Central Bank Reputation With Noise

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Abstract

This paper presents a simple model of central bank reputation with several appealing characteristics not common in reputation games. The basic model considers sellers in a monopolistic competition environment. Sellers have market power when setting posted prices, and are embedded in a cash-in-advance economy where fiat money creation is a noisy function of central bank actions. Our contribution is to add uncertainty by households over which type of central bank they are facing — a "good" central bank, with a relatively high penalty for taking inflationary actions, or a "bad" central, bank with a relatively low penalty for taking inflationary actions. This allows for reputation to be defined as the Bayesian posterior of households that they are facing a good central bank type. This feature also allows for the consideration of how a good central bank should act to maintain a good reputation if it has one or build a good reputation if it has lost one.

We show analytically that no Markov pooling equilibria can exist. We then argue that without sufficient noise, it is difficult for a pure symmetric Markov separating equilibria to exist. We do show computationally, however, that *with* sufficient noise, a pure symmetric Markov separating equilibrium of our reputation game exists and has appealing characteristics. In particular, in equilibrium, both good and bad central bank types choose lower inflationary actions than they would in the absence of reputation considerations, and both good and bad central banks are most aggressive in attempting to gain a good reputation when their current reputation is middling.

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1 Introduction

This paper presents a simple model of central bank reputation with several appealing characteristics not common in reputation games. In it, a benevolent central bank can be a “good” type, which receives a relatively large negative payoff to taking inflationary actions, or an otherwise identical “bad” type, which receives a lesser negative payoff to taking inflationary actions. Notably, neither type is behavioral, and each can continuously choose an action that affects the distribution of inflation outcomes. This implies that unlike games where the good type is behavioral, our game lets us consider how a good central bank “should” act to recover a good reputation if it has lost one, or maintain a good reputation if it has one.

The basic model builds on the work of Chari, Christiano and Eichenbaum (1998), where sellers in a monopolistic competition environment have market power when setting posted prices, and are embedded in a cash-in-advance economy where fiat money creation is a noisy function of central bank actions. The fundamental tension in such an economy is that because of market power and the cash-in-advance constraint, the real prices of consumption goods, relative to labor, are set inefficiently high, which creates a temptation for the central bank to use inflation to reduce these inefficiencies ex-post. (In equilibrium, however, such money-printing-induced inflation is anticipated by price setters and only exacerbates the inefficiencies in the game.) Our contribution is to add uncertainty by households over the type of central bank they are facing. This allows for reputation to be defined as the Bayesian posterior of households that they are facing the good central bank type.

We show first, analytically, that no Markov pooling equilibria to our game exist. We then argue that without sufficient noise disconnecting a central bank’s target inflation from realized inflation, it is difficult for a pure symmetric Markov separating equilibria to exist. Nevertheless, we then show computationally that with sufficient noise, a pure strategy symmetric Markov separating equilibrium exists and has appealing characteristics. In particular, in equilibrium, both types of central banks choose lower inflationary actions than they would in the absence of reputational considerations, and both good and bad central bank are most aggressive in attempting to gain a good reputation when their current reputation is middling.

2 Related Literature

There is a large literature on reputation and monetary policy. Building on models of central bank behavior that emphasize the time inconsistency of the optimal policy (Kydland and Prescott, 1977; Calvo, 1978; Barro and Gordon, 1983; Rogoff, 1985), this paper starts with the monopolistic competition framework of Chari et al. (1998). Similarly to many earlier contributions (Backus and

Driffill, 1985; Barro, 1986; Vickers, 1986), we introduce uncertainty among households regarding the type of central bank they face, allowing reputation to be defined as the Bayesian posterior of households identifying a "good" central bank type. We do so in an infinite horizon model where the types of the central bank are forever changing, as in Ball (1995) and Phelan (2006). We allow for imperfect monitoring of the central bank actions by introducing noise between the central bank monetary choice and its corresponding equilibrium outcome (Canzoneri, 1985; Cukierman and Meltzer, 1986; Cukierman, 2000; Faust and Svensson, 2001; Sleet, 2001; Laubach, 2003). We do not study announcements, but our setup is related to papers that analyze the role of monetary announcements with imperfect information about the central bank type (Cukierman and Liviatan, 1991; Schultz, 1996; Walsh, 2000). Work by King, Lu and Pastén (2008) extends these analyses by focusing on a model with a "strong" central bank type that can commit to its announcements and a "weak" one that can deviate. Lu, King and Pasten (2016) follows up with a New Keynesian model with imperfect monitoring where the "weak" central bank type follows a pre-specified behavioral rule. A more recent paper by Kostadinov and Roldán (2021) studies the role of announcements in a New Keynesian model where the "weak" type optimizes. Dovis and Kirpalani (2021) introduce a third agent, the rule designer, responsible for recommending the inflation target. Other recent works on monetary policy commitment are Afrouzi, Halac, Rogoff and Yared (2023) and Schreger, Yared and Zaratiegui (2023).

