the paper, we impose the following conditions on the primitives:

ASSUMPTION 1: *The following hold*: (i) *the function* $w : \Gamma \times \Pi \to \mathbb{R}$ *is continuous on* $\Gamma \times \Pi$; (ii) *for any* $\gamma_0 \in \Gamma$, *the function* $w(\gamma_0, \cdot)$ *is concave on* Π, *and twice differentiable on* $(0, \overline{\pi})$; (iii) *the function* $b : \Pi \to \mathbb{R}$ *is strictly concave on* Π, *and twice differentiable on* $(0, \overline{\pi})$; (iv) *there exists a twice differentiable function* $\pi_f : \Gamma \to (0, \overline{\pi})$ *such that*, *for all* $\gamma_0 \in \Gamma$, $\pi_f'(\gamma_0) > 0$ *and* $\pi_f(\gamma_0) \in \arg\max_{\pi \in \Pi}\{\gamma_0 \pi + b(\pi)\}$; *and* (v) *the function* $w_\pi : \Gamma \times (0, \overline{\pi}) \to \mathbb{R}$ *is continuous on* $\Gamma \times (0, \overline{\pi})$, *where* w_π *denotes the derivative of* w *in its second argument*.

Note that the function π_f indicates the agent's preferred, or flexible, action for any γ.

An allocation is a pair of functions $(\boldsymbol{\pi}, \mathbf{t})$, with $\boldsymbol{\pi} : \Gamma \to \Pi$ and $\mathbf{t} : \Gamma \to \mathbb{R}$, that represents the action and the amount of money burned as a function of the private information. The goal is simply to choose an allocation $(\boldsymbol{\pi}, \mathbf{t})$ so as to maximize the principal's welfare function:

$$
\text{(P)} \qquad \max \int_\Gamma \bigl(w\bigl(\gamma, \boldsymbol{\pi}(\gamma)\bigr) - \mathbf{t}(\gamma)\bigr)\, dF(\gamma) \quad \text{subject to:}
$$

$$
\gamma \in \arg\max_{\tilde{\gamma} \in \Gamma} \bigl\{\gamma \boldsymbol{\pi}(\tilde{\gamma}) + b\bigl(\boldsymbol{\pi}(\tilde{\gamma})\bigr) - \mathbf{t}(\tilde{\gamma})\bigr\}, \quad \text{for all } \gamma \in \Gamma,
$$

$$
\mathbf{t}(\gamma) \geq 0;\ \forall \gamma \in \Gamma,
$$

where the first constraint is an incentive-compatibility constraint that arises since the agent is privately informed of the value of γ.[16]

We will also consider the problem where money burning is ruled out by assumption, that is, where we impose on the above problem the additional constraint

$$
(1) \qquad \mathbf{t}(\gamma) = 0, \quad \forall \gamma \in \Gamma.
$$

Whether or not money burning is allowed, an important feature of the delegation problem defined above is that contingent transfers between the principal and agent are not allowed. This feature has significant implications for the set of incentive-compatible allocations. In the problem without money burning, for example, an incentive-compatible allocation can be strictly increasing and

[16]All integrals used in the paper are Lebesgue integrals.

continuous over an interval of values for γ only if the allocation over that interval is given by the agent's flexible allocation function, π_f. Incentive-compatible allocations may also exhibit jump discontinuities from a lower step to a higher step, where a step is a segment over which agent types are pooled (i.e., the allocation is independent of γ along a step).[17] To characterize an optimal allocation in this setting, we adopt a "guess-and-verify" approach, in which our guess is the optimal interval allocation and our verification process uses Lagrangian methods as described below.

3. OPTIMALITY OF INTERVAL DELEGATION

In this section, we define and characterize interval allocations. We then provide sufficient and necessary conditions for a solution to the problems stated in the previous section to be an interval allocation. We conclude the section with a discussion of the method of proof for our sufficiency and necessity propositions.

3.1. *Interval Allocation*

We begin with a definition of an interval allocation.

DEFINITION 1: An allocation $(\boldsymbol{\pi}, \mathbf{t})$ is an *interval allocation with bounds* a, b if $a, b \in \Gamma$; $a < b$; $\mathbf{t}(\gamma) = 0$ for all $\gamma \in \Gamma$; and

$$\boldsymbol{\pi}(\gamma) = \begin{cases} \pi_f(a); & \gamma \in [\underline{\gamma}, a], \\ \pi_f(\gamma); & \gamma \in (a, b), \\ \pi_f(b); & \gamma \in [b, \overline{\gamma}]. \end{cases}$$

Thus, when the principal utilizes an interval allocation, an agent that observes an intermediate value for γ can exercise flexibility and select the agent's preferred choice, $\pi_f(\gamma)$. An agent that observes a sufficiently high (low) value for γ then selects the highest (lowest) permissible action. An interval allocation satisfies the constraints of the problems stated in the previous section, since it is incentive compatible and burns no money. Note that an interval allocation takes the form of a *cap* if $a = \underline{\gamma}$.

We next characterize the interval allocation that is optimal within the class of interval allocations.

LEMMA 1—Optimal Interval: *The interval allocation with bounds* γ_L, γ_H *is optimal within the class of interval allocations only if the following conditions hold*:

[17]See Melumad and Shibano (1991) for a characterization of incentive-compatible allocations under quadratic preferences when money burning is not allowed.

(i) *if* $\gamma_H = \overline{\gamma}$, *then* $w_\pi(\overline{\gamma}, \pi_f(\overline{\gamma})) \geq 0$;
(ii) *if* $\gamma_H < \overline{\gamma}$, *then* $\int_{\gamma_H}^{\overline{\gamma}} w_\pi(\gamma, \pi_f(\gamma_H)) f(\gamma)\, d\gamma = 0$;
(iii) *if* $\gamma_L = \underline{\gamma}$, *then* $w_\pi(\underline{\gamma}, \pi_f(\underline{\gamma})) \leq 0$;
(iv) *if* $\gamma_L > \underline{\gamma}$, *then* $\int_{\underline{\gamma}}^{\gamma_L} w_\pi(\gamma, \pi_f(\gamma_L)) f(\gamma)\, d\gamma = 0$.

The proof appears in the Appendix.

To understand this lemma, observe that $w_\pi(\gamma, \pi_f(\gamma))$ indicates the direction of the bias of the agent with type γ. For example, when $w_\pi(\gamma, \pi_f(\gamma)) > 0$ for some γ, the principal would prefer a higher action than the one most preferred by the agent. Conditions (i) and (iii) imply that if agent types are not pooled at the extremes, it must be because the principal prefers an even more extreme action. Conditions (ii) and (iv) show that if agents are pooled at the extremes, then the interior boundary of any such pooling region must be such that the average bias among the pooled agents is zero (i.e., the action is efficient on average).

3.2. *Results*

We proceed next to determine conditions under which an interval allocation solves Problem (P), with and without the additional restriction (1). As we discuss in further detail below, the relative concavity of the principal's and agent's welfare functions is a key consideration in determining the optimality of an interval allocation. In particular, the following constant κ is helpful when stating the conditions for the optimality of an interval allocation.

DEFINITION 2: For the problem without money burning, that is, Problem (P) with the additional restriction (1), define κ to be

$$\kappa = \inf_{(\gamma,\pi)\in\Gamma\times\Pi} \left\{ \frac{w_{\pi\pi}(\gamma,\pi)}{b''(\pi)} \right\}. \tag{2}$$

For the problem with money burning, that is, Problem (P), define κ to be

$$\kappa = \min\left\{ \inf_{(\gamma,\pi)\in\Gamma\times\Pi} \left\{ \frac{w_{\pi\pi}(\gamma,\pi)}{b''(\pi)} \right\}, 1 \right\}. \tag{3}$$

For a given interval allocation with bounds γ_L, γ_H, consider the following conditions:

(c1) $\kappa F(\gamma) - w_\pi(\gamma, \pi_f(\gamma)) f(\gamma)$ is nondecreasing for all $\gamma \in [\gamma_L, \gamma_H]$.
(c2) If $\gamma_H < \overline{\gamma}$,

$$(\gamma - \gamma_H)\kappa \geq \int_\gamma^{\overline{\gamma}} w_\pi\big(\tilde{\gamma}, \pi_f(\gamma_H)\big) \frac{f(\tilde{\gamma})}{1 - F(\gamma)}\, d\tilde{\gamma}, \quad \forall \gamma \in [\gamma_H, \overline{\gamma}]$$

with equality at γ_H.

(c2′) If $\gamma_H = \overline{\gamma}$, $w_\pi(\overline{\gamma}, \pi_f(\overline{\gamma})) \geq 0$.

(c3) If $\gamma_L > \underline{\gamma}$,

$$(\gamma - \gamma_L)\kappa \leq \int_{\underline{\gamma}}^{\gamma} w_\pi\big(\tilde{\gamma}, \pi_f(\gamma_L)\big)\frac{f(\tilde{\gamma})}{F(\gamma)}\,d\tilde{\gamma}, \quad \forall\gamma \in [\underline{\gamma}, \gamma_L]$$

with equality at γ_L.

(c3′) If $\gamma_L = \underline{\gamma}$, $w_\pi(\underline{\gamma}, \pi_f(\underline{\gamma})) \leq 0$.

Note that conditions (c2′) and (c3′) correspond to conditions (i) and (iii), respectively, in Lemma 1; likewise, the equality requirements in conditions (c2) and (c3) correspond to conditions (ii) and (iv) in Lemma 1.

The next proposition shows that conditions (c1), (c2), (c2′), (c3), and (c3′) are sufficient for the optimality of an interval allocation (with and without money burning).

PROPOSITION 1—Sufficiency: *Consider $\gamma_L, \gamma_H \in \Gamma$ with $\gamma_L < \gamma_H$.*

(a) (No money burning) *If conditions* (c1), (c2), (c2′), (c3), *and* (c3′) *are satisfied with κ given by equation* (2), *then the interval allocation with bounds γ_L, γ_H solves the problem without money burning*, *that is*, *Problem* (P) *with the additional constraint* (1).

(b) (Money burning) *If conditions* (c1), (c2), (c2′), (c3), *and* (c3′) *are satisfied with κ given by equation* (3), *then the interval allocation with bounds γ_L, γ_H solves Problem* (P).

The proof appears in the Appendix.

Proposition 1 requires knowledge of the value of κ to be able to determine whether conditions (c1)–(c3′) hold. In some settings, however, it may be easier to determine bounds for κ. By inspection, conditions (c1)–(c3′) hold if they hold when κ is replaced by a lower value. Thus, the results of Proposition 1 remain the same if κ is replaced by a lower bound.

The next proposition characterizes situations where the conditions are also necessary.

PROPOSITION 2—Necessity: *Consider $\gamma_L, \gamma_H \in \Gamma$ with $\gamma_L < \gamma_H$.*

(a) (No money burning) *If $w(\gamma, \pi) = A[b(\pi) + B(\gamma) + C(\gamma)\pi]$ and f is differentiable*, *then conditions* (c1), (c2), (c2′), (c3), *and* (c3′) *with κ as given by equation* (2) *are necessary for the interval allocation with bounds γ_L, γ_H to solve the problem without money burning*, *that is*, *Problem* (P) *with the additional constraint* (1).

(b) (Money burning) *Let κ be given by equation* (3). *If either* (i) $\kappa = 1$, *or* (ii) $w(\gamma, \pi) = A[b(\pi) + B(\gamma) + C(\gamma)\pi]$ *and f is differentiable*, *then conditions* (c1), (c2), (c2′), (c3), *and* (c3′) *are necessary for the interval allocation with bounds γ_L, γ_H to solve Problem* (P).

The proof appears in the Appendix.

The following proposition specializes Proposition 1 to an important case in which the bias in the agent's preferences (i.e., the difference between the agent's utility and the principal's) is not directly affected by the agent's private information and always leads the principal to prefer an action that is lower than the action that the agent would select under flexibility (i.e., for given γ, the principal's preferred choice is below $\pi_f(\gamma)$).

PROPOSITION 3: *Assume that* (i) $w(\gamma, \pi) = v(\pi) + b(\pi) + \gamma\pi$ *for some function* $v : \Pi \to \mathbb{R}$; (ii) $\kappa \geq 1/2$ *with* κ *defined as in equation* (2) (*or alternatively*, *as in equation* (3)); *and* (iii) *there exists* $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$ *such that*

$$v'\big(\pi_f(\gamma_H)\big) + \mathbb{E}[\gamma|\gamma \geq \gamma_H] - \gamma_H = 0, \tag{4}$$

and $v'(\pi_f(\gamma)) \leq 0$ *for all* $\gamma \in [\underline{\gamma}, \gamma_H]$. *Then*, *for* f *nondecreasing*, *the interval allocation with bounds* $\underline{\gamma}$, γ_H *solves Problem* (P) *with the additional constraint* (1) (*or alternatively*, *solves Problem* (P)).

The proof appears in the Appendix.

Proposition 3 states that, to check whether an interval allocation in the form of a cap with $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$ is optimal when $w(\gamma, \pi) = v(\pi) + b(\pi) + \gamma\pi$, it suffices to check that $\kappa \geq 1/2$ and that the density is nondecreasing. Note that this monotonicity restriction on the density can be weakened if we have more knowledge of the payoff functions b and w.[18] Note also that if $v'(\pi_f(\gamma)) < 0$ for all $\gamma \in \Gamma$, then $v'(\pi_f(\underline{\gamma})) + \mathbb{E}[\gamma] - \underline{\gamma} > 0$ is sufficient for condition (iii) of the proposition.[19]

In Section 4, we show that Proposition 3 can be used to characterize the optimal trade agreement in standard trade models with perfect competition and monopolistic competition, respectively.

3.3. *A Discussion of the Results*

3.3.1. *The Sufficiency Proposition*

In what follows, we briefly describe the method of proof used for Proposition 1.

Consider first part (a) of Proposition 1, the problem without money burning. By writing the incentive constraints in their usual integral form plus a monotonicity restriction, we can rewrite Problem (P) with the additional constraint

[18] We illustrate this point in our trade-agreement application. See the discussion following Corollary 2.

[19] It is also possible to write a version of Proposition 3 that refers to the optimality of a floor allocation (rather than a cap) for the case where $v'(\pi_f(\gamma)) \geq 0$ for all $\gamma \in [\gamma_L, \overline{\gamma}]$. In that case, the sufficient condition is f nonincreasing.

(1) as

$$\max \int w(\gamma, \boldsymbol{\pi}(\gamma))\, dF(\gamma) \quad \text{subject to:} \tag{5}$$

$$\gamma\boldsymbol{\pi}(\gamma) + b(\boldsymbol{\pi}(\gamma)) = \int_{\underline{\gamma}}^{\gamma} \boldsymbol{\pi}(\tilde{\gamma})\, d\tilde{\gamma} + \underline{U}, \quad \text{for all } \gamma \in \Gamma, \tag{6}$$

$$\boldsymbol{\pi} \text{ nondecreasing}, \tag{7}$$

where $\underline{U} \equiv \underline{\gamma}\boldsymbol{\pi}(\underline{\gamma}) + b(\boldsymbol{\pi}(\underline{\gamma}))$, and where we use that $\mathbf{t}(\gamma) = 0$ for all γ.[20]

To prove part (a) of Proposition 1, we follow and extend the Lagrangian approach used by Amador, Werning, and Angeletos (2006). Differently from that paper, here we have to deal with a (possible) failure of concavity of the Lagrangian (which we discuss below), together with a continuum of equality constraints as captured in (6). We embed the monotonicity constraint (7) into the choice set of $\boldsymbol{\pi}$, and we write constraints (6) as two inequalities:

$$\int_{\underline{\gamma}}^{\gamma} \boldsymbol{\pi}(\tilde{\gamma})\, d\tilde{\gamma} + \underline{U} - \gamma\boldsymbol{\pi}(\gamma) - b(\boldsymbol{\pi}(\gamma)) \leq 0 \quad \text{for all } \gamma \in \Gamma, \tag{8}$$

$$-\int_{\underline{\gamma}}^{\gamma} \boldsymbol{\pi}(\tilde{\gamma})\, d\tilde{\gamma} - \underline{U} + \gamma\boldsymbol{\pi}(\gamma) + b(\boldsymbol{\pi}(\gamma)) \leq 0 \quad \text{for all } \gamma \in \Gamma. \tag{9}$$

The problem is then to choose a function $\boldsymbol{\pi} \in \Phi$ so as to maximize (5) subject to (8) and (9) and where the choice set is given by $\Phi \equiv \{\boldsymbol{\pi} | \boldsymbol{\pi} : \Gamma \to \Pi \text{ and } \boldsymbol{\pi} \text{ nondecreasing}\}$.

For the rest of the proof, we use Theorem 1 in Appendix B, which relies on a Lagrangian method. Basically, given an interval allocation, we construct Lagrange multiplier functions associated with constraints (8) and (9) that satisfy complementary slackness and are such that the proposed interval allocation maximizes the resulting Lagrangian over the choice set Φ. As usual, to check whether an allocation maximizes the Lagrangian, first-order conditions are particularly useful. Building on Amador, Werning, and Angeletos (2006), we are able to express the first-order conditions for maximizing the Lagrangian over the set of nondecreasing functions, Φ. A novel feature of our problem, however, is that the Lagrangian is not necessarily concave in $\boldsymbol{\pi}$. The first-order conditions are thus not, in general, sufficient for optimality. The key is to note

[20]In contrast to the standard principal-agent problem with a privately informed agent, a distinctive feature of our Problem (P) with the additional constraint (1) is that we do not allow for contingent transfers from the agent to the principal (or vice versa). The standard approach is to substitute constraint (6) into the objective (5), maximize the resulting expression, and check whether the solution is monotonic. This approach is infeasible here, since we do not have available a transfer function with which to ensure that the resulting solution satisfies the incentive-compatibility constraint (6).

that Theorem 1 requires that the optimal allocation maximizes the Lagrangian at some specific (and valid) Lagrangian multipliers. Hence, our objective is to explicitly construct Lagrange multipliers such that the resulting Lagrangian is concave in $\boldsymbol{\pi}$ *and* the first-order conditions are satisfied at the proposed interval allocation. We show in the Appendix that this objective can be achieved under the conditions stated in part (a) of Proposition 1.

Consider now part (b) of Proposition 1, the problem with money burning. Using the integral form for the incentive constraints, Problem (P) can be rewritten as

$$\max \int \big(w\big(\gamma, \boldsymbol{\pi}(\gamma)\big) - \mathbf{t}(\gamma)\big)\, dF(\gamma) \quad \text{subject to:}$$

$$\gamma\boldsymbol{\pi}(\gamma) + b\big(\boldsymbol{\pi}(\gamma)\big) - \mathbf{t}(\gamma) = \int_{\underline{\gamma}}^{\gamma} \boldsymbol{\pi}(\tilde{\gamma})\, d\tilde{\gamma} + \underline{U}, \quad \text{for all } \gamma \in \Gamma,$$

$$\boldsymbol{\pi} \text{ nondecreasing, and } \mathbf{t}(\gamma) \geq 0, \text{ for all } \gamma \in \Gamma,$$

where $\underline{U} \equiv \underline{\gamma}\boldsymbol{\pi}(\underline{\gamma}) + b(\boldsymbol{\pi}(\underline{\gamma})) - \mathbf{t}(\underline{\gamma})$.

Solving the integral equation for $\mathbf{t}(\gamma)$ and substituting into both the objective and the nonnegativity constraint, we obtain the following equivalent problem:

$$\max \int \big(v\big(\gamma, \boldsymbol{\pi}(\gamma)\big)f(\gamma) + \big(1 - F(\gamma)\big)\boldsymbol{\pi}(\gamma)\big)\, d\gamma + \underline{U} \quad \text{subject to:} \tag{10}$$

$$\gamma\boldsymbol{\pi}(\gamma) + b\big(\boldsymbol{\pi}(\gamma)\big) - \int_{\underline{\gamma}}^{\gamma} \boldsymbol{\pi}(\tilde{\gamma})\, d\tilde{\gamma} - \underline{U} \geq 0; \quad \text{for all } \gamma \in \Gamma, \tag{11}$$

$$\boldsymbol{\pi} \text{ nondecreasing,} \tag{12}$$

where v is defined such that $v(\gamma, \boldsymbol{\pi}(\gamma)) \equiv w(\gamma, \boldsymbol{\pi}(\gamma)) - b(\boldsymbol{\pi}(\gamma)) - \gamma\boldsymbol{\pi}(\gamma)$.[21]

In comparison to the problem analyzed in part (a), this problem has two novel features. First, the objective function in (10) is not necessarily concave in $\boldsymbol{\pi}$, as we have not imposed any assumptions on the function v. The second difference is that constraint (11) entails only one inequality constraint, whereas constraints (8) and (9) together capture two inequality constraints. Nevertheless, the sufficiency proof proceeds in a similar fashion to the one in part (a). We embed the monotonicity restriction, constraint (12), into the choice set of the problem. Then we guess a Lagrange multiplier for constraint (11) and form a Lagrangian. Using the conditions of part (b) of Proposition 1, we then show that our constructed Lagrange multiplier is valid, the resulting Lagrangian is concave in $\boldsymbol{\pi}$, and the proposed interval allocation satisfies the first-order conditions for maximizing the Lagrangian over the set of nondecreasing functions.