3 Environment

In this section, we present a version of the model in Chari et al. (1998), modified to allow for noisy signals and two types of central banks (and thus a role for reputation).

Time is discrete and infinite with $t \in \{0, 1, \ldots\}$. The real economy consists of a mass 1 continuum of households, with names $j \in [0, 1]$, split into shopper-seller pairs. In each period, each household j produces good j using only its individual effort. Households consume a composite good made up of the output off all households so that the consumption of the composite good by household j, c_j, is such that

$$c_j = \left(\int_k c_{j,k}^{\frac{\sigma-1}{\sigma}} \, dk \right)^{\frac{\sigma}{\sigma-1}},$$

where $c_{j,k}$ is the consumption of household j of good k, and where $\sigma > 1$. A household j that consumes c_j units of the composite good and produces y_j of its own good receives a static payoff $\log(c_j) - \alpha y_j$. In a symmetric *efficient* allocation, all households consume the same amount of each good at each date, $c^* = \frac{1}{\alpha}$.

Our solution concept will be pure symmetric Markov perfect equilibrium of a game between households and central banks.

3.1 Players

The players of our game consist of long-lived households and a countable number of central banks.

As stated previously, we assume a continuum of households of mass 1, with names $j \in [0,1]$, split into shopper-seller pairs, where each household j produces good j.

Second, we assume a countable list of potential central banks, where only one central bank on the list is active at any time. Each central bank is of type $i \in \{1,2\}$. With probability $\rho_0 \in [0,1]$, the first central bank is of type 1, the second is type 2, the third is type 1, and so on. With probability $1-\rho_0$, the first central bank is of type 2, the second is type 1, the third is type 2, and so on.

At the start of the game, the first central bank on the list is the active player. At the end of the first and each subsequent period, if the current central bank is of type 1, with probability $1-\delta \in [0,1]$ it remains in the game, and with probability δ, it is replaced by the next central bank in the alternating list (a central bank of type 2). If the central bank is of type 2, with probability $1-\epsilon \in [0,1]$ it remains in the game, and with probability ϵ, it is replaced by the next central bank in the alternating list (a central bank of type 1). At all times, central bank type is private.

3.2 Timing:

In each period t, each household j starts off with initial holdings of money. Without loss, let almost all households, j, start with money holdings $m_j = 1$. Each household j then chooses the nominal price of its product, P_j. A price P_j obligates the seller to produce whatever quantity other households choose to purchase at that price.

After households individually set their prices, the central bank (of type i) privately chooses an action $\mu_i \in [0, \overline{\mu}]$ (where $\overline{\mu}$ may be infinity).

After the central bank chooses μ_i, the actual rate of money growth, a random variable, μ_a, is determined by a distribution $F(\mu_a|\mu)$, with support $[0,\infty)$ for all μ and with density $f(\mu_a|\mu)$ differentiable in μ. Upon the realization of μ_a, each household's shopper is given μ_a units of newly created money lump sum.

Next, each shopper j with $1+\mu_a$ units of money chooses how much to purchase of each good k, $c_{j,k}$, subject to a cash-in-advance constraint that

$$\int_k P_k c_{j,k} dk \leq 1 + \mu_a.$$

The money holdings of household j at end of the period are then whatever was not spent by the household's shopper plus whatever cash was collected by the household's seller in return for producing good j.

Finally, if the central bank is of type $i = 1$, it leaves the game with probability δ and is replaced by a central bank of type $i = 2$. Likewise, if the central bank is of type $i = 2$, it leaves the game with probability ϵ and is replaced by a central bank of type $i = 1$.

3.3 Pure Symmetric Markov Strategies:

We will consider only **pure** symmetric Markov strategies where the state variable is the posterior that the current central bank is type 1, denoted by ρ. Here, a pure symmetric Markov strategy is a specification

$$(P(\rho), \mu_1(\rho), \mu_2(\rho), c(\mu_a, \rho)),$$

where $P(\rho)$ represents the common price set by all households (as a function of ρ), $\mu_i(\rho)$ represents the action of a central bank of type $i \in \{1, 2\}$, again as a function of ρ, and $c(\mu_a, \rho)$ represents household consumption (same across all goods) as a function of *beginning of period beliefs*, ρ, and the realized rate of money growth, μ_a.

3.4 Beliefs:

Our assumption that any μ_a is possible for any μ_i ensures that Bayes' rule can always be used to determine beliefs. To this end, let $\hat{\rho}(\mu_a, \rho)$ be the Bayesian posterior after μ_a is observed but before end-of-period type switches, and $\rho^+(\mu_a, \rho)$ be the Bayesian posterior at the beginning of the next period, after a possible type switch realization. Then,

$$\hat{\rho}(\mu_a, \rho) = \frac{\rho f(\mu_a|\mu_1(\rho))}{\rho f(\mu_a|\mu_1(\rho)) + (1-\rho) f(\mu_a|\mu_2(\rho))},$$

and

$$\rho^+(\mu_a, \rho) = (1-\delta)\hat{\rho}(\mu_a, \rho) + \epsilon(1 - \hat{\rho}(\mu_a, \rho)). \tag{1}$$

3.5 Payoffs:

Household j, which produces y_j and consumes $c_{j,k}$ for goods $k \in [0, 1]$, gets utility

$$\log(c_j) - \alpha y_j,$$

where

$$c_j = \left(\int_k c_{j,k}^{\frac{\sigma-1}{\sigma}} \, dk \right)^{\frac{\sigma}{\sigma-1}}.$$

Over time and uncertainty, households care about the expectation of its discounted, by $\beta \in (0,1)$, lifetime per-period utility stream.

If all households set the same price and consume the same amount of each good, then $y_j = y$ for all j, and $c_{j,k} = c = y$ for all (j,k). Given this, we assume a central bank of type i that chooses action μ_i gets static payoff

$$\log(c) - \alpha c - \gamma_i h(\mu_i).$$

Here, $h(\mu_i)$ represents a negative payoff to a central bank for taking an inflationary action against, say, a mandate not to do so. (We assume $h(\mu_i)$ is twice differentiable with $h(0) = 0$, $h_i'(0) = 0$, $h'(\mu_i) > 0$ and $h''(\mu_i) > 0$, for $\mu_i > 0$.) Central bank types i differ only by γ_i, the amount of disutility they get by choosing $\mu_i > 0$. We assume $\gamma_1 > \gamma_2$, which means that a type 1 central bank is the "good" type of central bank.

Over time and uncertainty, central banks care about the expectation of their discounted per-period stream of payoffs while they are active in the game. For type 1 banks, we define their discount factor, β_1, to be $\beta_1 = \beta(1-\delta)$, and for type 2 banks, we define their discount factor to be $\beta_2 = \beta(1-\epsilon)$.

3.6 Equilibrium Conditions:

The Household Problem. Consider the problem of a household that starts the period with m units of money when all other agents start with 1, and the aggregate state is ρ. Given an aggregate state ρ, we let $c(\mu_a, \rho)$ and $P(\rho)$ denote the equilibrium aggregate responses of consumption and prices, and let $\rho^+(\mu_a, \rho)$ denote its evolution. Abusing notation and letting $F(\mu_a|\rho) = \rho F(\mu_a|\mu_1(\rho)) + (1-\rho)F(\mu_a|\mu_2(\rho))$, we can write the problem of a household recursively as follows:

$$V(m|\rho) = \max_{\{\hat{c}(\mu_a), m^+(\mu_a), \hat{y}(\mu_a), \hat{P}\}} \int \{\log(\hat{c}(\mu_a)) - \alpha \hat{y}(\mu_a) + \beta V(m^+(\mu_a)|\rho^+(\mu_a, \rho))\}\, dF(\mu_a|\rho),$$

subject to:

$$P(\rho)\hat{c}(\mu_a) \leq m + \mu_a,$$

$$m^+(\mu_a) = \frac{m + \mu_a - P(\rho)\hat{c}(\mu_a) + \hat{P}\hat{y}(\mu_a)}{1+\mu_a},$$