[21]Note that once we have solved this program for $\boldsymbol{\pi}(\gamma)$ and $\mathbf{t}(\underline{\gamma})$, we can recover $\mathbf{t}(\gamma)$ via the integral equation above.

Appealing once more to Theorem 1 in Appendix B, we then conclude that the interval allocation is an optimal solution to the problem of maximizing (10) subject to (11) and (12).

3.3.2. *The Necessity Proposition*

We describe next the method of proof used in establishing Proposition 2.

When v is concave in π, the problem of maximizing (10) subject to (11) and (12) as stated above is a maximization problem with a concave objective function and a convex constraint set. We can then use standard Lagrangian techniques to characterize necessary conditions for a maximum and thereby strengthen our results to show that the sufficient conditions in this case are also necessary. This implies part (b) of Proposition 2 for $\kappa = 1$.

For the case without money burning, or when $\kappa < 1$ in the case with money burning, we cannot appeal to Lagrangian techniques to show that sufficient conditions are also necessary. In the latter case, the problem is that the objective function of (10) is not concave. In the former case, the difficulty is that (6) represents a continuum of equality constraints, which violates an interiority requirement.

To establish the necessity proposition for these cases, we use two sets of necessary conditions that arise from simple perturbations. The first set is associated with the interval (γ_L, γ_H) over which the allocation exhibits flexibility, while the second set applies to the intervals $[\underline{\gamma}, \gamma_L]$ and $[\gamma_H, \overline{\gamma}]$ over which the allocation entails pooling. Using these necessary conditions, we are able to identify a family of preferences for which conditions (c1), (c2), (c2′), (c3), and (c3′) are also necessary for the optimality of an interval allocation. As we confirm in Section 5, this class of preferences includes the standard quadratic preferences used in the delegation literature and the preferences studied by Amador, Werning, and Angeletos (2006).

The first perturbation is to remove a vanishing interval of choices within the flexibility region and check that the resulting change in welfare is nonpositive. Proceeding in this way, and imposing that f is differentiable, we show that an interval allocation is optimal only if

$$\left(\frac{w_{\pi\pi}(\gamma, \pi_f(\gamma))}{b''(\pi_f(\gamma))}\right) f(\gamma) - \frac{d}{d\gamma}\big[w_\pi\big(\gamma, \pi_f(\gamma)\big) f(\gamma)\big] \geq 0, \tag{13}$$

for all $\gamma \in [\gamma_L, \gamma_H]$.

We can illustrate this condition by considering the case in which the agent's bias is strictly positive in that $w_\pi(\gamma, \pi_f(\gamma)) < 0$. Over the region of flexibility, if we undertake a small perturbation in which the flexible actions are removed for types γ to $\gamma + \varepsilon > \gamma$, then there would be an indifferent type $\gamma(\varepsilon)$ such that types between γ and $\gamma(\varepsilon)$ select $\pi_f(\gamma)$ and types between $\gamma(\varepsilon)$ and $\gamma + \varepsilon$ select $\pi_f(\gamma + \varepsilon)$. First, notice that this perturbation induces types between γ and $\gamma(\varepsilon)$ to make a less-biased choice, while types between $\gamma(\varepsilon)$ and $\gamma + \varepsilon$

make a more-biased choice. As suggested by the second term in the left hand side of inequality (13), the perturbation is less likely to offer an improvement if the density and/or the bias is greater for higher types. Second, notice that the perturbation increases the variance of the allocation relative to the flexible allocation. Since $\pi_f'(\gamma) = -1/b''(\pi_f(\gamma))$, this variance effect is captured by the first term in the left hand side of inequality (13). As suggested by this term, the perturbation is less likely to offer an improvement when the concavity of the principal's welfare relative to the concavity of the agent's welfare is greater.

Now note that we can write (13) as

$$\left(\frac{w_{\pi\pi}(\gamma, \pi_f(\gamma))}{b''(\pi_f(\gamma))} - \kappa\right) f(\gamma) + \frac{d}{d\gamma}\big[\kappa F(\gamma) - w_\pi\big(\gamma, \pi_f(\gamma)\big) f(\gamma)\big] \geq 0, \tag{14}$$

where we know that

$$\kappa \leq \inf_{(\gamma,\pi)\in\Gamma\times\Pi} \left\{ \frac{w_{\pi\pi}(\gamma, \pi)}{b''(\pi)} \right\}.$$

It follows that the first term of (14) is nonnegative, and thus the necessary condition is weaker than condition (c1), as expected. For a family of preferences with the property that $\frac{w_{\pi\pi}(\gamma,\pi)}{b''(\pi)}$ is constant for all γ and π, however, the necessary condition (14) coincides with condition (c1) when money burning is not allowed or when $\kappa < 1$. The necessity proposition identifies a family of preferences for which this property holds.

The second perturbation is to take a pooling region (above γ_H or below γ_L) and offer a new choice of π that is attractive to some types lying within this region. The necessary conditions are obtained by checking that such a perturbation does not increase the principal's welfare. Arguing in this way, and for the family of preferences specified in the necessity proposition, we can establish that conditions (c2), (c2′), (c3), and (c3′) are also necessary.

4. APPLICATION TO TRADE POLICY

We now apply our findings and characterize an optimal trade agreement between governments with private political pressures. We assume that a trade agreement identifies a menu of permissible tariffs and is negotiated before private political pressures are realized. After a government learns its private information, it then applies its preferred tariff from the permissible set. An optimal trade agreement maximizes ex ante joint government welfare subject to incentive-compatibility constraints.

We consider two general settings in which a role for a trade agreement arises. The first setting is a standard two-country model with perfect competition. In this model, a trade agreement can generate mutual gains for governments, due to the familiar "terms-of-trade" externality. Following the "new-trade" theory of intra-industry trade, the second model captures a two-country setting with monopolistic competition. Building on Helpman and Krugman (1989), we consider a setting in which an import tariff switches expenditures toward domestic varieties, generating a negative international externality even though the import tariff does not alter the terms of trade. As noted in the Introduction, both settings have received significant attention in the literature on trade agreements. We consider both models and show that our propositions above can be successfully used to provide general conditions under which an optimal trade agreement takes the form of a tariff cap.

In particular, we show below that, in both settings, the problem of designing an optimal trade agreement can be represented as a special case of the general delegation problem discussed in Section 2, where $w(\gamma, \pi) = v(\pi) + b(\pi) + \gamma\pi$ and $v'(\pi) < 0$ for all $\pi \in [0, \overline{\pi})$. In other words, whether competition is perfect or monopolistic, we show that the trade model can be represented as a delegation problem in which the agent's choice of an action π generates a negative externality through a function v that is independent of the agent's private information. Proposition 3 can then be immediately used to provide a simple sufficient condition for the optimality of tariff caps.

In this way, for both perfect and monopolistic competition settings, we provide an interpretation for the central role of tariff caps in GATT/WTO rules and negotiations. Our findings also provide an interpretation for the phenomenon of binding overhang, since they indicate for both settings that applied tariffs are strictly below bound tariffs with positive probability in an optimal trade agreement. We conclude the section by showing that our main findings continue to hold when the model is extended to allow for imperfect ("leaky bucket") contingent transfers, provided that such transfers are sufficiently inefficient.

4.1. *Optimal Agreement Under Perfect Competition*

We first study the optimality of tariff caps in a two-country model with perfect competition. After presenting results for a general representation of this trade model, we develop further findings by focusing on two particular specifications: the linear-quadratic model analyzed by Bagwell and Staiger (2005) and an endowment model with log utility.

4.1.1. *Mapping Into Our Modeling Framework*

In our Supplemental Material (Amador and Bagwell (2013)), we describe the trade model in more detail. In what follows, we present the basic setup and results that allow us to write the trade agreement problem as a delegation problem. There are two countries, home and foreign, and two goods, x and n, where n is a numeraire. The home country imports good x, and we look for the optimal trade agreement for this good. We assume that consumers in both countries have a symmetric utility function that is quasi-linear in the numeraire: $u(c^x) + c^n$, where c^x and c^n represent the amounts consumed of goods x and n, respectively. We assume that u is strictly increasing, strictly concave, and thrice continuously differentiable. Letting p denote the home relative price of x to n, with p_* representing the relative price in the foreign country, we assume that there are competitive supply functions of good x in both countries, which we represent as $Q(p)$ and $Q_*(p_*)$, respectively. For prices that induce strictly positive supply, we assume that these functions are strictly increasing and twice continuously differentiable. We also assume that $Q(p) < Q_*(p)$ for any p such that there is strictly positive world supply.[22]

As is standard, we assume that the numeraire is produced in each country under constant returns to scale using labor (the only factor), where the supply of labor is inelastic. The wage and the price of the numeraire may then be set at unity. The numeraire is freely traded across countries so as to ensure that trade is balanced.

We abstract from export policies and assume that the home country may use a specific (i.e., per unit) import tariff, τ, for good x. As we describe in further detail in our Supplemental Material, the market-clearing prices in the home and foreign countries are then determined as functions of τ by the requirements that the home country import volume for good x equals the foreign country export volume for this good and that $p = p_* + \tau$.

We may now represent the welfare functions of both governments. Let $\pi \in [0, \overline{\pi}]$ denote the producer surplus (profit) at home for good x that is induced by a given tariff, where $\overline{\pi}$ is the producer surplus that is obtained when trade is prohibited. We can write the home government's welfare as

$$\gamma\pi + b(\pi),$$

where b is the sum of consumer surplus and tariff revenue in the home country. The shock $\gamma \in \Gamma$ represents a political economy shock that determines the weight that the home government puts on the welfare of its (import-competing) producers. The welfare of the foreign government, $v(\pi)$, is determined as the sum of consumer surplus and producer surplus in the foreign country.[23]

When trade volume is positive, a higher import tariff raises p and lowers p_*, where the latter effect is the traditional terms-of-trade externality. A higher import tariff is then associated with a higher level of profit in the home country and a lower level of foreign welfare. We assume henceforth that trade volume

[22]In our Supplemental Material, we consider a slightly more general and symmetric trade model with three goods. The foreign country then imports a good y from the home country, where the supply assumptions on good y are the mirror image of those stated here for good x. Separability in the utility function, plus quasi-linearity, allows us to study the two good problem independently, as we do here.

[23]See our Supplemental Material for additional details.

is positive at tariffs that deliver $\pi \in [0, \overline{\pi})$, from which it follows that $v'(\pi) < 0$ for all $\pi \in [0, \overline{\pi})$. We assume that $b''(\pi) < 0$; however, as we show in our Supplemental Material, if $Q'' \leq 0$, $Q''_* \leq 0$ and $u''' \geq 0$, then $v''(\pi) > 0$ for $\pi \in [0, \overline{\pi})$. We are thus careful below not to exclude the possibility of a strictly convex foreign welfare function.

The home and foreign governments negotiate a trade agreement before the political economy parameter, γ, is realized. Thus, at the time of negotiation, the home government is uncertain about its future preferences. We assume that γ is distributed over the support $\Gamma \equiv [\underline{\gamma}, \overline{\gamma}]$ according to a strictly positive density $f(\gamma)$. We represent the c.d.f. as $F(\gamma)$. Once the value of γ is realized, the home government is privately informed of this value.[24]

We may imagine that a trade agreement allows a higher import tariff only if certain wasteful bureaucratic procedures are followed by the importing country. In addition, given that there is a one-to-one relationship between profit and tariff levels, we can, without loss of generality, represent the trade agreement as choosing an allocation over the levels of profit in the home country, π, rather than on tariff levels directly. Hence, we model a trade agreement as a pair, $(\boldsymbol{\pi}(\gamma), \mathbf{t}(\gamma))$, that determines, for each γ, the profit allocated to the domestic producers and the level of wasteful activities or money burning. We look for a trade agreement that is incentive compatible and maximizes expected joint government welfare. The optimal trade agreement solves the following problem:

$$\text{(PT)} \qquad \max_{\boldsymbol{\pi}(\gamma), \mathbf{t}(\gamma)} \left\{ \int_{\gamma \in \Gamma} \big(\gamma \boldsymbol{\pi}(\gamma) + b\big(\boldsymbol{\pi}(\gamma)\big) + v\big(\boldsymbol{\pi}(\gamma)\big) - \mathbf{t}(\gamma)\big)\, dF(\gamma) \right\}$$

$$\text{subject to:} \quad \gamma \in \arg\max_{\widetilde{\gamma} \in \Gamma} \big\{ \gamma \boldsymbol{\pi}(\widetilde{\gamma}) + b\big(\boldsymbol{\pi}(\widetilde{\gamma})\big) - \mathbf{t}(\widetilde{\gamma}) \big\}.$$

Once the optimal profit function is determined, we can easily back out the associated tariff function.[25]

[24]In the three-good model that we describe in our Supplemental Material, the negotiation also concerns the foreign import tariff and precedes the realization of the foreign political economy parameter, γ_*, that defines the weight that the foreign government attaches to the profit of its import-competing producers (of good y). When the value of γ_* is realized, the foreign government is privately informed of its value. We assume that γ^* and γ are independent and identically distributed.

[25]The statement of the problem reflects our assumptions that governments do not have available contingent side-payments (monetary transfers) and that they seek a trade agreement that maximizes the sum of their expected welfares. The solution generates a particular outcome on the efficiency frontier when side-payments are not allowed. In the three-good model described in our Supplemental Material, an analogous solution applies for good y, where the foreign government has private information about the weight that it attaches to its import-competing industry. If the instrument space is expanded so that governments can make noncontingent side-payments during their negotiation, and thus before they obtain private information, then all efficient payoffs can be achieved by solving program (PT) and specifying an appropriate ex ante transfer. Grossman and Helpman (1995) made a similar point in their analysis of "trade talks."

Note that Problem (PT) maps into our general framework (and, in particular, the case considered in Proposition 3) by letting $w(\gamma, \pi) = \gamma\pi + b(\pi) + v(\pi)$. Assumption 1 then requires that $b''(\pi) < 0$, $v''(\pi) + b''(\pi) \leq 0$, and $\pi_f(\overline{\gamma}) < \overline{\pi}$.

Our next step is to determine sufficient conditions for the optimal tariff-cap allocation to represent an optimal trade agreement. Intuitively, the home government does not take into account the effect of its actions on foreign welfare. This leads to an upward bias in the home government tariff decisions, and hence the agreement should not pool the types at the bottom of the distribution, as these types are already choosing tariffs that are too high (i.e., $\gamma_L = \underline{\gamma}$). Instead, we expect now that $\gamma_H < \overline{\gamma}$, which follows from $v'(\pi_f(\overline{\gamma})) < 0$.

Let the proposed allocation be $(\boldsymbol{\pi}^\star, \mathbf{t}^\star(\gamma) \equiv 0)$, where

$$(15) \qquad \boldsymbol{\pi}^\star(\gamma) = \begin{cases} \pi_f(\gamma), & \text{for } \gamma < \gamma_H, \\ \pi_f(\gamma_H), & \text{for } \gamma \geq \gamma_H, \end{cases}$$

and where γ_H is as in (4).

In the present setting, the value κ takes the form

$$(16) \qquad \kappa \equiv \min\left\{ \min_{\pi \in [0, \overline{\pi}]} \left\{ \frac{v''(\pi) + b''(\pi)}{b''(\pi)} \right\}, 1 \right\}.$$

Note that, under Assumption 1, if we were to assume that v is a (weakly) concave function of π, then $\kappa = 1$ would follow. Recall, however, that under reasonable circumstances, v may be strictly convex. As discussed in Section 3.3.1, this highlights one of the novel contributions of the methods developed in the paper: the ability to handle a nonconcave v. Notice also that κ falls as the convexity of v increases relative to the concavity of b, and that a smaller value of κ makes conditions (c1)–(c3′) harder to satisfy.

When Assumption 1 holds, we can now apply Proposition 3 to show that, if $\kappa \geq 1/2$, the density is nondecreasing, and $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$ solves (4), then a tariff cap is an optimal trade agreement; that is, the proposed allocation $(\boldsymbol{\pi}^\star, \mathbf{t}^\star)$ then solves Problem (PT). We summarize the above discussion in the following corollary, stated without proof.

COROLLARY 1: *In the perfect competition trade model*, *suppose that* (i) *Assumption* 1 *holds for* $w(\gamma, \pi) = \gamma\pi + b(\pi) + v(\pi)$; (ii) $\kappa \geq 1/2$ *with* κ *defined as in* (16); *and* (iii) $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$ *solves* (4). *Then*, *the tariff-cap allocation defined in* (15) *is optimal for any nondecreasing density* f.

4.1.2. *Two Particular Specifications*

To further explore the optimality of tariff caps, we now consider two particular specifications for the trade model. The first specification is the linear-quadratic model of trade that Bagwell and Staiger (2005) utilized. We show that tariff caps are optimal for this model when the density function is nondecreasing, and we also utilize the structure of the model to establish the optimality of tariff caps under a condition that allows for decreasing densities. The second specification considers an endowment model with log utility. We show that our findings apply here as well, and confirm thereby that our analysis may be usefully applied to specific settings without quadratic payoffs.

Linear-Quadratic Example. Following Bagwell and Staiger (2005), we now assume $u(c) = c - c^2/2$, $Q(p) = p/2$, and $Q_*(p_*) = p_*$. The flexible or Nash tariff, $\tau_f(\gamma)$, is the tariff that maximizes domestic government welfare, given the realized value of the political economy parameter, γ. For a given value of γ, the fully efficient (i.e., first best) tariff, $\tau_e(\gamma)$, is the tariff that maximizes the sum of home and foreign government welfare. For $\gamma \in [1, 7/4)$, the flexible and efficient tariff functions satisfy $\tau_f(\gamma) > \tau_e(\gamma)$.[26] Thus, for political economy parameters in this range, the flexible tariff is higher than efficient. Intuitively, when contemplating a higher tariff, the domestic government does not internalize the negative terms-of-trade externality that is experienced by the foreign government. When $\gamma = 7/4$, the domestic political economy parameter is so high that the efficient tariff eliminates all trade. The flexible and efficient tariffs then agree: $\tau_f(7/4) = 1/6 = \tau_e(7/4)$, where 1/6 is the prohibitive tariff that eliminates all trade.

We provide here conditions for the linear-quadratic model under which the optimal trade agreement is given by an optimal tariff cap. To this end, we assume that political shocks are distributed over $[\underline{\gamma}, \overline{\gamma}]$, where $1 \leq \underline{\gamma} < \overline{\gamma} < 7/4$. Letting π denote domestic profits as before, we can now explicitly represent the welfare functions as

$$b(\pi) = \frac{1}{2}(-1 + 9\sqrt{\pi} - 17\pi), \quad v(\pi) = \frac{1}{4}(2 - 6\sqrt{\pi} + 9\pi), \tag{17}$$

where $\overline{\pi} = 1/9$.[27]

We confirm next that Assumption 1 holds in the linear-quadratic model.

LEMMA 2: *Let $\Pi = [0, 1/9]$ and $\Gamma = [\underline{\gamma}, \overline{\gamma}]$, for $1 \leq \underline{\gamma} < \overline{\gamma} < 7/4$. Let $b : \Pi \to \mathbb{R}$ be defined as in* (17). *Let $w : \Gamma \times \Pi \to \mathbb{R}$ be defined as $w(\gamma, \pi) = \gamma\pi + b(\pi) + v(\pi)$, where v is as in* (17). *Then Assumption* 1 *holds.*

[26]In particular, $\tau_f(\gamma) = (8\gamma - 5)/[4(17 - 2\gamma)]$, which strictly exceeds $\tau_e(\gamma) = 4(\gamma - 1)/(25 - 4\gamma)$ for $\gamma \in [1, 7/4)$.

[27]In this example, $\overline{\pi} = 1/9$ is obtained when the prohibitive tariff of 1/6 is imposed. The expressions for $b(\pi)$, $v(\pi)$, and $\overline{\pi}$ are derived in our Supplemental Material.

To prove Lemma 2, we observe that the function $w(\gamma, \pi) = \gamma\pi + b(\pi) + v(\pi)$, for b and v as defined in (17), is continuous in both arguments. For any γ_0, the function $w(\gamma_0, \cdot)$ is concave on Π, and twice differentiable on $(0, 1/9)$. The function b is strictly concave and twice differentiable on $(0, 1/9)$. The corresponding flexible allocation is $\pi_f(\gamma) = (9/(34 - 4\gamma))^2$, which is interior and strictly increasing for $\gamma \in \Gamma$. Finally, $w_\pi(\gamma, \pi) = \frac{3}{2\sqrt{\pi}} - 25/4 + \gamma$ is continuous on $\Gamma \times (0, 1/9)$.