$$\hat{y}(\mu_a) = c(\mu_a, \rho)\left(\frac{\hat{P}}{P(\rho)}\right)^{-\sigma},$$

and $\hat{P} \geq 0$, $\hat{c}(\mu_a) \geq 0$, $m^+(\mu_a) \geq 0$, where the first constraint is the cash-in-advance constraint of the shopper, the second constraint is the "re-normalized" budget constraint of the household,[1] and the third constraint is the restriction that the seller must satisfy the demand at the quoted price, $\hat{P}$. The measurability constraint on $\hat{P}$ (that it cannot be a function of μ_a) reflects the fact that the seller sets the price before the realization of μ_a.

Let $\hat{c}(\mu_a, m, \rho)$ and $\hat{P}(m, \rho)$ denote the associated optimal policy functions. Then, in an equilibrium, these must coincide with the aggregate behavior when $m = 1$; that is, $\hat{c}(\mu_a, 1, \rho) = c(\mu_a, \rho)$, and $\hat{P}(1, \rho) = P(\rho)$.

Let us define the expected marginal utility of consumption, $g(\rho)$, to be

$$g(\rho) \equiv \int \frac{1}{c(\mu_a, \rho)} dF(\mu_a|\rho).$$

Using the equilibrium condition, and the envelope condition of the household problem, we get that

$$V'(1|\rho) = \frac{g(\rho)}{P(\rho)}.$$

Using the first order condition with respect to $\hat{P}$, and imposing the equilibrium conditions, we obtain that

$$\int \left\{ \alpha\sigma \frac{c(\mu_a, \rho)}{P(\rho)} + \beta(1-\sigma)\frac{c(\mu_a, \rho)}{1+\mu_a} V'(1|\rho^+(\mu_a, \rho)) \right\} dF(\mu_a|\rho) = 0,$$

or equivalently,

$$\int \left\{ \frac{\alpha\sigma}{P(\rho)} + \frac{\beta(1-\sigma)}{1+\mu_a} \frac{g(\rho^+(\mu_a, \rho))}{P(\rho^+(\mu_a, \rho))} \right\} c(\mu_a, \rho) dF(\mu_a|\rho) = 0. \tag{2}$$

If the cash-in-advance constraint does not bind, $c(\mu_a, \rho) < \frac{1+\mu_a}{P(\rho)}$; thus, for $c(\mu_a, \rho)$ to satisfy household optimization, it must satisfy the first order condition:

$$\frac{1}{c(\mu_a, \rho)} = \frac{\beta}{1+\mu_a} \frac{P(\rho)}{P(\rho^+(\mu_a, \rho))} g(\rho^+(\mu_a, \rho)),$$

in which case

$$c(\mu_a, \rho) = \frac{1+\mu_a}{P(\rho)} \left(\frac{P(\rho^+(\mu_a, \rho))}{\beta} \frac{1}{g(\rho^+(\mu_a, \rho))} \right),$$

which implies that

$$\frac{P(\rho^+(\mu_a, \rho))}{\beta} \frac{1}{g(\rho^+(\mu_a, \rho))} < 1.$$

[1]The household money balances next period are normalized relative to the aggregate money balances, hence the division by $1 + \mu_a$ on the right hand side.

If the cash-in-advance constraint binds, $c(\mu_a, \rho) = \frac{1+\mu_a}{P(\rho)}$, then optimality requires that

$$\frac{1}{c(\mu_a, \rho)} \geq \frac{\beta}{1 + \mu_a} \frac{P(\rho)}{P(\rho^+(\mu_a, \rho))} g(\rho^+(\mu_a, \rho)),$$

in which case

$$c(\mu_a, \rho) \leq \frac{1 + \mu_a}{P(\rho)} \left(\frac{P(\rho^+(\mu_a, \rho))}{\beta} \frac{1}{g(\rho^+(\mu_a, \rho))} \right),$$

which implies

$$\frac{P(\rho^+(\mu_a, \rho))}{\beta} \frac{1}{g(\rho^+(\mu_a, \rho))} \geq 1.$$

Thus, the optimality condition for household consumption can be summarized by

$$c(\mu_a, \rho) = \frac{1 + \mu_a}{P(\rho)} \min \left\{ \frac{P(\rho^+(\mu_a, \rho))}{\beta} \frac{1}{g(\rho^+(\mu_a, \rho))}, 1 \right\}. \tag{3}$$

Equations (2) and (3) summarize the optimality conditions of the household problem.