Using (17), we may easily verify, for the linear-quadratic example, that $\kappa = 2/3 > 1/2$. If $\mathbb{E}[\gamma] > [7 + 8\underline{\gamma}]/12$, then we may also verify that γ_H is interior.[28] Thus, if f is nondecreasing, then we may conclude from Proposition 3 that the optimal trade agreement is represented by an optimal tariff cap with $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$.[29]

We now summarize the above discussion and extend the result to nonincreasing densities.

COROLLARY 2: *In the trade model under perfect competition, let $Q(p) = p/2$, $Q_*(p) = p$, $u(c) = c - c^2/2$, and let the political shocks be distributed over $[\underline{\gamma}, \overline{\gamma}]$, where $1 \le \underline{\gamma} < \overline{\gamma} < 7/4$. Assume that $\mathbb{E}[\gamma] > [7 + 8\underline{\gamma}]/12$. If either*:

(i) *f is nondecreasing on Γ, or*

(ii) *f is differentiable on Γ and $f(\gamma) + (\frac{7}{4} - \gamma)f'(\gamma) \ge 0$ for all $\gamma \in \Gamma$,*

then an optimal trade agreement is represented as an optimal tariff cap with $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$.

The proof appears in Appendix F.

Corollary 2 confirms that the optimal tariff cap identified by Bagwell and Staiger (2005) in fact represents an optimal trade agreement for a broad family of distributions and when the possibility of money burning is also allowed. Indeed, Corollary 2 holds as stated whether or not money burning is allowed in the analysis, since $\kappa = 2/3 < 1$ in the linear-quadratic model of trade.[30]

[28] To show that $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$ exists, we recall that a sufficient condition for an interior maximizer is that $v'(\pi_f(\underline{\gamma})) + \mathbb{E}[\gamma] - \underline{\gamma} > 0$. Calculations confirm that this inequality holds if and only if $\mathbb{E}[\gamma] > [7 + 8\underline{\gamma}]/12$. When $\underline{\gamma} = 1$, this inequality reduces to Bagwell and Staiger's (2005) assumption that $\mathbb{E}[\gamma] > 5/4$.

[29] In fact, in this case, we can confirm that γ_H is unique. Given that $\kappa > 1/2$, if $f(\gamma)$ is nondecreasing, we may see from the proof of Proposition 3 that $v'(\pi_f(\gamma^c)) - \gamma^c + \mathbb{E}[\gamma|\gamma > \gamma^c]$ is strictly decreasing, which implies that γ_H is unique.

[30] Bagwell and Staiger (2005) also considered the potential role of escape clauses in trade agreements. In one extension, they allowed that governments negotiate two bindings, where the lower (higher) binding applies during normal (exceptional) times. An incentive-compatible escape clause must entail some cost if it is to be used only when political pressure is high. As they explained, such a cost could be provided if the escape clause binding were set at a high level and if tariffs between the two bindings were not allowed. Our analysis allows for the possibility of such "strong" escape clauses. Due to the presence of money burning, which might correspond to administrative and potential legal costs, our analysis includes, as well, a more realistic escape

Note that part (ii) of Corollary 2 includes densities that are decreasing over ranges or even over the entire support, provided that the rate of decrease is not so great as to violate the inequality, $f(\gamma) + (\frac{7}{4} - \gamma)f'(\gamma) \geq 0$. As well, the condition holds for any concave density for which $f(\overline{\gamma}) + (\frac{7}{4} - \overline{\gamma})f'(\overline{\gamma}) \geq 0$, or for any convex density for which $f(\underline{\gamma}) + (\frac{7}{4} - \underline{\gamma})f'(\underline{\gamma}) \geq 0$.[31]

An interesting special case is that the density is uniform.[32] In this case, $\gamma_H = 3\overline{\gamma} - \frac{7}{2}$ and the optimal tariff cap is

$$\overline{\tau} = \frac{1}{6} - \frac{7(7 - 4\overline{\gamma})}{24(4 - \overline{\gamma})}.$$

Recalling that a tariff of 1/6 is prohibitive, we see that the optimal tariff cap allows for positive trade volume since $\overline{\gamma} < 7/4$. The optimal tariff cap binds for higher types (i.e., for $\gamma \geq \gamma_H$), while lower types apply their flexible (Nash) tariffs and thus exhibit binding overhang. Interestingly, as $\overline{\gamma}$ approaches 7/4, $\overline{\tau}$ approaches 1/6 and so γ_H approaches $\overline{\gamma}$. Thus, when the distribution is uniform and the highest level of support approaches the limiting case in which zero trade volume is efficient, the optimal trade agreement entails full flexibility for all types! In this limiting case, governments with private information are unable to design a trade agreement that improves upon the noncooperative (Nash) benchmark.

The results presented above also provide a foundation for two recent analyses of tariff caps. Beshkar, Bond, and Rho (2011) extended the linear-quadratic model to allow that countries have asymmetric sizes. Restricting attention to the family of tariff caps, they provided theoretical and empirical support for the prediction that smaller countries have higher optimal tariff caps and a greater probability of binding overhang. In this context, our analysis provides conditions ensuring that an optimal trade agreement indeed takes the form of a tariff cap. Amador and Bagwell (2012) applied the propositions above to consider the possibility that a government's private information concerns the value of tariff revenue. While most political economy models of trade policy assume that producer surplus receives greater relative weight in the government welfare function, their extension may be of special relevance for some developing countries.[33] The problem of designing an optimal trade agreement when the value of tariff revenue is private information does not immediately fit into the

clause binding that defines only a maximum tariff for exceptional times. In any case, under the conditions that we provide, an escape clause of this nature does not represent an improvement relative to the optimal tariff cap.

[31]To see this, note that the derivative of $f(\gamma) + (\frac{7}{4} - \gamma)f'(\gamma)$ with respect to γ is $(\frac{7}{4} - \gamma)f''(\gamma)$.

[32]For further discussion of the optimal tariff cap under a uniform distribution, see Bagwell and Staiger (2005).

[33]For further discussion of political economy models of trade policy, see, for example, Baldwin (1987), Feenstra and Lewis (1991), and Grossman and Helpman (1995).

framework presented above; however, they provided an approach for embedding this problem into the framework above when money burning is allowed.

A Log Utility With Endowments Example. The linear-quadratic example is tractable and offers a convenient setting with which to illustrate our findings. An important benefit of our general analysis is that we can employ our findings to characterize an optimal trade agreement for other examples, too. In Appendix G, we consider an example with log utility and endowments (inelastic supply), where the endowment of good x in the foreign country exceeds that in the home country. Specifically, we assume that $u(c) = \log(c)$, $Q(p) = 1$, and $Q_*(p_*) = \overline{Q}_*$, where $\overline{Q}_* > 1$. For $\overline{Q}_*$ sufficiently close to 1, we find that $\kappa \approx 2/3$. Applying Proposition 3, we conclude that a tariff cap is then optimal for distributions with nondecreasing densities when $\overline{Q}_*$ is close to 1.

We note that we could have obtained a version of Corollary 2 by applying results from Alonso and Matouschek (2008). In Section 5, we discuss in detail the relationship between our Propositions 1 and 2 and the findings of Alonso and Matouschek (2008) and the earlier delegation literature. Here, we make two points that are of particular relevance for the trade-agreement application. First, the results from Alonso and Matouschek (2008) apply only if money burning is ruled out. Second, the results from the earlier delegation literature also fail to apply when preferences are not linear-quadratic, as in the log utility example, and as in the model with monopolistic competition that we discuss next.

4.2. *Optimal Agreement Under Monopolistic Competition*

We consider now the optimality of tariff caps in a new-trade setting featuring monopolistic competition and intra-industry trade with a fixed number of firms. In particular, we analyze trade policy in a two-country setting with an outside good in which consumer demand exhibits a constant elasticity of substitution (CES) across differentiated varieties. The outside good is freely traded and serves to fix wages. As Helpman and Krugman (1989) discussed, an import tariff in this setting does not alter the terms of trade but does switch expenditures toward domestic varieties. An import tariff then "shifts profit" from foreign to domestic firms, as Ossa (2012) argued. Allowing that the domestic government faces private political pressures, we use Proposition 3 to establish conditions for the new-trade setting under which an optimal trade agreement takes the form of a tariff cap.[34]

[34]Chang (2005) provided micro-foundations for the government welfare functions that we consider. In Chang's model, an exogenous and inelastic supply of a sector-specific factor determines the number of varieties produced in a country, and the owners of the sector-specific input may engage in lobbying. We abstract from the lobbying game and posit directly that the domestic gov-

Consider a setting with two countries, each with a representative household. Let us consider the following utility function for the home country's consumer:

$$U = \log D + N,$$

where D is a CES aggregate composed of differentiated goods. We define D as

$$D = \left(\sum_{i=1}^{n} (D_{iH})^{\alpha} + \sum_{i=1}^{n_*} (D_{iF})^{\alpha} \right)^{1/\alpha},$$

where D_{iH} denotes the amount consumed of variety i produced at home and D_{iF} denotes the amount consumed of variety i produced abroad. The value of N represents the consumption of a numeraire or outside good.

There are n and n_* monopolistically competitive firms at home and abroad, respectively, where n and n_* exceed unity. Each firm produces a single variety and hires labor in a competitive domestic labor market. We assume that the number of firms in each country is fixed so that firms earn positive ex ante profit.[35] We can then consider the profit-shifting role of trade policy and allow for political pressures stemming from producer interests. We also assume that $\alpha \in (0, 1)$, so as to ensure a well-defined profit-maximizing price for each firm.

We consider a simple production technology in which labor is the only factor of production in each country. Labor is immobile across countries and is used in the production of both the differentiated goods and the numeraire. The production of the numeraire good has a constant unit labor requirement equal to unity, while the production of each differentiated variety has a constant unit labor requirement equal to $\lambda > 0$. The total amount of labor in each country is denoted by L and L_*, respectively. The numeraire good is produced in each country and freely traded, which ensures that, in each country, trade is balanced and the wage rate is unity.

The domestic price index (i.e., the price index faced by consumers at home) is

$$P = \left(\sum_{i=1}^{n} (p_i)^{\alpha/(\alpha-1)} + \sum_{i=1}^{n_*} \big(p_{i*}(1+\tau)\big)^{\alpha/(\alpha-1)} \right)^{(\alpha-1)/\alpha},$$

where p_i represents the price of home variety i at home and p_{i*} represents the price of foreign variety i abroad. The value of τ represents an (ad valorem)

ernment maximizes a welfare function in which profits in the differentiated sector may receive greater weight. We assume further that the domestic government is privately informed about its preferences.

[35]Ossa (2012) also fixed the number of firms in each country. Instead of directly fixing the number of firms, we could obtain the same result by expanding the model to require a specific factor for the differentiated sector, where the supply of the factor is exogenous and inelastic. For further discussion, see Chang (2005) and Helpman and Krugman (1989).

import tariff that the home country imposes on foreign goods, so that $p_{i*}(1+\tau)$ is the consumption price in the home country of a foreign-produced variety i. The price index for foreign consumers is symmetric, with a foreign import tariff τ_* affecting the consumption prices in the foreign country of home-produced varieties. We restrict τ and τ_* to reside in $(-1, \infty)$.

We consider next the determination of prices and the resulting price indices. The home demands for home- and foreign-produced goods, respectively, are isoelastic and given by

$$D_{iH}(p_i, P) = P^{\alpha/(1-\alpha)}(p_i)^{-1/(1-\alpha)} \quad \text{and}$$
$$D_{iF}(p_{i*}, P) = P^{\alpha/(1-\alpha)}\big(p_{i*}(1+\tau)\big)^{-1/(1-\alpha)}.$$

Under monopolistic competition, each firm ignores the impact of its price on price indices. Each firm then maximizes its profit by setting a price equal to a constant markup over marginal cost:

$$p_i = p_{i*} = \frac{\lambda}{\alpha}.$$

Thus, for a home-produced variety, the consumption prices for home and foreign consumers, respectively, are $\frac{\lambda}{\alpha}$ and $\frac{\lambda}{\alpha}(1+\tau_*)$. As Helpman and Krugman (1989) emphasized, an important feature of the model is thus that an import tariff does not generate a terms-of-trade externality: the world or export price for a given variety is not affected by the magnitude of the import tariff. With profit-maximizing prices now determined, we see that the price indexes at home and abroad are given as follows:

$$P = \frac{\lambda}{\alpha}\big(n + n_*(1+\tau)^{\alpha/(\alpha-1)}\big)^{(\alpha-1)/\alpha} \quad \text{and}$$
$$P_* = \frac{\lambda}{\alpha}\big(n_* + n(1+\tau_*)^{\alpha/(\alpha-1)}\big)^{(\alpha-1)/\alpha}.$$

We can now define the following components of home welfare:

$$\mathrm{CS} = \frac{1-\alpha}{\alpha}\log\big(n + n_*(1+\tau)^{\alpha/(\alpha-1)}\big) + \log(\alpha/\lambda) - 1 + L,$$
$$\mathrm{PS} = (1-\alpha)\left(\frac{n}{n + n_*(1+\tau)^{\alpha/(\alpha-1)}} + \frac{n(1+\tau_*)^{1/(\alpha-1)}}{n(1+\tau_*)^{\alpha/(\alpha-1)} + n_*}\right),$$
$$\mathrm{TR} = 1 - \frac{n + n_*(1+\tau)^{1/(\alpha-1)}}{n + n_*(1+\tau)^{\alpha/(\alpha-1)}},$$

where CS denotes consumer surplus enjoyed by home consumers, PS represents producer surplus or profit for differentiated-goods home firms, and

TR indicates the tariff revenue generated by τ for the home country. Notice that CS and TR are functions of the home tariff τ but are independent of the foreign tariff τ_*. By contrast, PS depends in a separable fashion on both tariffs. Intuitively, a higher home tariff shifts the expenditures of home consumers toward home-produced varieties, which increases profit for home firms on domestic sales. A higher foreign tariff similarly shifts expenditures of foreign consumers away from home-produced varieties, which reduces profit for home firms on foreign sales. These respective effects are separable, since the import tariff imposed in one country does not affect the terms of trade and thus does not alter the price index in the other country. The respective functions for the foreign country (CS_*, PS_*, and TR_*) are symmetric.

We posit that the home government welfare is a weighted sum of home consumer surplus, profit, and tariff revenue, where profit receives an extra weight denoted by $\gamma \geq 1$. The welfare function for the home government is then

$$W_H = \mathrm{CS} + \mathrm{TR} + \gamma \mathrm{PS}.$$

As before, the political economy weight of the home government, γ, is unknown at the time that the agreement is formed and is privately known by the home government before the home tariff is applied. We assume that γ has a continuous distribution $F(\gamma)$ with bounded support $\Gamma = [\underline{\gamma}, \overline{\gamma}]$, and with associated density $f(\gamma)$.

To focus attention on the problem of the home government, we assume that private information is one-sided. Thus, the political economy weight of the foreign government, γ_*, is fixed and commonly known. For simplicity, we set $\gamma_* = 1$ and thus assume that the foreign government maximizes national real income.[36] The welfare function for the foreign government is thus

$$W_F = \mathrm{CS}_* + \mathrm{TR}_* + \mathrm{PS}_*.$$

To map this setup into our framework, it is convenient to do the following change of variables. Define π and π_* to be

$$\pi = \frac{n}{n + n_*(1+\tau)^{\alpha/(\alpha-1)}} \quad \text{and} \quad \pi_* = \frac{n_*}{n_* + n(1+\tau_*)^{\alpha/(\alpha-1)}}.$$

[36] As in the perfect competition setting, it is straightforward to consider a slightly more general model with a symmetric structure. For the monopolistic competition setting, a symmetric model obtains if we include a second differentiated sector with a CES aggregate that enters the utility function in a symmetric and additively separable fashion. The foreign government is privately informed about its political economy weight for foreign firms in this sector, and the home government has a known political economy weight of unity for home firms in this sector. Given the existence of an outside good, we can study the optimal trade agreement across sectors independently, as we do here.

Notice that π strictly increases with τ and that π_* likewise strictly increases with τ_*. Since τ and τ_* are restricted to lie in $(-1, \infty)$, it follows that π and π_* must lie in $\Pi = (0, 1)$. To simplify the algebra that follows, we also impose that $n = n_*$. We can write

$$\frac{\mathrm{CS}}{1-\alpha} = -\frac{1}{\alpha}\log\pi + \frac{1}{\alpha}\log n + \frac{\log(\alpha/\lambda) - 1 + L}{1-\alpha},$$
$$\frac{\mathrm{PS}}{1-\alpha} = \pi + (\pi_*)^{(\alpha-1)/\alpha}(1-\pi_*)^{1/\alpha},$$
$$\frac{\mathrm{TR}}{1-\alpha} = \frac{1}{1-\alpha}\bigl(1 - \pi - (\pi)^{(\alpha-1)/\alpha}(1-\pi)^{1/\alpha}\bigr),$$

where we have used the assumption that $n = n_*$. Henceforth, we work with π and π_* as our choice variables. Let us then define the following functions:

$$b(\pi) = -\frac{1}{\alpha}\log\pi - \frac{1}{1-\alpha}\pi - \frac{1}{1-\alpha}(\pi)^{(\alpha-1)/\alpha}(1-\pi)^{1/\alpha}, \tag{18}$$

$$v(\pi) = (\pi)^{(\alpha-1)/\alpha}(1-\pi)^{1/\alpha}, \tag{19}$$

where $v'(\pi) < 0 < v''(\pi)$ for all $\pi \in \Pi$.

It follows that the welfare functions can be written as

$$W_H = (1-\alpha)\bigl(\gamma\pi + b(\pi) + \gamma v(\pi_*)\bigr) + C_H,$$
$$W_F = (1-\alpha)\bigl(\pi_* + b(\pi_*) + v(\pi)\bigr) + C_F,$$

for some constants C_H and C_F. Note that this representation exploits the separable fashion in which home and foreign tariffs affect the profit of any firm, as described above.

The problem of designing an optimal trade agreement is then to find an allocation for the home country $\boldsymbol{\pi}: \Gamma \to \Pi$ that is incentive compatible and maximizes the total welfare of the governments. Given the separable manner in which π and π_* enter the welfare functions, the optimal trade agreement solves Problem (P) subject to (1), with $w(\gamma, \pi) = \gamma\pi + b(\pi) + v(\pi)$. We have the following result.

LEMMA 3: *Let $\Pi = (0, 1)$ and $\Gamma = [\underline{\gamma}, \overline{\gamma}]$. Let $b: \Pi \to \mathbb{R}$ be defined as in* (18). *Let $w: \Gamma \times \Pi \to \mathbb{R}$ be defined as $w(\gamma, \pi) = \gamma\pi + b(\pi) + v(\pi)$, where v is as in* (19). *If $1 > \alpha > 1/2$ and $\alpha(1-\alpha)\overline{\gamma} < 1$, then Assumption 1 holds.*

The proof appears in the Appendix.

For $\alpha \in (1/2, 1)$, the value of κ, according to equation (2), is[37]

$$\kappa = \inf_{\pi \in \Pi} \left\{ \frac{\alpha - \alpha(1-\pi)^{(1-2\alpha)/\alpha}\pi^{(\alpha-1)/\alpha}}{\alpha - (1-\pi)^{(1-2\alpha)/\alpha}\pi^{(\alpha-1)/\alpha}} \right\}$$
$$= 1 - \left(\frac{1}{1-\alpha} - \left(\frac{2\alpha - 1}{1-\alpha} \right)^{(2\alpha-1)/\alpha} \right)^{-1} \in (0, 1).$$

We can further show that $\kappa \geq 1/2$ for $\alpha \geq 2/3$. Recall as well that $v'(\pi) < 0$. Together with the results from Lemma 3, these findings indicate that we can apply Proposition 3 to obtain the following corollary.

COROLLARY 3: *Let* $1 > \alpha \geq 2/3$ *and* $\alpha(1-\alpha)\overline{\gamma} < 1$. *For any nondecreasing density* f, *if there exists* $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$ *that solves equation* (4), *then the optimal trade agreement in the monopolistic competition model is a tariff cap*.

Corollary 3 requires the existence of an interior γ_H for a cap allocation to be optimal. A simpler condition to check is $\mathbb{E}[\gamma] - \alpha\underline{\gamma} - (2-\alpha)/\alpha > 0$, which guarantees that there exists an interior γ_H. Under the assumptions of Corollary 3, the simpler condition holds as $\underline{\gamma} \to 1$ and $\overline{\gamma} \to 1/(\alpha(1-\alpha))$, and is thus satisfied if the support for γ is sufficiently wide. We note further that an interior γ_H ensures that the corresponding tariff cap is strictly positive.[38]

The estimation performed by Eaton and Kortum (2002) implies a value of $\alpha = 0.89$ (corresponding to an elasticity of substitution across varieties of 9.28), and the estimation of Bernard, Eaton, Jensen, and Kortum (2003) implies a value of $\alpha = 0.78$ (corresponding to an elasticity of 4.6).[39] Both of these estimates lie above the critical cutoff of 2/3 identified in Corollary 3 for the optimality of a cap under a nondecreasing density. Related to this, Broda and Weinstein (2006) estimated an average elasticity of substitution for 10-digit (TSUSA) goods of around 8 (and around 4 for within 3-digit (TSUSA) goods), also generating a value of α above the cutoff.

[37]The result follows from the fact that $(1-\pi)^{(2\alpha-1)/\alpha}\pi^{(1-\alpha)/\alpha}$ achieves a maximum at $\pi = \frac{1-\alpha}{\alpha}$ if $1 > \alpha > 1/2$.

[38]Notice that $\pi_f(\underline{\gamma}) > 1/2$, given that $b'(1/2) = 0$; it thus follows from the definition of π that the flexible tariff is strictly positive for $\gamma = \underline{\gamma}$ and thus for all higher values of γ, including γ_H. To derive the simpler condition for an interior γ_H, we note further that $v'(\pi_f(\underline{\gamma})) = (1-\alpha)\underline{\gamma} - (1-\alpha)/(\alpha\pi_f(\underline{\gamma})) - 1$. One can then use the condition $v'(\pi_f(\underline{\gamma})) + \mathbb{E}[\gamma] - \underline{\gamma} > 0$, discussed right after Proposition 3. Finally, a nondecreasing density implies that $\mathbb{E}[\gamma] \geq (\underline{\gamma} + \overline{\gamma})/2$, and we may use this inequality to verify that the simpler condition then holds as $\underline{\gamma} \to 1$ and $\overline{\gamma} \to 1/(\alpha(1-\alpha))$.

[39]See Dekle, Eaton, and Kortum (2007), Ossa (2011), and Ossa (2012), for example, for further use of these estimates.

4.3. *Imperfect Transfers*

Our findings above establish conditions for the optimality of tariff caps in trade models with perfect and monopolistic competition. These findings are derived in settings that allow for the burning of resources; however, we assume that contingent transfers of resources from one government to the other are infeasible. In this subsection, we relax this assumption.

Suppose now that instead of no transfers, there is imperfect transferability from the home to the foreign government. In particular, let t represent a transfer from the home government to the foreign one, and suppose that only a fraction $\rho \in [0, 1]$ of this transfer actually reaches the foreign government. We thus posit a "leaky bucket" model of transfers, in which a fraction $(1 - \rho)$ of any transfer is lost.

The setup with money burning stated in Problem (P) corresponds to the case where $\rho = 0$. More generally, for any $\rho \in [0, 1]$, the welfare function under the agreement becomes

$$\int_{\Gamma} \big(\gamma \boldsymbol{\pi}(\gamma) + b\big(\boldsymbol{\pi}(\gamma)\big) + v\big(\boldsymbol{\pi}(\gamma)\big) - (1 - \rho)\mathbf{t}(\gamma)\big)\, dF(\gamma). \tag{20}$$

The goal of the agreement is then to maximize (20) subject to the same constraint set as in Problem (P).

We may use our findings to analyze the problem with imperfect transferability as well. The key point is that we may multiply (20) by $\frac{1}{1-\rho}$ and rewrite the objective as

$$\int_{\Gamma} \big(\tilde{w}\big(\gamma, \boldsymbol{\pi}(\gamma)\big) - \mathbf{t}(\gamma)\big)\, dF(\gamma),$$

where $\tilde{w}(\gamma, \pi) = \frac{1}{1-\rho}(\gamma\pi + b(\pi) + v(\pi)) = \frac{1}{1-\rho} w(\gamma, \pi)$. Hence, the problem with imperfect transferability maps into Problem (P), after a rescaling of the principal's objective function. Note that parts (b) of Propositions 1 and 2 apply unchanged to this problem, once w has been replaced by $\tilde{w}$ in conditions (c1)–(c3′) and in the definition of κ given by equation (3).

The optimal interval allocation is independent of ρ, as money burning is by definition not used. We can thus study how changes in ρ affect the sufficient conditions by checking how conditions (c1)–(c3′) change for given values of γ_L and γ_H. Note first that if κ is initially less than 1, then a marginal increase in ρ does not affect the sufficient conditions at all. For sufficiently high values of ρ, however, $\kappa = 1$ is necessary. Further, when $\kappa = 1$, an increase in ρ makes the sufficient conditions harder to satisfy. In the limit, as $\rho \to 1$, the first best agreement can be implemented if it is monotonic (as it is in the trade applications above). In this limiting case, an interval allocation is optimal only if it is first best, and hence optimal for all $\rho \in [0, 1]$.

In the trade model with perfect competition, $\kappa < 1$ obtains if $v''(\pi) > 0$, which, as noted, in turn holds if $Q'' \leq 0$, $Q''_* \leq 0$, and $u''' \geq 0$. In particular, $\kappa = 2/3$ in the linear-quadratic example, while $\kappa \approx 2/3$ in the example with log utility and endowments when $\overline{Q}_*$ is sufficiently close to 1. Likewise, in the trade model with monopolistic competition, $\kappa \in (0, 1)$ for $\alpha \in (1/2, 1)$. Thus, in all of these settings, the optimality of a tariff cap is robust to the possibility that resources may be transferred in a sufficiently inefficient manner (i.e., for ρ sufficiently close to zero). At the same time, our discussion in this subsection implies that the optimal tariff cap does not maximize expected government welfare when a sufficiently efficient transfer instrument is available.

5. RELATION TO PREVIOUS LITERATURE

In this section, we discuss how our propositions can be used to obtain previous results found in the delegation literature. In particular, we show that our Propositions 1 and 2 deliver, as special cases, the main results on interval delegation obtained by Alonso and Matouschek (2008), Amador, Werning, and Angeletos (2006), and Ambrus and Egorov (2009).

5.1. *Relation to Alonso and Matouschek*

Alonso and Matouschek (2008) studied the optimal delegation problem in the absence of money burning. We show that our Propositions 1 and 2 can be used to derive Alonso and Matouschek's (2008) characterization of sufficient and necessary conditions for interval delegation to be optimal.

In their main analysis, Alonso and Matouschek (2008) assumed that the principal's welfare function is quadratic, and that, for any given state of nature, the agent's welfare function is single-peaked and symmetric around the agent's preferred action.[40] Alonso and Matouschek (2008) solved the following problem:

$$\max_{\boldsymbol{\pi}:\Gamma\to\mathbb{R}} \int_{\gamma\in\Gamma} w\big(\gamma, \boldsymbol{\pi}(\gamma)\big)\, dF(\gamma),$$

where $w(\gamma, \pi) \equiv -(\pi - \pi_P(\gamma))^2/2$ and subject to the agent choosing according to any utility function of the form $v_A(\pi - \pi_f(\gamma), \gamma)$, where v_A is single-peaked and symmetric around zero with respect to its first argument and where π_f is strictly increasing.

[40] Alonso and Matouschek (2008) also identified sufficient conditions for interval delegation when the principal's preferences take a more general form, while maintaining the symmetry of the agent's preferences. However, the preferences that they allowed require the absence of any bias for an intermediate type. This requirement is not met in many applications, including the trade-agreement application that we discuss previously. Our approach permits weaker sufficient conditions, holds for a more general class of preferences, and identifies a family of preferences for which the sufficient conditions are also necessary.

In this setup without money burning, it is without loss of generality to choose a quadratic utility for the agent, $v_A(x,\gamma) = -x^2/2$.[41] It is also without loss of generality to assume that $\pi_f(\gamma) = \gamma$.[42] It follows that the agent's utility can be written, after removing parts not affected by choices, as $\gamma\pi + b(\pi)$, where $b(\pi) \equiv -\pi^2/2$.

This utility specification satisfies the conditions for Proposition 2, part (a), with $A = 1$, $C(\gamma) = \pi_P(\gamma)$, and $B(\gamma) = -(\pi_P(\gamma))^2/2$. Hence, the conditions we provide in Proposition 1 are both sufficient and necessary. We thus obtain Alonso and Matouschek's (2008) result regarding the optimality of what they called *threshold delegation* as a special case of our results.[43]

5.2. *Relation to Amador, Werning, and Angeletos*

Amador, Werning, and Angeletos (2006) studied the following hyperbolic consumption-savings problem: choose $\mathbf{u} : \Gamma \to \mathbb{R}$ and $\mathbf{w} : \Gamma \to \mathbb{R}$ such that

$$\max_{\mathbf{u},\mathbf{w}} \left\{ \int_\Gamma \big(\gamma \mathbf{u}(\gamma) + \beta \mathbf{w}(\gamma)\big)\, dF(\gamma) \right\} \quad \text{subject to:}$$

$$\gamma \in \arg\max_{\tilde{\gamma}\in\Gamma} \big\{ \gamma \mathbf{u}(\tilde{\gamma}) + \beta\delta \mathbf{w}(\tilde{\gamma}) \big\}, \tag{21}$$

$$C\big(\mathbf{u}(\gamma)\big) + K\big(\mathbf{w}(\gamma)\big) \le y;\ \forall \gamma \in \Gamma, \tag{22}$$

where C and K are strictly increasing and convex cost functions. The value of β represents the standard discount factor, and $\delta \in (0,1)$ captures the hyperbolic adjustment. The value of γ is a shock to the marginal utility of current consumption, which is private information to the agent. The constraint (21) is the incentive-compatibility constraint and the constraint (22) is the budget constraint.

We map this into our setting with money burning. To do this, we let $\mathbf{t}(\gamma) \equiv \beta\delta(W(y - C(\mathbf{u}(\gamma))) - \mathbf{w}(\gamma))$, where W is defined to be the inverse of K. Letting $\boldsymbol{\pi} = \mathbf{u}$, we can write the problem as

$$\max \left\{ \int_\Gamma \big(\gamma\delta \boldsymbol{\pi}(\gamma) + \beta\delta W\big(y - C\big(\boldsymbol{\pi}(\gamma)\big)\big) - \mathbf{t}(\gamma)\big)\, dF(\gamma) \right\} \quad \text{subject to:}$$

$$\gamma \in \arg\max_{\tilde{\gamma}\in\Gamma} \big\{ \gamma \boldsymbol{\pi}(\tilde{\gamma}) + \beta\delta W\big(y - C\big(\boldsymbol{\pi}(\tilde{\gamma})\big)\big) - \mathbf{t}(\tilde{\gamma}) \big\},$$

$$\mathbf{t}(\gamma) \ge 0;\ \forall \gamma \in \Gamma.$$

[41] Under a single-peaked and symmetric utility specification, the agent with type γ prefers π_0 to π_1 if and only if $|\pi_0 - \pi_f(\gamma)| < |\pi_1 - \pi_f(\gamma)|$. This ranking also holds for the quadratic specification, and hence guarantees that any allocation satisfies incentive compatibility under the original utility specification if and only if it does so under the quadratic one.

[42] Alonso and Matouschek (2008) showed that it is without loss of generality to assume $\pi_f(\gamma) = \alpha + \beta\gamma$ for any α and $\beta > 0$. We can then choose $\alpha = 0$ and $\beta = 1$.

[43] See our Supplemental Material for an exact statement of the above.

Using our notation, the above problem is equivalent to our problem with money burning, (P2′), with

$$b(\pi) \equiv \beta\delta W\bigl(y - C(\pi)\bigr); \quad w(\gamma, \pi) \equiv \gamma\delta\pi + b(\pi).$$

Note that, under this mapping, b is strictly concave, and $w(\gamma_0, \cdot)$ is also strictly concave for any $\gamma_0 \in \Gamma$ and satisfies the conditions of Proposition 2, part (b), with $A = 1$. Hence, we can use Propositions 1 and 2 to derive necessary and sufficient conditions for the optimality of interval delegation, which delivers the minimum-savings results of Amador, Werning, and Angeletos (2006).

5.3. *Relation to Ambrus and Egorov*

Ambrus and Egorov (2009) analyzed a delegation problem with a principal and a privately informed agent. An initial transfer between the principal and the agent is used to satisfy the agent's ex ante participation constraint, but transfers between the principal and agent are otherwise infeasible.[44] A contract specifies incentive-compatible actions and money burning levels for the agent as functions of the agent's private information or type. They explicitly solved for optimal contracts in the *quadratic-uniform model*, in which the principal and agent have quadratic utility functions and the type is distributed uniformly over $[0, 1]$.

The quadratic-uniform model studied by Ambrus and Egorov (2009) can be mapped into our problem with money burning, Problem (P), by assuming that

$$w(\gamma, \pi) = -\frac{\alpha+1}{2}\left(\pi - \gamma - \frac{\beta}{\alpha+1}\right)^2, \quad \text{and} \quad b(\pi) = \beta\pi - \frac{\pi^2}{2},$$

where $\alpha > 0$ and $0 < \beta < 1$.[45] It follows that $\pi_f(\gamma) = \gamma + \beta$. Note that the preferences above satisfy our conditions in Proposition 2 with $A = 1 + \alpha$, $B(\gamma) = -[\gamma + \beta/(1+\alpha)]^2/2$, and $C(\gamma) = \gamma - \alpha\beta/(1+\alpha)$, so the conditions (c1), (c2), (c2′), (c3), and (c3′) are both necessary and sufficient for the optimality of the interval delegation allocation. Ambrus and Egorov (2009) also assumed that $F(\gamma) = \gamma$ with $\underline{\gamma} = 0$ and $\overline{\gamma} = 1$.

We can now obtain Ambrus and Egorov's (2009) characterization of optimal interval delegation using our propositions. When $\alpha \le 1$, we can use Proposition 1, part (b), to show that the cap allocation where $\gamma_L = \underline{\gamma}$ and $\gamma_H = 1 - 2\alpha\beta/(1+\alpha)$ is optimal.

To illustrate how our propositions can be applied to generalize previous research, we maintain the quadratic preferences of Ambrus and Egorov (2009)

[44]They also considered an extended model in which contingent transfers are allowed.

[45]See our Supplemental Material for details. While Ambrus and Egorov highlighted cases in which $\beta < 1$, they also discussed the possibility that $\beta \ge 1$. Below, we maintain the assumption that $0 < \beta < 1$.

but relax their uniform distribution assumption. When $\alpha \leq 1$, we can use Proposition 1 to show that a cap allocation is optimal for a more general class of distributions. As we show in our Supplemental Material, if (i) $\mathbb{E}[\gamma] - \underline{\gamma} > \frac{\alpha\beta}{\alpha+1}$, then there exists $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$ such that $\mathbb{E}[\gamma|\gamma \geq \gamma_H] - \gamma_H = \alpha\beta/(1+\alpha)$. If, in addition, (ii) $F(\gamma) + \alpha\beta f(\gamma)$ is nondecreasing for $\gamma \in [\underline{\gamma}, \gamma_H]$, and (iii) $(1+\alpha)\mathbb{E}[\widetilde{\gamma}|\widetilde{\gamma} \geq \gamma] - \gamma \leq \alpha(\gamma_H + \beta)$ for $\gamma \in [\gamma_H, \overline{\gamma}]$, then a cap allocation with $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$ is optimal. We show further that hypothesis (iii) holds if $d(\gamma) \equiv \mathbb{E}[\widetilde{\gamma}|\widetilde{\gamma} \geq \gamma] - \gamma$ is convex, which, in turn, holds if f is twice differentiable, nondecreasing, and not too convex.[46] Using this, we can show, for example, that in the case of a power distribution, where $F(\gamma) = \gamma^n$ and $0 = \underline{\gamma} < \overline{\gamma} = 1$, a cap allocation is optimal if $n/(n+1) > \alpha\beta/(1+\alpha)$, $n \geq 1$ and $\alpha \leq 1$. The case of a uniform distribution is then captured as a special case when $n = 1$.

6. CONCLUSION

We consider a general representation of the delegation problem, and we provide conditions under which an interval allocation is an optimal solution to this problem. We analyze both the delegation problem without money burning and the delegation problem with money burning. As we show, important characterizations of optimal delegation in previous work can be captured as special cases of our findings. We also develop a new application of delegation theory to the theory of trade agreements among privately informed governments. For both perfect and monopolistic competition settings, we establish conditions under which tariff caps are optimal and thereby provide interpretations of negotiations over tariff bindings and also binding overhang.

To establish our findings, we utilize and extend the Lagrangian methods developed by Amador, Werning, and Angeletos (2006). Our analysis allows that the Lagrangian may fail to be concave with respect to the action, which is a possibility that arises naturally in the trade application, for example. We expect that our techniques will be useful for other studies of applied mechanism design when contingent transfers are infeasible.

Our work suggests several promising directions for future research. For example, the general representation of the delegation problem that we analyze assumes a single agent. We expect that our Lagrangian methods can be extended to characterize optimal delegation in settings with multiple agents. Likewise, our analysis of optimal tariff caps provides a foundation for further analysis of GATT/WTO rules concerning exceptions for contingent protection. We plan to pursue these and other extensions in future research.

[46]As we show in the Supplemental Material, the exact result is that d is convex if (i) $f'(\gamma) \geq 0$ for all $\gamma \in \Gamma$, (ii) if there exists $\gamma \in (\underline{\gamma}, \overline{\gamma})$ such that $f'(\gamma) > 0$, then $f'(\overline{\gamma}) > 0$, and (iii) $f''(\gamma) \leq 2f'(\gamma)^2/f(\gamma) + f'(\gamma)f(\gamma)/(1-F(\gamma))$ for all $\gamma \in \Gamma$.

APPENDIX A: PROOF OF LEMMA 1

The welfare function can, under an interval allocation, be written as

$$\mathrm{Obj}(\gamma_L, \gamma_H) = \int_{\underline{\gamma}}^{\gamma_L} w\big(\gamma, \pi_f(\gamma_L)\big) f(\gamma)\, d\gamma + \int_{\gamma_L}^{\gamma_H} w\big(\gamma, \pi_f(\gamma)\big) f(\gamma)\, d\gamma + \int_{\gamma_H}^{\overline{\gamma}} w\big(\gamma, \pi_f(\gamma_H)\big) f(\gamma)\, d\gamma.$$

If γ_L is interior (i.e., we are in part (iv)), then the necessary first-order condition for an interior γ_L is

$$\frac{d\mathrm{Obj}}{d\gamma_L} = \left(\int_{\underline{\gamma}}^{\gamma_L} w_\pi\big(\gamma, \pi_f(\gamma_L)\big) f(\gamma)\, d\gamma\right) \pi_f'(\gamma_L) = 0.$$

Using that $\pi_f'(\gamma_L) > 0$ by assumption, we have that condition (iv) is necessary.

Computing the second derivative of the welfare function we have that

$$\begin{aligned}\frac{d^2\mathrm{Obj}}{d\gamma_L^2} &= \left(\int_{\underline{\gamma}}^{\gamma_L} w_\pi\big(\gamma, \pi_f(\gamma_L)\big) f(\gamma)\, d\gamma\right) \pi_f''(\gamma_L) \\ &\quad + \big(w_\pi\big(\gamma_L, \pi_f(\gamma_L)\big) f(\gamma_L)\big) \pi_f'(\gamma_L) \\ &\quad + \left(\int_{\underline{\gamma}}^{\gamma_L} w_{\pi\pi}\big(\gamma, \pi_f(\gamma_L)\big) f(\gamma)\, d\gamma\right) \big(\pi_f'(\gamma_L)\big)^2.\end{aligned}$$

Now note that if $\gamma_L = \underline{\gamma}$, then $d\mathrm{Obj}/d\gamma_L = 0$ and $d^2\mathrm{Obj}/d\gamma_L^2 = w_\pi(\underline{\gamma}, \pi_f(\underline{\gamma})) \times f(\underline{\gamma})\pi_f'(\underline{\gamma})$. So if $\gamma_L = \underline{\gamma}$ is optimal, it must be that $w_\pi(\underline{\gamma}, \pi_f(\underline{\gamma})) \leq 0$. This implies that condition (iii) is necessary. The proofs for conditions (i) and (ii) follow a similar argument, so we omit them. *Q.E.D.*

APPENDIX B: A MODIFIED VERSION OF LUENBERGER'S SUFFICIENCY THEOREM

Here we provide a slightly modified version of Theorem 1 in Section 8.4 of Luenberger (1969, p. 220) that makes explicit the complementary slackness condition.