Here is a fake equation.

$$c(\mu_a, \rho) = \frac{1 + \mu_a}{P(\rho)} \min \left\{ \frac{P(\rho^+(\mu_a, \rho))}{\beta} \frac{1}{g(\rho^+(\mu_a, \rho))}, 1 \right\}.$$

Central Bank's Problem. Consider a central bank of type $i \in \{1, 2\}$ that must choose μ_i when its reputation is ρ. Let $V_i(\rho)$ be the lifetime payoff to a central bank of type i. Its decision problem when choosing μ_i is

$$V_i(\rho) = \max_{\mu_i \in [0, \overline{\mu}]} \int ((1 - \beta_i)(\log(c(\mu_a, \rho)) - \alpha c(\mu_a, \rho) - \gamma_i h(\mu_i)) + \beta_i V_i(\rho^+(\mu_a, \rho)))\, dF(\mu_s|\mu_i). \tag{4}$$

Thus, the local optimality condition for $\mu_i(\rho)$, if we assume that $\mu_i(\rho)$ is interior, is

$$(1 - \beta_i) \frac{\partial \mathbb{E}[\log(c(\mu_a, \rho)) - \alpha c(\mu_a, \rho) | \mu = \mu_i(\rho)]}{\partial \mu} + \beta_i \frac{\partial \mathbb{E}[V_i(\rho^+(\mu_a, \rho)) | \mu = \mu_i(\rho)]}{\partial \mu} = (1 - \beta_i) \gamma_i h'(\mu_i(\rho)), \tag{5}$$

where $\frac{\partial \mathbb{E}[\phi(\mu_a)|\mu = x]}{\partial \mu} \equiv \frac{\partial}{\partial \mu} \left[\int \phi(\mu_a) dF(\mu_a|\mu) \right] \Big|_{\mu = x}$.

3.7 Equilibrium Definition

We are now ready to define an equilibrium.

Definition 1. *A pure symmetric Markov equilibrium is a strategy* $(P(\rho), \mu_1(\rho), \mu_2(\rho), c(\mu_a, \rho))$*, and a belief update function,* $\rho^+(\mu_a, \rho)$*, where* $\rho \in [\epsilon, 1-\delta]$ *and* $\mu_a \in [0, \infty)$*, such that (i) Bayes' rule holds, that is, equation* (1) *holds given* $\mu_1(\rho)$*, and* $\mu_2(\rho)$*; (ii) the household optimality conditions* (2) *and* (3) *hold; (iii)* $\mu_i(\rho)$ *solves Problem* (4) *for* $i \in \{1, 2\}$*.*

4 A Single Central Bank Type

In this section, we consider the case where there is only one type of central bank to make clearer the basic forces of the model when reputation is not an issue, both with and without noise.

In the simplest special case, the central bank's type is known and permanently fixed, or $\rho_0 = 1$ and $\epsilon = \delta = 0$, and there is no noise, or $\mu_a = \mu$ with certainty. (The latter violates our assumption that the density $f(\mu_a|\mu)$ exists, but we can nevertheless analyze this case.)

Here, a pure symmetric Markov strategy is simply a specification of two scalars and function, $(P, \mu, c(\mu_a))$ with $c(\mu_a) > 0$ and $P > 0$. With every date the same, P represents the constant price, relative to the beginning period money supply, μ represents the constant action of the central bank — in this case, the percent increase in the money supply — and $c(\mu_a)$ represents the consumption of households not only for the specified increase in the money supply, μ, but also for all possible values, μ_a. (The specification of the consumption response of households to off-path increases in the money supply is necessary to check the incentives of the central bank.)

In this simplified case, the optimality condition for household consumption, from equation (3), is given by:

$$c(\mu_a) = \frac{1+\mu_a}{P} \min\left\{\frac{Pc(\mu)}{\beta}, 1\right\},$$

where we used that $g = \frac{1}{c(\mu)}$. Here, $c(\mu_a)$ on the left hand side is the specification of the optimal consumption of the household for arbitrary μ_a, whereas $c(\mu)$ on the right hand side is the specification of consumption in the subsequent period for the specified central bank action μ.