THEOREM 1: *Let f be a real valued functional defined on a subset Ω of a linear space X. Let G be a mapping from Ω into the linear space Z having nonempty positive cone P. Suppose that* (i) *there exists a linear function $T : Z \to \mathbb{R}$ such that $T(z) \geq 0$ for all $z \in P$,* (ii) *there is an element $x_0 \in \Omega$ such that*

$$f(x_0) + T\big(G(x_0)\big) \leq f(x) + T\big(G(x)\big) \quad \textit{for all } x \in \Omega,$$

(iii) $-G(x_0) \in P$, *and* (iv) $T(G(x_0)) = 0$. *Then* x_0 *solves*

$$\min f(x) \quad \textit{subject to:} \quad -G(x) \in P, \quad x \in \Omega.$$

PROOF: Note that from (ii) and (iii), x_0 is in the constraint set of the minimization problem. Suppose that there exists an $x_1 \in \Omega$ with $f(x_1) < f(x_0)$ and $-G(x_1) \in P$, so that x_0 is not a minimizer. Then, by hypothesis (i), $T(-G(x_1)) \geq 0$. Linearity implies that $T(G(x_1)) \leq 0$. Using this together with (iv), it follows that $f(x_1) + T(G(x_1)) < f(x_0) = f(x_0) + T(G(x_0))$, which contradicts hypothesis (ii). *Q.E.D.*

APPENDIX C: PROOF OF PROPOSITION 1

We prove each of the parts of this proposition separately. For both cases, let $\boldsymbol{\pi}^\star : \Gamma \to \Pi$ denote the proposed interval allocation with bounds γ_L, γ_H such that conditions (c1), (c2), (c2′), (c3), and (c3′) are satisfied.

Proof of Part (a) of Proposition 1

Our objective here is to be able to apply Theorem 1 in Appendix B, which is a modified version of the sufficiency Theorem 1 of Section 8.4 in Luenberger (1969, p. 220).

By writing the incentive constraints in their usual integral form plus a monotonicity restriction,[47] we can rewrite Problem (P) with constraint (1) as

$$\text{(P1}'\text{)} \qquad \max_{\boldsymbol{\pi}:\Gamma\to\Pi} \int w\big(\gamma, \boldsymbol{\pi}(\gamma)\big)\, dF(\gamma) \quad \text{subject to:}$$

$$\text{(23)} \qquad \gamma\boldsymbol{\pi}(\gamma) + b\big(\boldsymbol{\pi}(\gamma)\big) = \int_{\underline{\gamma}}^{\gamma} \boldsymbol{\pi}(\tilde{\gamma})\, d\tilde{\gamma} + \underline{U} \quad \text{for all } \gamma \in \Gamma,$$

$$\text{(24)} \qquad \boldsymbol{\pi} \text{ nondecreasing,}$$

where $\underline{U} \equiv \underline{\gamma}\boldsymbol{\pi}(\underline{\gamma}) + b(\boldsymbol{\pi}(\underline{\gamma}))$.

We follow and extend the Lagrangian approach used by Amador, Werning, and Angeletos (2006). Following Amador, Werning, and Angeletos (2006), we first embed the monotonicity constraint (24) into the choice set of $\boldsymbol{\pi}(\gamma)$. Then, we write constraint (23) as two inequalities:

$$\text{(25)} \qquad \int_{\underline{\gamma}}^{\gamma} \boldsymbol{\pi}(\tilde{\gamma})\, d\tilde{\gamma} + \underline{U} - \gamma\boldsymbol{\pi}(\gamma) - b\big(\boldsymbol{\pi}(\gamma)\big) \leq 0 \quad \text{for all } \gamma \in \Gamma,$$

$$\text{(26)} \qquad -\int_{\underline{\gamma}}^{\gamma} \boldsymbol{\pi}(\tilde{\gamma})\, d\tilde{\gamma} - \underline{U} + \gamma\boldsymbol{\pi}(\gamma) + b\big(\boldsymbol{\pi}(\gamma)\big) \leq 0 \quad \text{for all } \gamma \in \Gamma.$$

[47]See Milgrom and Segal (2002).

The problem is then to choose a function $\boldsymbol{\pi} \in \Phi$ so as to maximize (P1′) subject to (25) and (26) and where the choice set is given by $\Phi \equiv \{\boldsymbol{\pi} | \boldsymbol{\pi} : \Gamma \to \Pi$ and $\boldsymbol{\pi}$ nondecreasing$\}$.

By assigning cumulative Lagrange multiplier functions Λ_1 and Λ_2 to constraints (25) and (26), respectively, we can write the Lagrangian for the problem:

$$\begin{aligned}(27)\quad & \mathcal{L}(\boldsymbol{\pi}|\Lambda_1, \Lambda_2) \\ & \equiv \int_\Gamma w(\gamma, \boldsymbol{\pi}(\gamma))\, dF(\gamma) \\ & \quad - \int_\Gamma \left(\int_{\underline{\gamma}}^{\gamma} \boldsymbol{\pi}(\gamma')\, d\gamma' + \underline{U} - \gamma \boldsymbol{\pi}(\gamma) - b(\boldsymbol{\pi}(\gamma)) \right) d(\Lambda_1(\gamma) - \Lambda_2(\gamma)).\end{aligned}$$

The Lagrange multipliers Λ_1 and Λ_2 are restricted to be nondecreasing functions. Let $\Lambda(\gamma) \equiv \Lambda_1(\gamma) - \Lambda_2(\gamma)$. Integrating by parts the Lagrangian, we get[48]

$$\begin{aligned}\mathcal{L}(\boldsymbol{\pi}|\Lambda) = & \int_\Gamma \left[w(\gamma, \boldsymbol{\pi}(\gamma)) f(\gamma) - (\Lambda(\overline{\gamma}) - \Lambda(\gamma)) \boldsymbol{\pi}(\gamma) \right] d\gamma \\ & + \int_\Gamma (\gamma \boldsymbol{\pi}(\gamma) + b(\boldsymbol{\pi}(\gamma)))\, d\Lambda(\gamma) - \underline{U}(\Lambda(\overline{\gamma}) - \Lambda(\underline{\gamma})).\end{aligned}$$

A proposed multiplier

Let us propose some nondecreasing multipliers Λ_1 and Λ_2 so that their difference, Λ, satisfies:

$$\Lambda(\gamma) = \begin{cases} 1 + \kappa(1 - F(\gamma)), & \gamma \in [\gamma_H, \overline{\gamma}], \\ 1 - w_\pi(\gamma, \pi_f(\gamma)) f(\gamma), & \gamma \in (\gamma_L, \gamma_H), \\ 1 - \kappa F(\gamma), & \gamma \in [\underline{\gamma}, \gamma_L], \end{cases}$$

where κ is given by (2).

Note that Λ is well defined even when γ_L and γ_H are not interior. Below, we will show that the hypothesis of Proposition 1, part (a), guarantees that $\kappa F(\gamma) + \Lambda(\gamma) \equiv R(\gamma)$ is nondecreasing; hence, it follows that $\Lambda(\gamma)$ can be written as the difference of two nondecreasing functions, $R(\gamma) - \kappa F(\gamma)$.[49]

[48]Note that $h(\gamma) \equiv \int_{\underline{\gamma}}^{\gamma} \boldsymbol{\pi}(\gamma')\, d\gamma'$ exists (as $\boldsymbol{\pi}$ is bounded and measurable by monotonicity) and is absolutely continuous; and $\Lambda(\gamma) \equiv \Lambda_1(\gamma) - \Lambda_2(\gamma)$ is a function of bounded variation, as it is the difference between two nondecreasing and bounded functions. It follows then that $\int_{\underline{\gamma}}^{\overline{\gamma}} h(\gamma)\, d\Lambda(\gamma)$ exists (it is the Riemann–Stieltjes integral), and integration by parts can be done as follows: $\int_{\underline{\gamma}}^{\overline{\gamma}} h(\gamma)\, d\Lambda(\gamma) = h(\overline{\gamma})\Lambda(\overline{\gamma}) - h(\underline{\gamma})\Lambda(\underline{\gamma}) - \int_{\underline{\gamma}}^{\overline{\gamma}} \Lambda(\gamma)\, dh(\gamma)$. Since $h(\gamma)$ is absolutely continuous, we can replace $dh(\gamma)$ with $\boldsymbol{\pi}(\gamma)\, d\gamma$.

[49]For our purposes, only the difference between the multipliers matters: we just need to know that there exist some nondecreasing functions whose difference delivers Λ.

Concavity of the Lagrangian

We now check that the Lagrangian, when evaluated at the multipliers, is indeed concave. First, we check that the jumps in Λ at γ_L and γ_H are nonnegative. The jumps are

$$\begin{aligned}&1-\kappa F(\gamma_L)\leq 1-w_\pi\big(\gamma_L,\pi_f(\gamma_L)\big)f(\gamma_L),\\&1-w_\pi\big(\gamma_H,\pi_f(\gamma_H)\big)f(\gamma_H)\leq 1+\kappa\big(1-F(\gamma_H)\big).\end{aligned}$$

To show this, we use conditions (c2), (c2′), (c3), and (c3′) as follows.

If $\gamma_L>\underline{\gamma}$, we know that the inequality in (c3) must be satisfied with equality at γ_L. Hence we can sign the derivative at γ_L, and we get that

$$w_\pi\big(\gamma_L,\pi_f(\gamma_L)\big)\frac{f(\gamma_L)}{F(\gamma_L)}\leq\kappa,$$

which delivers that the jump at γ_L is nonnegative. If $\gamma_L=\underline{\gamma}$, then (c3′) directly implies that the jump at $\underline{\gamma}$ is nonnegative. A similar argument, using (c2) and (c2′), works to show that the jump at γ_H is nonnegative.

Using that $\Lambda(\overline{\gamma})=\Lambda(\underline{\gamma})=1$, we can write the Lagrangian as

$$\begin{aligned}\mathcal{L}(\boldsymbol{\pi}|\Lambda)=&\int_\Gamma\big[w\big(\gamma,\boldsymbol{\pi}(\gamma)\big)-\kappa\big(\gamma\boldsymbol{\pi}(\gamma)+b\big(\boldsymbol{\pi}(\gamma)\big)\big)\big]f(\gamma)\,d\gamma\\&-\int_\Gamma\big(1-\Lambda(\gamma)\big)\boldsymbol{\pi}(\gamma)\,d\gamma\\&+\int_\Gamma\big(\gamma\boldsymbol{\pi}(\gamma)+b\big(\boldsymbol{\pi}(\gamma)\big)\big)\,d\big(\kappa F(\gamma)+\Lambda(\gamma)\big).\end{aligned}\tag{28}$$

By the definition of κ in equation (2), we see that $w(\gamma,\boldsymbol{\pi}(\gamma))-\kappa b(\boldsymbol{\pi}(\gamma))$ is concave in $\boldsymbol{\pi}(\gamma)$; further, condition (c1) and the fact that jumps at γ_H and γ_L are nonnegative imply that $\kappa F(\gamma)+\Lambda(\gamma)$ is nondecreasing. Hence, the above Lagrangian is concave at the proposed multiplier.

Maximizing the Lagrangian

We now proceed to show that the proposed allocation $\boldsymbol{\pi}^\star$ maximizes the Lagrangian. For this, we use the sufficiency part of Lemma A.2 in Amador, Werning, and Angeletos (2006), which concerns the maximization of concave functionals in a convex cone.

First, let us extend b and w to the entire positive ray of the real line in the following way[50]:

$$\hat{w}(\gamma,\pi)=\begin{cases} w(\gamma,\pi_0)+w_\pi(\gamma,\pi_0)(\pi-\pi_0), & \text{for } \pi\in[0,\pi_0),\\ w(\gamma,\pi), & \text{for } \pi\in[\pi_0,\pi_1],\\ w(\gamma,\pi_1)+w_\pi(\gamma,\pi_1)(\pi-\pi_1), & \text{for } \pi\in(\pi_1,\infty), \end{cases}$$

for all $\gamma\in\Gamma$ and $\pi\in[0,\infty)$, and for some values π_0 and π_1 such that $\pi_0\in(0,\pi_f(\underline{\gamma}))$ and $\pi_1\in(\pi_f(\overline{\gamma}),\overline{\pi})$. We similarly define $\hat{b}$. These extensions are possible as a result of Assumption 1, which ensures the interiority of the flexible allocation and the continuity of the derivative in $(0,\overline{\pi})$. Then we let $\hat{\Phi}=\{\boldsymbol{\pi}|\boldsymbol{\pi}:\Gamma\to\mathbb{R}_+ \text{ and } \boldsymbol{\pi} \text{ nondecreasing}\}$. Note that $\hat{\Phi}$ is a convex cone, and both $\hat{b}$ and $\hat{w}$ are continuous, differentiable, and concave. We then define the extended Lagrangian, $\hat{\mathcal{L}}(\boldsymbol{\pi}|\Lambda)$, as in (28) but using $\hat{w}$ and $\hat{b}$ instead of w and b. Note that, by concavity of the Lagrangian, $\hat{\mathcal{L}}(\boldsymbol{\pi}|\Lambda)\geq\mathcal{L}(\boldsymbol{\pi}|\Lambda)$ for $\boldsymbol{\pi}\in\Phi$. By the definitions of $\hat{w}$ and $\hat{b}$, it follows that $\hat{\mathcal{L}}(\boldsymbol{\pi}^\star|\Lambda)=\mathcal{L}(\boldsymbol{\pi}^\star|\Lambda)$. So if $\hat{\mathcal{L}}$ is maximized at $\boldsymbol{\pi}^\star$, so is $\mathcal{L}$.

We can now use Lemma A.2 in Amador, Werning, and Angeletos (2006), which states that the Lagrangian $\hat{\mathcal{L}}$ is maximized at $\boldsymbol{\pi}^\star$ if $\hat{\mathcal{L}}$ is a concave functional defined in a convex cone $\hat{\Phi}$; $\partial\hat{\mathcal{L}}(\boldsymbol{\pi}^\star;\boldsymbol{\pi}^\star|\Lambda)=0$; and $\partial\hat{\mathcal{L}}(\boldsymbol{\pi}^\star;\mathbf{x}|\Lambda)\leq 0$ for all $\mathbf{x}\in\hat{\Phi}$, where the first-order conditions are in terms of Gateaux differentials.[51] Now note that $\partial\hat{\mathcal{L}}(\boldsymbol{\pi}^\star;\mathbf{x}|\Lambda)=\partial\mathcal{L}(\boldsymbol{\pi}^\star;\mathbf{x}|\Lambda)$ for all $\mathbf{x}\in\hat{\Phi}$. This follows from the definition of the Gateaux differential, the interiority of $\boldsymbol{\pi}^\star$, and the definitions of $\hat{w}$ and $\hat{b}$, which taken together imply that, for any $\mathbf{x}\in\hat{\Phi}$, there exists $\varepsilon>0$ such that $\hat{\mathcal{L}}(\boldsymbol{\pi}^\star+\alpha\mathbf{x})=\mathcal{L}(\boldsymbol{\pi}^\star+\alpha\mathbf{x})$ for all $0<\alpha<\varepsilon$. We can then say that if

$$\partial\mathcal{L}\big(\boldsymbol{\pi}^\star;\boldsymbol{\pi}^\star|\Lambda\big)=0,$$

$$\partial\mathcal{L}\big(\boldsymbol{\pi}^\star;\mathbf{x}|\Lambda\big)\leq 0;\quad \text{for all } \mathbf{x}\in\hat{\Phi},$$

then $\boldsymbol{\pi}^\star$ maximizes the Lagrangian $\mathcal{L}$.

[50]Note that if $\Pi=[0,\infty)$, then this extension is not necessary as Φ is a convex cone.

[51]Given a function $T:\Omega\to Y$, where $\Omega\subset X$ and X and Y are normed spaces, if for $x\in\Omega$ and $h\in X$ the limit

$$\lim_{\alpha\downarrow 0}\frac{1}{\alpha}\big[T(x+\alpha h)-T(x)\big]$$

exists, then it is called the Gateaux differential at x with direction h and is denoted by $\partial T(x;h)$.

For our problem, taking the Gateaux differential in direction $\mathbf{x} \in \hat{\Phi}$ and using that $b'(\pi_f(\gamma)) = -\gamma$, we get that[52]

$$\begin{aligned}(29)\qquad \partial\mathcal{L}\big(\boldsymbol{\pi}^\star;\mathbf{x}|\Lambda\big) &= \int_\Gamma \big[w_\pi\big(\gamma,\boldsymbol{\pi}^\star(\gamma)\big)f(\gamma) - \big(1-\Lambda(\gamma)\big)\big]\mathbf{x}(\gamma)\,d\gamma \\ &\quad + \int_{\underline{\gamma}}^{\gamma_L} (\gamma-\gamma_L)\mathbf{x}(\gamma)\,d\Lambda(\gamma) + \int_{\gamma_H}^{\overline{\gamma}} (\gamma-\gamma_H)\mathbf{x}(\gamma)\,d\Lambda(\gamma),\end{aligned}$$

which can be rewritten as

$$\begin{aligned}\partial\mathcal{L}\big(\boldsymbol{\pi}^\star;\mathbf{x}|\Lambda\big) &= \int_{\underline{\gamma}}^{\gamma_L} \big[w_\pi\big(\gamma,\pi_f(\gamma_L)\big)f(\gamma) \\ &\quad - \kappa F(\gamma) - \kappa(\gamma-\gamma_L)f(\gamma)\big]\mathbf{x}(\gamma)\,d\gamma \\ &\quad + \int_{\gamma_H}^{\overline{\gamma}} \big[w_\pi\big(\gamma,\pi_f(\gamma_H)\big)f(\gamma) + \kappa\big(1-F(\gamma)\big) \\ &\quad - \kappa(\gamma-\gamma_H)f(\gamma)\big]\mathbf{x}(\gamma)\,d\gamma.\end{aligned}$$

Integrating by parts, we have[53]

$$\begin{aligned}\partial\mathcal{L}\big(\boldsymbol{\pi}^\star;\mathbf{x}|\Lambda\big) &= \bigg[\int_{\underline{\gamma}}^{\gamma_L} \big[w_\pi\big(\gamma,\pi_f(\gamma_L)\big)f(\gamma) - \kappa F(\gamma) \\ &\quad - \kappa(\gamma-\gamma_L)f(\gamma)\big]\,d\gamma\bigg]\mathbf{x}(\gamma_L) \\ &\quad - \int_{\underline{\gamma}}^{\gamma_L}\bigg[\int_{\underline{\gamma}}^{\gamma} \big[w_\pi\big(\tilde{\gamma},\pi_f(\gamma_L)\big)f(\tilde{\gamma}) - \kappa F(\tilde{\gamma}) \\ &\quad - \kappa(\tilde{\gamma}-\gamma_L)f(\tilde{\gamma})\big]\,d\tilde{\gamma}\bigg]\,d\mathbf{x}(\gamma)\end{aligned}$$

[52] Existence of the Gateaux differential follows from Lemma A.1 of Amador, Werning, and Angeletos (2006). To be able to use that lemma, first note that the Lagrangian (28) is written as the sum of three terms. The middle one is linear in $\boldsymbol{\pi}$, so we can obtain directly the Gateaux differential. The remaining two terms are then integrals with integrands that are concave and satisfy the hypothesis of Lemma A.1. Existence of the integrals in the right hand side of equation (29) follows from Λ being of bounded variation and x being monotone in γ, and thus integrable, together with $w_\pi(\gamma,\boldsymbol{\pi}^\star(\gamma))$ bounded and continuous in γ. The continuity of w_π follows from Assumption 1 and that $\boldsymbol{\pi}^\star$ is continuous. It follows also that w_π is bounded as it is a continuous real function in a compact set.

[53] Integration by parts works, as one of the functions involved in each case is continuous. Existence of the integrals follows from $w_\pi(\gamma,\boldsymbol{\pi}^\star(\gamma))$ being bounded and continuous in γ, as stated in footnote 52.

$$
\begin{aligned}
&+\left[\int_{\gamma_H}^{\overline{\gamma}} \left[w_\pi\big(\gamma, \pi_f(\gamma_H)\big) f(\gamma) + \kappa\big(1 - F(\gamma)\big)\right.\right. \\
&\left.\left. - \kappa(\gamma - \gamma_H) f(\gamma)\right] d\gamma\right] \mathbf{x}(\gamma_H) \\
&+ \int_{\gamma_H}^{\overline{\gamma}} \left[\int_{\gamma}^{\overline{\gamma}} \left[w_\pi\big(\tilde{\gamma}, \pi_f(\gamma_H)\big) f(\tilde{\gamma}) + \kappa\big(1 - F(\tilde{\gamma})\big)\right.\right. \\
&\left.\left. - \kappa(\tilde{\gamma} - \gamma_H) f(\tilde{\gamma})\right] d\tilde{\gamma}\right] d\mathbf{x}(\gamma).
\end{aligned}
$$

We require that this differential be nonpositive for all nondecreasing $\mathbf{x}$ and zero when evaluated at $\mathbf{x} = \boldsymbol{\pi}^\star$. Note that for $\gamma \in [\underline{\gamma}, \gamma_L] \cup [\gamma_H, \overline{\gamma}]$, if $\mathbf{x} = \boldsymbol{\pi}^\star$, then $d\mathbf{x}(\gamma) = 0$. So we need that

$$
\begin{aligned}
&\int_{\underline{\gamma}}^{\gamma} \left[w_\pi\big(\tilde{\gamma}, \pi_f(\gamma_L)\big) f(\tilde{\gamma}) - \kappa F(\tilde{\gamma}) - \kappa(\tilde{\gamma} - \gamma_L) f(\tilde{\gamma})\right] d\tilde{\gamma} \geq 0 \\
&\quad \forall \gamma \in [\underline{\gamma}, \gamma_L] \text{ with equality at } \gamma_L, \\
&\int_{\gamma}^{\overline{\gamma}} \left[w_\pi\big(\tilde{\gamma}, \pi_f(\gamma_H)\big) f(\tilde{\gamma}) + \kappa\big(1 - F(\tilde{\gamma})\big) - \kappa(\tilde{\gamma} - \gamma_H) f(\tilde{\gamma})\right] d\tilde{\gamma} \leq 0 \\
&\quad \forall \gamma \in [\gamma_H, \overline{\gamma}] \text{ with equality at } \gamma_H.
\end{aligned}
$$

Note that the above equations are implied by

$$
\begin{aligned}
&\int_{\underline{\gamma}}^{\gamma} \left[w_\pi\big(\tilde{\gamma}, \pi_f(\gamma_L)\big) \frac{f(\tilde{\gamma})}{F(\gamma)}\right] d\tilde{\gamma} \geq \kappa(\gamma - \gamma_L) \\
&\quad \forall \gamma \in [\underline{\gamma}, \gamma_L] \text{ with equality at } \gamma_L, \\
&\int_{\gamma}^{\overline{\gamma}} \left[w_\pi\big(\tilde{\gamma}, \pi_f(\gamma_H)\big) \frac{f(\tilde{\gamma})}{1 - F(\gamma)}\right] d\tilde{\gamma} \leq \kappa(\gamma - \gamma_H) \\
&\quad \forall \gamma \in [\gamma_H, \overline{\gamma}] \text{ with equality at } \gamma_H,
\end{aligned}
$$

if γ_H or γ_L is interior, respectively. Thus if γ_H or γ_L is interior, then (c2) or (c3) is sufficient for the satisfaction of the respective above equation. If not, that is, if $\gamma_L = \underline{\gamma}$ or $\gamma_H = \overline{\gamma}$, then the respective above equation is automatically satisfied.

Hence, using concavity of Lagrangian plus Lemma A.2 in Amador, Werning, and Angeletos (2006), we have shown that the proposed allocation $\boldsymbol{\pi}^\star$ maximizes the Lagrangian (27) given the multipliers.

Applying Luenberger's Sufficiency Theorem

We now apply Theorem 1 in Appendix B. To apply this theorem, we set (i) $x_0 \equiv \boldsymbol{\pi}^\star$; (ii) $X \equiv \{\boldsymbol{\pi} | \boldsymbol{\pi} : \Gamma \to \Pi\}$; (iii) f to be given by the negative of the objective function, $\int_\Gamma w(\gamma, \boldsymbol{\pi}(\gamma))\, dF(\gamma)$, as a function of $\boldsymbol{\pi}$; (iv) $Z \equiv \{(z_1, z_2) | z_1 : \Gamma \to \mathbb{R}$ and $z_2 : \Gamma \to \mathbb{R}$ with z_1, z_2 integrable$\}$; (v) $\Omega \equiv \Phi$; (vi) $P \equiv \{(z_1, z_2) | (z_1, z_2) \in Z$ such that $z_1(\gamma) \geq 0$ and $z_2(\gamma) \geq 0$ for all $\gamma \in \Gamma\}$; (vii) G to be the mapping from Φ to Z given by the left hand sides of inequalities (25) and (26); (viii) the linear operator T is given by

$$T\big((z_1, z_2)\big) \equiv \int_\Gamma z_1(\gamma)\, d\Lambda_1(\gamma) + \int_\Gamma z_2(\gamma)\, d\Lambda_2(\gamma),$$

and Λ_1 and Λ_2 being nondecreasing functions implies that $T(z) \geq 0$ for $z \in P$. We have that

$$\begin{aligned} T\big(G(x_0)\big) \equiv \int_\Gamma \bigg(\int_{\underline{\gamma}}^{\gamma} \boldsymbol{\pi}^\star\big(\gamma'\big)\, d\gamma' + \underline{U} - \gamma \boldsymbol{\pi}^\star(\gamma) \\ - b\big(\boldsymbol{\pi}^\star(\gamma)\big) \bigg)\, d\big(\Lambda_1(\gamma) - \Lambda_2(\gamma)\big) = 0, \end{aligned}$$

where the last equality follows from the fact that there is no money burned. We have found conditions under which the proposed allocation, $\boldsymbol{\pi}^\star$, minimizes $f(x) + T(G(x))$ for $x \in \Omega$. Given that $T(G(x_0)) = 0$, then the conditions of Theorem 1 hold, and it follows that $\boldsymbol{\pi}^\star$ solves $\min_{x \in \Omega} f(x)$ subject to $-G(x) \in P$, which is Problem (P) with the additional constraint (1).

Proof of Part (b) of Proposition 1

Using the integral form for the incentive constraints, Problem (P) becomes

$$\max_{\left\{ \substack{\boldsymbol{\pi} : \Gamma \to \Pi, \\ \mathbf{t} : \Gamma \to \mathbb{R}} \right\}} \int \big(w\big(\gamma, \boldsymbol{\pi}(\gamma)\big) - \mathbf{t}(\gamma)\big)\, dF(\gamma) \quad \text{subject to:}$$

$$\gamma \boldsymbol{\pi}(\gamma) + b\big(\boldsymbol{\pi}(\gamma)\big) - \mathbf{t}(\gamma) = \int_{\underline{\gamma}}^{\gamma} \boldsymbol{\pi}(\tilde{\gamma})\, d\tilde{\gamma} + \underline{U} \quad \text{for all } \gamma \in \Gamma,$$

$$\boldsymbol{\pi} \text{ nondecreasing, and } \mathbf{t}(\gamma) \geq 0 \quad \text{for all } \gamma \in \Gamma,$$

where $\underline{U} \equiv \underline{\gamma} \boldsymbol{\pi}(\underline{\gamma}) + b(\boldsymbol{\pi}(\underline{\gamma})) - \mathbf{t}(\underline{\gamma})$.

Solving the integral equation for $\mathbf{t}(\gamma)$ and substituting into both the objective and the nonnegativity constraint, we get the following equivalent problem:

$$\text{(P2}'\text{)}\qquad \max_{\left\{\substack{\boldsymbol{\pi}:\Gamma\to\Pi,\\ \mathbf{t}(\underline{\gamma})\geq 0}\right\}} \int \big(v\big(\gamma,\boldsymbol{\pi}(\gamma)\big)f(\gamma)+\big(1-F(\gamma)\big)\boldsymbol{\pi}(\gamma)\big)\,d\gamma+\underline{U}\quad \text{subject to:}$$

$$\gamma\boldsymbol{\pi}(\gamma)+b\big(\boldsymbol{\pi}(\gamma)\big)-\int_{\underline{\gamma}}^{\gamma}\boldsymbol{\pi}(\tilde{\gamma})\,d\tilde{\gamma}-\underline{U}\geq 0\quad \text{for all } \gamma\in\Gamma, \tag{30}$$

$$\boldsymbol{\pi}\text{ nondecreasing,} \tag{31}$$

where v is defined such that $v(\gamma,\boldsymbol{\pi}(\gamma))\equiv w(\gamma,\boldsymbol{\pi}(\gamma))-b(\boldsymbol{\pi}(\gamma))-\gamma\boldsymbol{\pi}(\gamma)$.

Note that once we have solved this program for $\boldsymbol{\pi}(\gamma)$ and $\mathbf{t}(\underline{\gamma})$, we can recover $\mathbf{t}(\gamma)$ via

$$\mathbf{t}(\gamma)=\gamma\boldsymbol{\pi}(\gamma)+b\big(\boldsymbol{\pi}(\gamma)\big)-\int_{\underline{\gamma}}^{\gamma}\boldsymbol{\pi}(\tilde{\gamma})\,d\tilde{\gamma}-\underline{U}.$$

Let the associated Lagrangian be defined as

$$\begin{aligned}&\mathcal{L}\big(\boldsymbol{\pi},\mathbf{t}(\underline{\gamma})|\tilde{\Lambda}\big)\\&\quad\equiv\int_{\gamma\in\Gamma}\big(v\big(\gamma,\boldsymbol{\pi}(\gamma)\big)f(\gamma)+\big(1-F(\gamma)\big)\boldsymbol{\pi}(\gamma)\big)\,d\gamma+\underline{U}\\&\qquad-\int_{\gamma\in\Gamma}\left(\int_{\underline{\gamma}}^{\gamma}\boldsymbol{\pi}\big(\gamma'\big)\,d\gamma'+\underline{U}-\gamma\boldsymbol{\pi}(\gamma)-b\big(\boldsymbol{\pi}(\gamma)\big)\right)d\tilde{\Lambda}(\gamma),\end{aligned}$$

where $\tilde{\Lambda}$ is the (cumulative) Lagrange multiplier associated with equation (30). It is required that $\tilde{\Lambda}$ be nondecreasing.

Integrating by parts, and setting $\tilde{\Lambda}(\overline{\gamma})=1$ without loss of generality, we get

$$\begin{aligned}\mathcal{L}\big(\boldsymbol{\pi},\mathbf{t}(\underline{\gamma})|\tilde{\Lambda}\big)&\equiv\int_{\gamma\in\Gamma}\big(v\big(\gamma,\boldsymbol{\pi}(\gamma)\big)f(\gamma)+\big(\tilde{\Lambda}(\gamma)-F(\gamma)\big)\boldsymbol{\pi}(\gamma)\big)\,d\gamma\\&\quad+\int_{\gamma\in\Gamma}\big(\gamma\boldsymbol{\pi}(\gamma)+b\big(\boldsymbol{\pi}(\gamma)\big)\big)\,d\tilde{\Lambda}(\gamma)+\tilde{\Lambda}(\underline{\gamma})\underline{U}.\end{aligned} \tag{32}$$

A Proposed Multiplier

Our proposed multiplier in this case is

$$\tilde{\Lambda}(\gamma)=\begin{cases}(1-\kappa)F(\gamma)+\kappa, & \text{for } \gamma\in[\gamma_H,\overline{\gamma}],\\ F(\gamma)-w_{\pi}\big(\gamma,\pi_f(\gamma)\big)f(\gamma), & \text{for } \gamma\in(\gamma_L,\gamma_H),\\ (1-\kappa)F(\gamma), & \text{for } \gamma\in[\underline{\gamma},\gamma_L],\end{cases}$$

where κ is given by definition (3).

Monotonicity of the Lagrange Multiplier

Now let us show that the Lagrange multiplier is nondecreasing. In the flexibility region, $\gamma \in (\gamma_L, \gamma_H)$, the Lagrange multiplier can be written as $\tilde{\Lambda}(\gamma) = \kappa F(\gamma) - w_\pi(\gamma, \pi_f(\gamma))f(\gamma) + (1-\kappa)F(\gamma)$. Under (c1) and the definition of κ in equation (3), which ensures $\kappa \le 1$, this is the sum of two nondecreasing functions and hence is nondecreasing. In the interior of the pooling regions, $\gamma < \gamma_L$ or $\gamma > \gamma_H$, the Lagrange multiplier is also nondecreasing since $\kappa \le 1$, by equation (3). We now need to check that at the jumps, $\{\gamma_L, \gamma_H\}$, the Lagrange multiplier is nondecreasing. At γ_L, we have that the Lagrange multiplier has a jump of size

$$\kappa F(\gamma_L) - w_\pi\big(\gamma_L, \pi_f(\gamma_L)\big)f(\gamma_L).$$

We need to consider two cases. If $\gamma_L = \underline{\gamma}$, then the jump is equal to $-w_\pi(\underline{\gamma}, \pi_f(\underline{\gamma}))f(\underline{\gamma})$, which is nonnegative by (c3′). If $\gamma_L > \underline{\gamma}$, then condition (c3) holds at γ_L with equality. Taking the derivative of that condition, it must then be that

$$\kappa - w_\pi\big(\gamma_L, \pi_f(\gamma_L)\big)f(\gamma_L)/F(\gamma_L) \ge 0,$$

which implies that the jump in the multiplier is nonnegative. A similar argument, using (c2) and (c2′), shows that the Lagrange multiplier has a nonnegative jump at γ_H, and so we have shown that the proposed Lagrange multiplier is nondecreasing.

Concavity of the Lagrangian

We first check that the Lagrangian is concave in the allocation at the proposed multiplier. Note that we can write the Lagrangian as

$$\begin{aligned}\mathcal{L}\big(\boldsymbol{\pi}, \mathbf{t}(\underline{\gamma})|\tilde{\Lambda}\big) \equiv \int_{\gamma\in\Gamma} &\big\{\big[w\big(\gamma, \boldsymbol{\pi}(\gamma)\big) - \kappa\gamma\boldsymbol{\pi}(\gamma) - \kappa b\big(\boldsymbol{\pi}(\gamma)\big)\big]f(\gamma)\\ &+ \big(\tilde{\Lambda}(\gamma) - F(\gamma)\big)\boldsymbol{\pi}(\gamma)\big\}\,d\gamma\\ &+ \int_{\gamma\in\Gamma}\big(\gamma\boldsymbol{\pi}(\gamma) + b\big(\boldsymbol{\pi}(\gamma)\big)\big)\,d\big((\kappa-1)F(\gamma) + \tilde{\Lambda}(\gamma)\big),\end{aligned}$$

where we use that $\tilde{\Lambda}(\underline{\gamma}) = 0$. By the definition of κ, we have that $w(\gamma, \pi) - \kappa b(\pi)$ is concave in π. To see this, note that the second derivative is $w_{\pi\pi}(\gamma, \pi) - \kappa b''(\pi) = b''(\pi)(w_{\pi\pi}(\gamma, \pi)/b''(\pi) - \kappa)$. The last term in brackets is nonnegative given our definition of κ, and hence the function $w(\gamma, \pi) - \kappa b(\pi)$ is concave in π. Finally, we note that, from (c1) and the fact that jumps in the multiplier are nonnegative, it follows that $(\kappa-1)F(\gamma) + \tilde{\Lambda}(\gamma)$ is nondecreasing, which is needed in the second integral to guarantee that the concavity of b is not reversed.

Maximizing the Lagrangian

That the Lagrangian is maximized at the proposed allocation is similar to the argument used in our proof for part (a) of Proposition 1 (the no money burning case). To see this, first note that $\mathbf{t}(\underline{\gamma})$ does not appear in the Lagrangian, given the proposed Lagrange multiplier. This implies that we can restrict attention to maximizing the Lagrangian over just $\boldsymbol{\pi}(\gamma)$ for $\gamma \in \Gamma$. Now, let $\Lambda(\gamma) = 1 - F(\gamma) + \tilde{\Lambda}(\gamma)$. Then the Lagrangian can be rewritten as

$$\begin{aligned}\mathcal{L}(\boldsymbol{\pi}, \mathbf{t}(\underline{\gamma})|\Lambda) \equiv &\int_{\gamma\in\Gamma} \{[w(\gamma, \boldsymbol{\pi}(\gamma)) - \kappa(\gamma\boldsymbol{\pi}(\gamma) + b(\boldsymbol{\pi}(\gamma)))]f(\gamma) \\ &- (1 - \Lambda(\gamma))\boldsymbol{\pi}(\gamma)\}\,d\gamma \\ &+ \int_{\gamma\in\Gamma} (\gamma\boldsymbol{\pi}(\gamma) + b(\boldsymbol{\pi}(\gamma)))\,d(\kappa F(\gamma) + \Lambda(\gamma)),\end{aligned}$$

which is equivalent to the Lagrangian in the proof of part (a) of Proposition 1 with κ given by equation (3), and where Λ is the Lagrange multiplier as defined in that section. The same argument used there shows that, given the conditions of the proposition, which are written in terms of κ given by equation (3), the Lagrangian is maximized at an interval allocation.

Applying Luenberger's Sufficiency Theorem

We now apply Theorem 1. To apply this theorem, we set (i) $x_0 \equiv (\boldsymbol{\pi}^\star, 0)$; (ii) f to be given by the negative of the objective function, $f \equiv -\int_\Gamma (v(\gamma, \boldsymbol{\pi}(\gamma))f(\gamma) + (1 - F(\gamma))\boldsymbol{\pi}(\gamma))\,d\gamma - \underline{U}$; (iii) $X \equiv \{\boldsymbol{\pi}, \underline{t}|\underline{t} \in \mathbb{R}_+ \text{ and } \boldsymbol{\pi} : \Gamma \to \Pi\}$; (iv) $Z \equiv \{z|z : \Gamma \to \mathbb{R} \text{ with } z \text{ integrable}\}$; (v) $\Omega \equiv \{\boldsymbol{\pi}, \underline{t}|\underline{t} \in \mathbb{R}_+, \boldsymbol{\pi} : \Gamma \to \Pi;\ \text{and } \boldsymbol{\pi} \text{ nondecreasing}\}$; (vi) $P \equiv \{z|z \in Z \text{ such that } z(\gamma) \geq 0 \text{ for all } \gamma \in \Gamma\}$; (vii) G to be the mapping from Ω to Z given by the negative of the left hand side of inequality (30); (viii) $T(z)$ be the linear mapping

$$T(z) = \int_\Gamma z(\gamma)\,d\tilde{\Lambda}(\gamma),$$

where $T(z) \geq 0$ for $z \in P$ follows from $\tilde{\Lambda}$ nondecreasing. We have that

$$\begin{aligned}T(G(x_0)) &\equiv \int_\Gamma \left(\int_{\underline{\gamma}}^{\gamma} \boldsymbol{\pi}^\star(\tilde{\gamma})\,d\tilde{\gamma} + \underline{U} - \gamma\boldsymbol{\pi}^\star(\gamma) - b(\boldsymbol{\pi}^\star(\gamma)) \right) d\tilde{\Lambda}(\gamma) \\ &= 0,\end{aligned}$$

which follows from $\mathbf{t}(\underline{\gamma}) = 0$ for all γ. We have found conditions under which the proposed allocation, $x_0 = (\boldsymbol{\pi}^\star, 0)$, minimizes $f(x) + T(G(x))$ for $x \in \Omega$. Given that $T(G(x_0)) = 0$, the conditions of Theorem 1 hold, and it follows that $(\boldsymbol{\pi}^\star, 0)$ solves $\min_{x\in\Omega} f(x)$ subject to $-G(x) \in P$, which is Problem (P).

Q.E.D.

APPENDIX D: PROOF OF PROPOSITION 2

The proof of this proposition will proceed through a series of lemmas.

We first show that, in the case with money burning, if $\kappa = 1$, then the conditions in Proposition 1 are also necessary.

LEMMA 4—Necessity With Money Burning When $\kappa = 1$: *Let κ be given by equation* (3). *If $\kappa = 1$, then conditions* (c1), (c2), (c2′), (c3), *and* (c3′) *are necessary for an interval allocation with bounds γ_L, γ_H to solve Problem* (P).

PROOF: The proof here follows the proof of Proposition 4, the necessity result, in Amador, Werning, and Angeletos (2006). We proceed to use Theorem 1 (a necessity theorem) in Luenberger (1969, p. 217). Let (i) f to be given by the negative of the objective function, $f \equiv -\int_\Gamma (v(\gamma, \boldsymbol{\pi}(\gamma))f(\gamma) + (1 - F(\gamma))\boldsymbol{\pi}(\gamma))\,d\gamma - \underline{U}$; (ii) $X \equiv \{\boldsymbol{\pi}, \underline{t} | \underline{t} \in \mathbb{R}_+ \text{ and } \boldsymbol{\pi} : \Gamma \to \Pi\}$; (iii) $Z \equiv \{z | z : \Gamma \to \mathbb{R} \text{ and } z \text{ continuous}\}$ with the norm $\|z\| = \sup |z(\gamma)|$; (iv) $\Omega \equiv \{\boldsymbol{\pi}, \underline{t} | \underline{t} \in \mathbb{R}_+, \boldsymbol{\pi} : \Gamma \to \Pi, \text{ nondecreasing and continuous}\}$; (v) $P \equiv \{z | z \in Z \text{ such that } z(\gamma) \geq 0 \text{ for all } \gamma \in \Gamma\}$; (vi) G to be the mapping from Ω to Z given by the negative of the left hand side of inequality (30). Note that we are restricting the choice set to be the set of nondecreasing and *continuous* functions $\boldsymbol{\pi}$. This is because we are looking for necessary conditions for the optimal allocation to be an interval, which is continuous.

Note that, given that $\kappa = 1$ (by the hypothesis of the proposition), f is convex. Note as well that G is convex, Ω is convex, P contains an interior point (e.g., $z(\gamma) = 1$ for all $\gamma \in \Gamma$), and that the positive dual of Z is isomorphic to the space of nondecreasing functions on Γ by the Riesz representation theorem (see Luenberger (1969, chapter 5, p. 113)). Note as well that if $(\boldsymbol{\pi}, \underline{t})$ is optimal and lies in Ω, then it must be optimal within Ω. To see that there exists an interior point to the constraint set, just consider the allocation x_1 that bunches every type at some π_1 and burns a strictly positive amount. That allocation is in Ω and generates a function $G(x_1)$ that is in the interior of the negative cone $-P$. Given that the proposed allocation is continuous, it follows that the hypothesis of Theorem 1 of Luenberger (1969, p. 217) holds and there exists a nondecreasing function Λ_0, such that the Lagrangian, $\mathcal{L}(\boldsymbol{\pi}, \underline{t} | \Lambda_0)$, is maximized at $(\boldsymbol{\pi}^\star, \mathbf{t}^\star(\underline{\gamma}))$ within Ω.[54] Without loss of generality, we normalize $\Lambda_0(\overline{\gamma}) = 1$.

In a similar fashion as in the proof of part (a) of Proposition 1, we can now use Lemmas A.1 and A.2 of Amador, Werning, and Angeletos (2006), and argue that if the Lagrangian is maximized at some $(\boldsymbol{\pi}^\star, \mathbf{t}^\star(\underline{\gamma})) \in \Omega$, then it must

[54] Note that the Lagrangian in our case corresponds to the negative of the Lagrangian as defined in Luenberger (1969).

be the case that

$$\partial\mathcal{L}\big(\boldsymbol{\pi}^\star, \mathbf{t}^\star(\underline{\gamma}); \boldsymbol{\pi}^\star, \mathbf{t}^\star(\underline{\gamma})|\Lambda_0\big) = 0,$$
$$\partial\mathcal{L}\big(\boldsymbol{\pi}^\star, \mathbf{t}^\star(\underline{\gamma}); \mathbf{x}, y|\Lambda_0\big) \leq 0,$$

for all $(\mathbf{x}, y) \in \Omega$, and where, as before, the derivative is in terms of Gateaux differentials.

Taking the Gateaux differential of the Lagrangian in (32), we get[55]

$$\begin{aligned}
&\partial\mathcal{L}\big(\boldsymbol{\pi}^\star, \mathbf{t}^\star(\underline{\gamma}); \mathbf{x}, y|\Lambda_0\big) \\
&\quad = \int_{\gamma\in\Gamma} \big(v_\pi\big(\gamma, \boldsymbol{\pi}^\star(\gamma)\big) f(\gamma) + \big(\Lambda_0(\gamma) - F(\gamma)\big)\big)\mathbf{x}(\gamma)\, d\gamma \\
&\qquad + \int_{\underline{\gamma}}^{\gamma_L} (\gamma - \gamma_L)\mathbf{x}(\gamma)\, d\Lambda_0(\gamma) + \int_{\gamma_H}^{\overline{\gamma}} (\gamma - \gamma_H)\mathbf{x}(\gamma)\, d\Lambda_0(\gamma) \\
&\qquad + \Lambda_0(\underline{\gamma})\big((\underline{\gamma} - \gamma_L)\mathbf{x}(\underline{\gamma}) - y\big).
\end{aligned}$$

Let us define g to be such that:

$$\begin{aligned}
g(\gamma) \equiv &\int_{\gamma}^{\overline{\gamma}} \big(v_\pi\big(\tilde{\gamma}, \boldsymbol{\pi}^\star(\tilde{\gamma})\big) f(\tilde{\gamma}) + \big(\Lambda_0(\tilde{\gamma}) - F(\tilde{\gamma})\big)\big)\, d\tilde{\gamma} \\
&+ \int_{\gamma}^{\overline{\gamma}} \big[\mathbb{I}(\tilde{\gamma} < \gamma_L)(\tilde{\gamma} - \gamma_L) + \mathbb{I}(\tilde{\gamma} > \gamma_H)(\tilde{\gamma} - \gamma_H)\big]\, d\Lambda_0(\tilde{\gamma}).
\end{aligned}$$

Integrating by parts the derivative above (which can be done given that $\mathbf{x}$ is continuous), it follows that

$$\begin{aligned}
\partial\mathcal{L}\big(\boldsymbol{\pi}^\star, \mathbf{t}^\star(\underline{\gamma}); \mathbf{x}, y|\Lambda_0\big) = &\big[g(\underline{\gamma}) + \Lambda_0(\underline{\gamma})(\underline{\gamma} - \gamma_L)\big]\mathbf{x}(\underline{\gamma}) \\
&+ \int_{\underline{\gamma}}^{\overline{\gamma}} g(\gamma)\, d\mathbf{x}(\gamma) - \Lambda_0(\underline{\gamma})y.
\end{aligned}$$

The first-order conditions require the above to be nonpositive for all nondecreasing and nonnegative functions $\mathbf{x}$ and nonnegative y, and hence:

$$g(\underline{\gamma}) + \Lambda_0(\underline{\gamma})(\underline{\gamma} - \gamma_L) \leq 0; \quad g(\gamma) \leq 0; \quad \text{and} \quad \Lambda_0(\underline{\gamma}) \geq 0.$$

Using that $\partial\mathcal{L}(\boldsymbol{\pi}^\star, \mathbf{t}^\star(\underline{\gamma}); \boldsymbol{\pi}^\star, \mathbf{t}^\star(\underline{\gamma})|\Lambda_0) = 0$, it follows that (i) $g(\gamma) = 0$ for all $\gamma \in [\gamma_L, \gamma_H]$; (ii) $g(\underline{\gamma}) + \Lambda_0(\underline{\gamma})(\underline{\gamma} - \gamma_L) = 0$. From (i), we get that

$$\Lambda_0(\gamma) = F(\gamma) - v_\pi\big(\gamma, \boldsymbol{\pi}^\star(\gamma)\big) f(\gamma), \tag{33}$$

[55]See footnote 52 for existence of the Gateaux differential.

for $\gamma \in [\gamma_L, \gamma_H]$.[56] And using (ii) as well:

$$\int_{\underline{\gamma}}^{\gamma_L} \big(v_\pi\big(\tilde{\gamma}, \boldsymbol{\pi}^\star(\tilde{\gamma})\big)f(\tilde{\gamma}) + \big(\Lambda_0(\tilde{\gamma}) - F(\tilde{\gamma})\big)\big)\, d\tilde{\gamma} + \int_{\underline{\gamma}}^{\gamma_L} (\tilde{\gamma} - \gamma_L)\, d\Lambda_0(\tilde{\gamma}) + \Lambda_0(\underline{\gamma})(\underline{\gamma} - \gamma_L) = 0. \tag{34}$$

From $g(\gamma) \leq 0$ for $\gamma \in [\underline{\gamma}, \gamma_L) \cup (\gamma_H, \overline{\gamma}]$ and $g(\gamma) = 0$ for $\gamma \in [\gamma_L, \gamma_H]$, it follows that

$$\int_{\gamma}^{\overline{\gamma}} \big(v_\pi\big(\tilde{\gamma}, \boldsymbol{\pi}^\star(\tilde{\gamma})\big)f(\tilde{\gamma}) + \big(\Lambda_0(\tilde{\gamma}) - F(\tilde{\gamma})\big)\big)\, d\tilde{\gamma} + \int_{\gamma}^{\overline{\gamma}} (\tilde{\gamma} - \gamma_H)\, d\Lambda_0(\tilde{\gamma}) \leq 0 \quad \text{for } \gamma \in (\gamma_H, \overline{\gamma}], \tag{35}$$

$$\int_{\gamma}^{\gamma_L} \big(v_\pi\big(\tilde{\gamma}, \boldsymbol{\pi}^\star(\tilde{\gamma})\big)f(\tilde{\gamma}) + \big(\Lambda_0(\tilde{\gamma}) - F(\tilde{\gamma})\big)\big)\, d\tilde{\gamma} + \int_{\gamma}^{\gamma_L} (\tilde{\gamma} - \gamma_L)\, d\Lambda_0(\tilde{\gamma}) \leq 0 \quad \text{for } \gamma \in [\underline{\gamma}, \gamma_L). \tag{36}$$

Now note that $\int_\gamma^{\overline{\gamma}} \Lambda_0(\tilde{\gamma})\, d\tilde{\gamma} = \Lambda_0(\tilde{\gamma})(\tilde{\gamma} - \gamma)|_\gamma^{\overline{\gamma}} - \int_\gamma^{\overline{\gamma}} (\tilde{\gamma} - \gamma)\, d\Lambda_0(\tilde{\gamma}) = (\overline{\gamma} - \gamma) - \int_\gamma^{\overline{\gamma}} (\tilde{\gamma} - \gamma)\, d\Lambda_0(\tilde{\gamma})$. And from the first of the two inequalities above, we get that

$$\int_{\gamma}^{\overline{\gamma}} \big(v_\pi\big(\tilde{\gamma}, \pi_f(\gamma_H)\big)f(\tilde{\gamma}) + 1 - F(\tilde{\gamma})\big)\, d\tilde{\gamma} + (\gamma - \gamma_H)\big(1 - \Lambda_0(\gamma)\big) \leq 0 \quad \text{for } \gamma \in (\gamma_H, \overline{\gamma}].$$

And thus, the best chance we have for the above inequality to hold, given that Λ_0 is nondecreasing and $\Lambda_0(\overline{\gamma}) = 1$, is that $\Lambda_0(\gamma) = 1$ for all $\gamma \in (\gamma_H, \overline{\gamma})$. Hence, a necessary condition is that

$$\int_{\gamma}^{\overline{\gamma}} \big(v_\pi\big(\tilde{\gamma}, \pi_f(\gamma_H)\big)f(\tilde{\gamma}) + 1 - F(\tilde{\gamma})\big)\, d\tilde{\gamma} \leq 0 \quad \text{for } \gamma \in (\gamma_H, \overline{\gamma}]. \tag{37}$$

[56]For all $\gamma \in [\gamma_L, \gamma_H]$, we note that $g(\gamma) = \int_\gamma^{\gamma_H} h(\gamma)\, d\gamma$ for some integrable function h. From properties of absolute continuity, it follows that $g(\gamma) = 0$ for all $\gamma \in [\gamma_L, \gamma_H]$ only if $h(\gamma) = 0$ almost everywhere in $[\gamma_L, \gamma_H]$. For simplicity's sake, we do not write the "a.e." conditioning in what follows, although the reader should keep it in mind.

Now subtracting equation (36) from equation (34), we get

$$\int_{\underline{\gamma}}^{\gamma} \big(v_\pi\big(\tilde{\gamma}, \boldsymbol{\pi}^\star(\tilde{\gamma})\big)f(\tilde{\gamma}) - F(\tilde{\gamma})\big)\, d\tilde{\gamma} + \Lambda_0(\gamma)(\gamma - \gamma_L) \geq 0 \quad \text{for } \gamma \in [\underline{\gamma}, \gamma_L).$$

The best chance of satisfying this equation is when $\Lambda_0(\gamma) = 0$ for all $\gamma < \gamma_L$ (given that $\Lambda_0(\gamma)$ is nondecreasing and we have shown above that $\Lambda_0(\underline{\gamma}) \geq 0$). Then a necessary condition for optimality is that

$$\int_{\underline{\gamma}}^{\gamma} \big(v_\pi\big(\tilde{\gamma}, \pi_f(\gamma_L)\big)f(\tilde{\gamma}) - F(\tilde{\gamma})\big)\, d\tilde{\gamma} \geq 0 \quad \text{for } \gamma \in [\underline{\gamma}, \gamma_L). \tag{38}$$

Note that (c1) follows from (33) and the restriction that Λ_0 must be nondecreasing. Condition (c2) follows from (37), where the equality restriction follows from Lemma 1. Condition (c2′) follows also from Lemma 1. Condition (c3) follows from (38), where the equality restriction follows from Lemma 1. Condition (c3′) follows from Lemma 1. Hence, when $\kappa = 1$, the sufficient conditions (c1), (c2), (c2′), (c3), and (c3′) are also necessary. *Q.E.D.*

Now we proceed to obtain another set of necessary conditions. Suppose that, within the flexibility region, we were to remove the prescribed allocations for some types x to $x+\varepsilon$. This is a feasible change to the allocation and is incentive compatible as follows: there is a type $\gamma(\varepsilon)$ that is now indifferent between the allocations for types x and $x + \varepsilon$, and $\gamma(\varepsilon)$ satisfies

$$\gamma(\varepsilon) = -\frac{b(\pi_f(x+\varepsilon)) - b(\pi_f(x))}{\pi_f(x+\varepsilon) - \pi_f(x)}. \tag{39}$$

All types between x and $\gamma(\varepsilon)$ now choose the allocation for type x, while all types between $\gamma(\varepsilon)$ and $x + \varepsilon$ choose the allocation for type $x + \varepsilon$.

The following lemma characterizes some useful properties of the γ function.

LEMMA 5: *Let* $\gamma(\cdot)$ *be defined as in* (39). *Then*, (i) $\gamma(0) = x$; (ii) $\gamma'(0) = 1/2$; *and* (iii) $\gamma''(0) = \frac{1}{6}\frac{\pi_f''(x)}{\pi_f'(x)}$.

PROOF: Recall also that, in the flexible allocation, we have that $b'(\pi_f(x)) = -x$. The first point follows from taking the limit of (39). The second follows from

$$\gamma'(\varepsilon) = \frac{x+\varepsilon-\gamma(\varepsilon)}{\pi_f(x+\varepsilon) - \pi_f(x)}\pi_f'(x+\varepsilon),$$

and taking the limit. Using L'Hôpital's rule, we get $\gamma'(0) = -\gamma'(0) + 1$ and so $\gamma'(0) = 1/2$. Taking another derivative, we get that

$$\gamma''(\varepsilon) = \frac{1 - 2\gamma'(\varepsilon)}{\pi_f(x+\varepsilon) - \pi_f(x)} \pi_f'(x+\varepsilon) + \gamma'(\varepsilon) \frac{\pi_f''(x+\varepsilon)}{\pi_f'(x+\varepsilon)}.$$

And taking limits as $\varepsilon \to 0$, we get $\gamma''(0) = -2\gamma''(0) + \frac{1}{2}\frac{\pi_f''(x)}{\pi_f'(x)}$, which delivers that $\gamma''(0) = \frac{1}{6}\frac{\pi_f''(x)}{\pi_f'(x)}$. *Q.E.D.*

We can now compute the effect on welfare of removing the allocations prescribed for types x to $x+\varepsilon$: all types between x and $\gamma(\varepsilon)$ choose $\pi_f(x)$ in the new allocation, while all types between $\gamma(\varepsilon)$ and $x+\varepsilon$ choose $\pi_f(x+\varepsilon)$. The change in welfare, ΔW, is given by the following equation:

$$\Delta W(\varepsilon) = \int_x^{\gamma(\varepsilon)} w\big(\gamma, \pi_f(x)\big)\, dF(\gamma) + \int_{\gamma(\varepsilon)}^{x+\varepsilon} w\big(\gamma, \pi_f(x+\varepsilon)\big)\, dF(\gamma) - \int_x^{x+\varepsilon} w\big(\gamma, \pi_f(\gamma)\big)\, dF(\gamma). \tag{40}$$

Then we can prove the following result.

LEMMA 6: *Let f be differentiable, and $\Delta W(\cdot)$ be defined as in* (40). *Then* (i) $\Delta W(0) = 0$; (ii) $\Delta W'(0) = 0$; (iii) $\Delta W''(0) = 0$; *and* (iv) $\Delta W'''(0) = \frac{\pi_f'(x)}{4}[\frac{d}{dx} w_\pi(x, \pi_f(x)) f(x))] + \frac{1}{4}\pi_f'(x) f(x) w_{\pi\pi}(x, \pi_f(x))\pi_f'(x)$.

PROOF: Taking the first derivative with respect to ε, we get that

$$\Delta W'(\varepsilon) = \big(w\big(\gamma(\varepsilon), \pi_f(x)\big) - w\big(\gamma(\varepsilon), \pi_f(x+\varepsilon)\big)\big) f\big(\gamma(\varepsilon)\big)\gamma'(\varepsilon) + \int_{\gamma(\varepsilon)}^{x+\varepsilon} w_\pi\big(\gamma, \pi_f(x+\varepsilon)\big)\pi_f'(x+\varepsilon)\, dF(\gamma).$$

Thus, $\Delta W'(0) = 0$ since $\gamma(0) = x$ by Lemma 5. Taking one more derivative, we get

$$\begin{aligned}
&\Delta W''(\varepsilon) \\
&\quad = \left[\frac{d}{d\varepsilon}\big(f\big(\gamma(\varepsilon)\big)\gamma'(\varepsilon)\big)\right]\big(w\big(\gamma(\varepsilon), \pi_f(x)\big) - w\big(\gamma(\varepsilon), \pi_f(x+\varepsilon)\big)\big) \\
&\qquad + f\big(\gamma(\varepsilon)\big)\big(\gamma'(\varepsilon)\big)^2\big[w_\gamma\big(\gamma(\varepsilon), \pi_f(x)\big) - w_\gamma\big(\gamma(\varepsilon), \pi_f(x+\varepsilon)\big)\big] \\
&\qquad - 2f\big(\gamma(\varepsilon)\big)\gamma'(\varepsilon) w_\pi\big(\gamma(\varepsilon), \pi_f(x+\varepsilon)\big)\pi_f'(x+\varepsilon)
\end{aligned}$$

$$
\begin{aligned}
&+ w_\pi\big(x+\varepsilon, \pi_f(x+\varepsilon)\big)\pi_f'(x+\varepsilon)f(x+\varepsilon)\\
&+ \int_{\gamma(\varepsilon)}^{x+\varepsilon} \big[w_{\pi\pi}\big(\gamma, \pi_f(x+\varepsilon)\big)\big(\pi_f'(x+\varepsilon)\big)^2\\
&+ w_\pi\big(\gamma, \pi_f(x+\varepsilon)\big)\pi_f''(x+\varepsilon)\big]\,dF(\gamma),
\end{aligned}
$$

from where, using $\gamma(0)=x$ and $\gamma'(0)=1/2$ by Lemma 5, it follows that $\Delta W''(0)=0$.

Taking another derivative, and using our knowledge of $\gamma(\varepsilon)$ through Lemma 5, we can get, after some algebra, that

$$
\begin{aligned}
\Delta W'''(0) = {} & \frac{1}{4} w_\pi\big(x, \pi_f(x)\big)\pi_f'(x)f'(x) + \frac{1}{4} w_{\pi\gamma}\big(x, \pi_f(x)\big)\pi_f'(x)f(x)\\
&+ \frac{1}{2} w_{\pi\pi}\big(x, \pi_f(x)\big)\big(\pi_f'(x)\big)^2 f(x),
\end{aligned}
$$

which delivers the result. *Q.E.D.*

A necessary condition for optimality is that $\Delta W'''(0) \le 0$. Now, using that $\pi_f'(x) = -1/b_{\pi\pi}(\pi_f(x))$ and that $\pi_f'(\gamma) > 0$, the following has been proved.

LEMMA 7—Necessity in the Flexible Region: *Let f be differentiable. Then, an interval allocation with bounds γ_L, γ_H is optimal only if*

$$
\left(\frac{w_{\pi\pi}(\gamma, \pi_f(\gamma))}{b''(\pi_f(\gamma))}\right) f(\gamma) - \frac{d}{d\gamma}\big[w_\pi\big(\gamma, \pi_f(\gamma)\big) f(\gamma)\big] \ge 0,
$$

for all $\gamma \in [\gamma_L, \gamma_H]$.

Now we proceed to obtain a necessary condition that will apply at the pooling regions.

LEMMA 8—Necessity in the Pooling Region: *Let $g(\pi_0|\pi) \equiv \frac{b(\pi)-b(\pi_0)}{\pi_0-\pi}$. An interval allocation with bounds γ_L, γ_H is optimal only if*:

(a) *if $\gamma_H < \overline{\gamma}$, then*

$$
\int_{g(\pi_0|\pi_f(\gamma_H))}^{\overline{\gamma}} \left(\frac{w(\tilde{\gamma}, \pi_0) - w(\tilde{\gamma}, \pi_f(\gamma_H))}{\pi_0 - \pi_f(\gamma_H)}\right) f(\tilde{\gamma})\, d\tilde{\gamma} \le 0, \tag{41}
$$

for all $\pi_0 \in [\pi_f(\gamma_H), \pi_f(\overline{\gamma})]$, and with equality at $\pi_0 = \pi_f(\gamma_H)$,

(b) *if $\gamma_H = \overline{\gamma}$, $w_\pi(\overline{\gamma}, \pi_f(\overline{\gamma})) \ge 0$,*

(c) *if $\gamma_L > \underline{\gamma}$,*

$$
\int_{\underline{\gamma}}^{g(\pi_0|\pi_f(\gamma_L))} \left(\frac{w(\tilde{\gamma}, \pi_0) - w(\tilde{\gamma}, \pi_f(\gamma_L))}{\pi_0 - \pi_f(\gamma_L)}\right) f(\tilde{\gamma})\, d\tilde{\gamma} \ge 0, \tag{42}
$$

for all $\pi_0 \in [\pi_f(\underline{\gamma}), \pi_f(\gamma_L)]$, *and with equality at* $\pi_0 = \pi_f(\gamma_L)$,

(d) *if* $\gamma_L = \underline{\gamma}$, $w_\pi(\underline{\gamma}, \pi_f(\underline{\gamma})) \le 0$.

PROOF: Let us just prove (a) and (b), as (c) and (d) follow a similar argument. That condition (b) is a necessary condition follows directly from Lemma 1. For condition (a), the fact that the condition must hold with equality at γ_H follows immediately from Lemma 1, as the allocation in the general case must be optimal as well when restricted to the class of interval allocations. To see this, note that when $\pi_0 = \pi_f(\gamma_H)$, we have that $g(\pi_f(\gamma_H)|\pi_f(\gamma_H)) = -b'(\pi_f(\gamma_H)) = \gamma_H$, and the term inside the brackets in the integral in condition (a) then becomes $w_\pi(\tilde{\gamma}, \pi_f(\gamma_H))$; and thus the condition is the same as in Lemma 1. To prove that the inequality in condition (a) must hold, consider the perturbation that introduces the choice π_0 (with no money burned) into the allocation. All agents between γ_H and $\gamma_0 \equiv g(\pi_0|\pi_f(\gamma_H))$ will remain with their old choice, $\pi_f(\gamma_H)$, while all agents between γ_0 and $\overline{\gamma}$ will now choose the new choice π_0. (This follows from noticing that type γ_0 remains indifferent between the two.) The effect on welfare of this perturbation is equal to the left hand side of inequality (41), multiplied by $\pi_0 - \pi_f(\gamma_H)$, and hence the inequality must hold or we would have found an improvement. *Q.E.D.*

Using the necessity results in Lemmas 4, 7, and 8, we can show that there is a family of utility functions for which (c1), (c2), (c2′), (c3), and (c3′) are necessary for the optimality of an interval allocation. The proof of Proposition 2, which follows below, shows this.

Proof of Part (a) of Proposition 2 (The No Money Burning Case)

For this case, we know that $A = \kappa$. We next note that the following inequalities are each equivalent to the sufficient condition (c2):

$$\int_{\gamma}^{\overline{\gamma}} w_\pi\big(\tilde{\gamma}, \pi_f(\gamma_H)\big)\frac{f(\tilde{\gamma})}{1-F(\gamma)}\,d\tilde{\gamma} \le (\gamma - \gamma_H)A,$$

$$\int_{\gamma}^{\overline{\gamma}} \big(w_\pi\big(\tilde{\gamma}, \pi_f(\gamma_H)\big) - A(\gamma - \gamma_H)\big)\frac{f(\tilde{\gamma})}{1-F(\gamma)}\,d\tilde{\gamma} \le 0,$$

$$\int_{\gamma}^{\overline{\gamma}} w_\pi\big(\tilde{\gamma}, \pi_f(\gamma)\big)f(\tilde{\gamma})\,d\tilde{\gamma} \le 0,$$

which should hold for all $\gamma \in [\gamma_H, \overline{\gamma}]$ and with equality at γ_H. Note that we have used that $w_\pi(\tilde{\gamma}, \pi_f(\gamma_H)) - A(\gamma - \gamma_H) = w_\pi(\tilde{\gamma}, \pi_f(\gamma))$, given our assumption about w and using that $b'(\pi_f(\gamma)) = -\gamma$.

The necessary condition (41) of Lemma 8 is now

$$A\int_{\gamma_0}^{\overline{\gamma}} \left(\left(\frac{b(\pi_0) - b(\pi_f(\gamma_H))}{\pi_0 - \pi_f(\gamma_H)}\right) + C(\tilde{\gamma})\right) f(\tilde{\gamma})\,d\tilde{\gamma} \le 0,$$

which should hold for all $\pi_0 \in [\pi_f(\gamma_H), \pi_f(\overline{\gamma})]$ and with equality at $\pi_0 = \pi_f(\gamma_H)$, where $\gamma_0 = g(\pi_0|\pi_f(\gamma_H))$. Using the definition of g, we have that the above is equivalent to

$$A\int_{\gamma_0}^{\overline{\gamma}} \big(-\gamma_0 + C(\tilde{\gamma})\big) f(\tilde{\gamma})\, d\tilde{\gamma} \leq 0.$$

Using that $b'(\pi_f(\gamma_0)) = -\gamma_0$, and thus that $w_\pi(\tilde{\gamma}, \pi_f(\gamma_0)) = A(-\gamma_0 + C(\tilde{\gamma}))$, it follows that this is the same as the sufficient condition (c2) as represented above. Note that (c2′) is the same as condition (b) of Lemma 8.

A similar argument shows that (c3) and (c3′) are equivalent to (c) and (d) of Lemma 8. Finally, under differentiability of f, the necessary condition in Lemma 7 is equivalent to condition (c1) using the preferences specified above. Taken together, the above shows that the conditions (c1), (c2), (c2′), (c3), and (c3′) are also necessary.

Proof of Part (b) of Proposition 2 (The Money Burning Case)

There are two cases to consider. The first case, where $A \geq 1$, implies that $\kappa = 1$, and this is already covered by our Lemma 4. For the second case, where $A < 1$, the result is the same as for part (a) above, as $A = \kappa$. *Q.E.D.*

APPENDIX E: PROOF OF PROPOSITION 3

Let $d(x) \equiv \mathbb{E}[\gamma|\gamma > x] - x = \int_x^{\overline{\gamma}} \frac{1-F(\gamma)}{1-F(x)}\, d\gamma$. The following lemma is useful.

LEMMA 9: *If f is nondecreasing, then $g(x) \equiv \frac{d(x)}{1-F(x)}$ is such that $g(x) \leq \frac{1}{2f(x)}$.*

PROOF: Note that

$$\begin{aligned} g'(x) &= \frac{d'(x)}{1-F(x)} + \frac{d(x)}{1-F(x)}\frac{f(x)}{1-F(x)} \\ &= \frac{g(x)f(x)-1}{1-F(x)} + \frac{g(x)f(x)}{1-F(x)} = \frac{2g(x)f(x)-1}{1-F(x)}, \end{aligned}$$

where we used that $d'(x) = -1 + d(x)\frac{f(x)}{1-F(x)}$. We also know that $\lim_{x\to\overline{\gamma}} g(x) = \frac{1}{2f(\overline{\gamma})}$ (which follows from applying L'Hôpital's rule on $d(x)/(1-F(x))$). From the ODE, it follows then that if $g(x_0) > \frac{1}{2f(x_0)}$ for some x_0, then $g'(x_0) > 0$, and given that $f(x)$ is nondecreasing, this implies that $g(x) > \frac{1}{2f(x_0)} \geq \frac{1}{2f(\overline{\gamma})}$ for all $x > x_0$, which is a contradiction of the limit condition. *Q.E.D.*

Let us now prove Proposition 3. Let $H(\gamma) = \kappa F(\gamma) - w_\pi(\gamma, \pi_f(\gamma))f(\gamma)$. Note that condition (c1) is equivalent to requiring that H be nondecreasing in $[\gamma_L, \gamma_H]$. The hypothesis of the proposition implies that $w_\pi(\gamma, \pi_f(\gamma)) = v'(\pi_f(\gamma))$. Using integration by parts, we can rewrite H as

$$
\begin{aligned}
H(\gamma) &= \kappa F(\gamma) - \int_{\underline{\gamma}}^{\gamma} v'\big(\pi_f(\tilde{\gamma})\big)\, df(\tilde{\gamma}) \\
&\quad - \int_{\underline{\gamma}}^{\gamma} f(\tilde{\gamma}) v''\big(\pi_f(\tilde{\gamma})\big)\pi_f'(\tilde{\gamma})\, d\tilde{\gamma} - v'\big(\pi_f(\underline{\gamma})\big) f(\underline{\gamma}) \\
&= \int_{\underline{\gamma}}^{\gamma} f(\tilde{\gamma}) \left(\kappa + \frac{v''(\pi_f(\tilde{\gamma}))}{b''(\pi_f(\tilde{\gamma}))}\right) d\tilde{\gamma} \\
&\quad + \int_{\underline{\gamma}}^{\gamma} \big(-v'\big(\pi_f(\tilde{\gamma})\big)\big)\, df(\tilde{\gamma}) - v'\big(\pi_f(\underline{\gamma})\big) f(\underline{\gamma}),
\end{aligned}
$$

where we used that $\pi_f'(\gamma) = -1/b''(\pi_f(\gamma))$. By the hypothesis that $\kappa \geq 1/2$, it follows that

$$
\frac{v''(\pi_f(\tilde{\gamma})) + b''(\pi_f(\tilde{\gamma}))}{b''(\pi_f(\tilde{\gamma}))} \geq \kappa \geq \frac{1}{2} \quad \Rightarrow \quad \frac{v''(\pi_f(\tilde{\gamma}))}{b''(\pi_f(\tilde{\gamma}))} + \kappa \geq 2\kappa - 1 \geq 0.
$$

And thus the first integral above is increasing in γ. The second integral is also increasing in γ as $-v' \geq 0$ and f is nondecreasing. It follows that H is the sum of a constant plus two nondecreasing functions in γ, so H is also nondecreasing. Hence condition (c1) holds.

Condition (c3′) holds, given that $w_\pi(\underline{\gamma}, \pi_f(\underline{\gamma})) = v'(\pi_f(\underline{\gamma})) \leq 0$ by the hypothesis of the proposition.

Finally, let $G(\gamma^c) = \int_{\gamma^c}^{\overline{\gamma}} w_\pi(\gamma, \pi_f(\gamma_H)) \frac{f(\gamma)}{1-F(\gamma^c)}\, d\gamma - (\gamma^c - \gamma_H)\kappa$. Condition (c2) requires that $G(\gamma^c) \leq 0$ for all $\gamma^c > \gamma_H$ and $G(\gamma_H) = 0$. Now note that, using (4), G can be written as

$$
G\big(\gamma^c\big) = \mathbb{E}\big[\gamma|\gamma > \gamma^c\big] - \mathbb{E}[\gamma|\gamma > \gamma_H] - \kappa\big(\gamma^c - \gamma_H\big). \tag{43}
$$

Hence, $G(\gamma_H) = 0$. Also, we can get that

$$
\begin{aligned}
G'\big(\gamma^c\big) &= \frac{d}{d\gamma^c}\big(\mathbb{E}\big[\gamma|\gamma > \gamma^c\big]\big) - \kappa \\
&= d'\big(\gamma^c\big) + 1 - \kappa = \frac{d(\gamma^c) f(\gamma^c)}{1 - F(\gamma^c)} - \kappa \leq \frac{1}{2} - \kappa,
\end{aligned}
$$

where the last inequality follows from f nondecreasing and Lemma 9. Letting $\kappa \geq \frac{1}{2}$ implies that $G'(\gamma^c) \leq 0$ for all $\gamma^c > \gamma_H$, which delivers that $G(\gamma^c) \leq 0$ for all $\gamma^c > \gamma_H$. And hence condition (c2) holds.

Given that Assumption 1 holds by hypothesis, we can then invoke Proposition 1 to complete the proof of Proposition 3. *Q.E.D.*

APPENDIX F: PROOF OF COROLLARY 2

Part (i) of the corollary was proved in the text.

To prove part (ii), the following lemma will be used.

LEMMA 10: *In the linear-quadratic case, if the function defined by* $\kappa F(\gamma) - v'(\pi_f(\gamma))f(\gamma)$ *for all* $\gamma \in \Gamma$ *is nondecreasing, then conditions* (c1) *and* (c2) *are satisfied.*

PROOF: Let $X(\gamma) = (1 - F(\gamma))G(\gamma)$, where G is as in (43). Then we can show that

$$X'(\gamma) = -\kappa + \kappa F(\gamma) - \big[v'\big(\pi_f(\gamma_H)\big) + (1-\kappa)(\gamma - \gamma_H)\big]f(\gamma).$$

In our linear-quadratic case, we have that $v'(\pi_f(\gamma)) = v'(\pi_f(\gamma_H)) + (1-\kappa)(\gamma - \gamma_H)$, with $\kappa = 2/3$, and thus

$$X'(\gamma) = -\kappa + \kappa F(\gamma) - v'\big(\pi_f(\gamma)\big)f(\gamma),$$

which is nondecreasing by the hypothesis of the lemma. This implies then that $X(\gamma)$ is a convex function of γ. Note that $X(\overline{\gamma}) = 0$ and $X'(\overline{\gamma}) = -v'(\pi_f(\overline{\gamma}))f(\overline{\gamma}) > 0$. It then follows that $X(\gamma)$ has at most another 0 for $\gamma < \overline{\gamma}$, which corresponds to γ_H. This also implies that $X(\gamma) < 0$ for all $\gamma \in (\gamma_H, \overline{\gamma})$ and thus $G(\gamma) < 0$ as well, which proves that condition (c2) holds. The hypothesis of the lemma directly implies condition (c1). *Q.E.D.*

To prove Corollary 2, we recall that $\mathbb{E}[\gamma] > [7 + 8\underline{\gamma}]/12$ ensures that γ_H is interior. We thus just need to show that $\kappa F(\gamma) - v'(\pi_f(\gamma))f(\gamma)$ is nondecreasing in $\gamma \in \Gamma$ and invoke Lemma 10. Assuming differentiability of f, and using that $\kappa = 2/3$ and that $v''/b'' = -1/3$, we get that $\kappa F(\gamma) - v'(\pi_f(\gamma))f(\gamma)$ is nondecreasing in γ if

$$\frac{2}{3}f(\gamma) + \frac{v''(\pi_f(\gamma))}{b''(\pi_f(\gamma))}f(\gamma) - v'\big(\pi_f(\gamma)\big)f'(\gamma) \geq 0,$$

or equivalently, $\frac{1}{3}f(\gamma) - v'(\pi_f(\gamma))f'(\gamma) \geq 0$. Substituting $v'(\pi_f(\gamma)) = -\frac{1}{3}[\frac{7}{4} - \gamma]$, we get the condition of part (ii) of Corollary 2. *Q.E.D.*

APPENDIX G: AN EXAMPLE WITH ENDOWMENT AND LOGARITHMIC UTILITY

In what follows, we develop an endowment example and show that Proposition 3 allows us to characterize the optimal trade agreement. Assume that $u(c) = \log(c)$ and that $Q(p) = 1$ and $Q_\star(p) = \overline{Q}_*$, where $\overline{Q}_* > 1$. Then we can write home consumer surplus plus tariff revenue, foreign welfare, and home profit, respectively, as

$$B = -p - p_\star z - \log(p), \quad V = p_\star z - \log(p_\star), \quad \Pi = p,$$

where $p = (1+z)^{-1}$, $p_\star = (\overline{Q}_* - z)^{-1}$, and z is the volume of trade.

Note that free trade is $z = \frac{1}{2}(\overline{Q}_* - 1)$. Writing everything in terms of π delivers

$$b(\pi) = -\pi + \frac{\pi - 1}{(\overline{Q}_* + 1)\pi - 1} - \log(\pi), \quad \text{and}$$

$$v(\pi) = \frac{1-\pi}{(\overline{Q}_* + 1)\pi - 1} - \log\left(\frac{\pi}{(\overline{Q}_* + 1)\pi - 1}\right),$$

and where $z = \frac{1}{\pi} - 1$.

The free trade allocation corresponds to $\pi^{ft} = \frac{2}{1+\overline{Q}_*}$. Zero trade corresponds to $\overline{\pi} = 1$. We will restrict attention to a set of admissible $\pi \in [\pi^{ft}, 1]$, which is equivalent to restricting tariffs to be nonnegative.

Note that $v'(\pi) = \frac{\pi - 1}{\pi((\overline{Q}_* + 1)\pi - 1)^2} \le 0$, and note as well that

$$b''(\pi) = \frac{1}{\pi^2} - \frac{2\overline{Q}_*(1+\overline{Q}_*)}{((\overline{Q}_* + 1)\pi - 1)^3},$$

which is negative for all $\pi \in [\pi^{ft}, 1]$ if $1 \le \overline{Q}_* < 1 + \sqrt{3}$. Similarly, one can show that

$$v''(\pi) + b''(\pi) = \frac{2 + (\overline{Q}_* + 1)\pi((\overline{Q}_* + 1)\pi - 4)}{\pi^2((\overline{Q}_* + 1)\pi - 1)^2},$$

from which it follows that $w(\gamma, \pi)$ is concave in π for all $\pi \in [\pi^{ft}, 1]$ if $1 \le \overline{Q}_* \le 1 + \sqrt{2}$. This last condition guarantees that Assumption 1 is satisfied.

Using the above, the value of κ can be found to be

$$\kappa = \begin{cases} \dfrac{2(\overline{Q}_* + 1)}{7\overline{Q}_* - 1}, & \text{for } 1 \le \overline{Q}_* \le \dfrac{4 + \sqrt{41}}{5}, \\[2ex] \dfrac{-1 - 2\overline{Q}_* + \overline{Q}_*^2}{-2 - 2\overline{Q}_* + \overline{Q}_*^2}, & \text{for } \dfrac{4 + \sqrt{41}}{5} \le \overline{Q}_* \le 1 + \sqrt{2}. \end{cases}$$

Note that $\kappa \ge 0$ for all $\overline{Q}_* \in [1, 1 + \sqrt{2}]$. Also for $\overline{Q}_*$ close to 1, $\kappa \approx 2/3$, which implies that we can apply Proposition 3, which requires $\kappa \ge 1/2$, and show that a tariff cap is optimal for distributions with nondecreasing densities when $\overline{Q}_*$ is close to 1. *Q.E.D.*

APPENDIX H: PROOF OF LEMMA 3

We proceed by showing that each part of Assumption 1 holds.

Part (i) of Assumption 1: This follows from the definition of w.

Part (ii) of Assumption 1: Let us show that $v''(\pi) + b''(\pi) < 0$ for $\pi \in (0, 1)$. To see this, note that

$$\begin{aligned} &v''(\pi) + b''(\pi) \\ &\quad = \frac{1 - (1-\pi)^{(1-2\alpha)/\alpha}\pi^{(\alpha-1)/\alpha}}{\alpha\pi^2} \\ &\quad \leq \frac{1 - \left(\left(1 - \left(\frac{1}{\alpha} - 1\right)\right)^{(2\alpha-1)/\alpha}\left(\frac{1}{\alpha} - 1\right)^{(1-\alpha)/\alpha}\right)^{-1}}{\alpha\pi^2} < 0, \end{aligned}$$

where the first inequality follows from the fact that $(1-\pi)^{(2\alpha-1)/\alpha}\pi^{(1-\alpha)/\alpha}$ achieves a maximum at $\pi = \frac{1-\alpha}{\alpha}$ if $1 > \alpha > 1/2$. The second inequality follows from the fact that $0 < 1 - (1/\alpha - 1) < 1$ and $0 < 1/\alpha - 1 < 1$ for $1 > \alpha > 1/2$, and thus $(1 - (\frac{1}{\alpha} - 1))^{(2\alpha-1)/\alpha}(\frac{1}{\alpha} - 1)^{(1-\alpha)/\alpha} < 1$ for $1/2 < \alpha < 1$.

Part (iii) of Assumption 1: First we show that $v''(\pi) > 0$ for $\pi \in (0, 1)$. To see this, note that $v''(\pi) = (1-\alpha)(1-\pi)^{(1-2\alpha)/\alpha}\pi^{-(1+\alpha)/\alpha}/\alpha^2 > 0$. Now note that $b'' < 0$ when $\alpha \in (1/2, 1)$ follows immediately from combining $v'' > 0$ and $b'' + v'' < 0$.

Part (iv) of Assumption 1: Given the problem of maximizing $b(\pi) + \gamma\pi$, we know that first-order conditions are sufficient for optimality, as b is concave. Thus, if $b'(\pi_0) = -\gamma$ for some $\pi_0 \in \Pi$, then π_0 is a maximizer of $b(\pi) + \gamma\pi$. Note that $\lim_{\pi\to 0} b'(\pi) = \infty$ and that $\lim_{\pi\to 1} b'(\pi) = -\frac{1}{\alpha(1-\alpha)}$. Hence if $\gamma < \frac{1}{\alpha(1-\alpha)}$, then an interior solution exists. Given that $\overline{\gamma} < \frac{1}{\alpha(1-\alpha)}$, it follows that this condition holds and an $\pi_f(\gamma)$ is interior for all γ.

To prove that $\pi_f'(\gamma) > 0$, we note that, by the implicit function theorem, $\pi_f'(\gamma) = -1/b''(\pi_f(\gamma)) > 0$, where the inequality follows from $b'' < 0$ in $(0, 1)$.

Part (v) of Assumption 1 follows directly from the definition of w, which completes the proof of the lemma. *Q.E.D.*

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