This implies

$$\frac{Pc(\mu_a)}{1+\mu_a} = \min\left\{\frac{Pc(\mu)}{\beta}, 1\right\}.$$

Note that the left hand side is the fraction of cash spent for arbitrary μ_a and the right hand side is independent of μ_a, and thus household optimality implies the cash-in-advance constraint binds for either all μ_a or no μ_a. It follows that the cash-in-advance constraint must bind for all μ_a.[2] Then,

$$c(\mu_a) = \frac{1+\mu_a}{P}.$$

The equilibrium condition (2), the determination of the price, can be written as

$$\frac{\alpha\sigma}{P} + \frac{\beta(1-\sigma)}{1+\mu}\frac{g}{P} = 0.$$

Using again that $g = \frac{1}{c(\mu)}$, we obtain that

$$c(\mu) = \left(\frac{\sigma-1}{\sigma}\frac{\beta}{1+\mu}\right)\frac{1}{\alpha}. \tag{6}$$

Using now that $c(\mu) = \frac{1+\mu}{P}$, we see that the equilibrium P must be such that

$$P = (1+\mu)^2\frac{\sigma}{\sigma-1}\frac{\alpha}{\beta}.$$

Equation (6) implies that equilibrium prices are set so that consumption and production are inefficiently low (since $\frac{\sigma-1}{\sigma}\frac{\beta}{1+\mu} < 1$ and efficiency requires $c = \frac{1}{\alpha}$). This condition demonstrates two inefficiencies. First, since $\sigma \in (1,\infty)$, goods are less than perfect substitutes for each other, and consequently each household symmetrically exploits its monopoly power when setting prices. The second inefficiency is the standard cash-in-advance inefficiency. The term $\frac{\beta}{1+\mu} < 1$ reflects the effect on price setting today of the fact that the dollars received in a transaction today cannot be spent until tomorrow, when the price level is $(1+\mu)$ times higher than today.

For the equilibrium determination of μ, note first that without noise, the equilibrium condition (5) becomes

$$(1-\beta)\frac{\partial[\log(c(\mu)) - \alpha c(\mu)]}{\partial\mu} = (1-\beta)\gamma h'(\mu),$$

which simplifies to

$$\frac{1}{P}\left(\frac{1}{c(\mu)} - \alpha\right) = \gamma h'(\mu).$$

Hence, with an arbitrary P fixed, as the central bank increases μ, the marginal benefit is that it increases both consumption and production by $\frac{1}{P}$, which marginally affects its payoff by $\frac{1}{c(\mu)} - \alpha$, which it trades off with the penalty associated with choosing a higher μ. But note that from (6), the left hand side of (4) is strictly positive, and thus given $h'(0) = 0$, $\mu > 0$ in any pure symmetric

[2] If the cash-in-advance constraint were not to bind, then we would have that $c(\mu) = \frac{1+\mu}{P}\frac{P}{\beta}c(\mu) = \frac{1+\mu}{\beta}c(\mu)$, which is impossible given $c(\mu) > 0$ and $(1+\mu)/\beta > 1$.

Markov perfect equilibrium. Put simply, the central bank's ability to inflate makes an inefficient equilibrium even worse. The ability of a central bank to print money increases the wedge between the marginal utility of consumption and the marginal disutility to production present in (6).

Next, allow noise, but still require the central bank's type to be known and permanently fixed. In this case, a pure symmetric Markov strategy is again a specification, $(P, \mu, c(\mu_a))$. Here, it is still the case that the cash-in-advance constraint binds for all μ_a realizations. To see this, note from (3) that

$$\frac{Pc(\mu_a)}{1+\mu_a} = \min\left\{\frac{P}{\beta g}, 1\right\}, \tag{7}$$

where $g = \mathbb{E}[\frac{1}{c(\mu_a)}|\mu]$. Here, the left hand side is the fraction of money holdings spent on current consumption, while the right hand side is a constant. Thus, as with the case with no noise (and a constant central bank type), either the cash-in-advance constraint binds for all μ_a or it binds for no μ_a. An argument similar to the above shows that the cash-in-advance constraint must bind for any realization of μ_a.[3]

We next introduce an assumption on the effect on μ_a of increasing μ from $\mu = 0$.

Assumption 1. *The density $f(\mu_a|\mu)$ and the elasticity of substitution parameter σ are such that*

$$\frac{\partial \mathbb{E}[\log(1+\mu_a)|\mu=0]}{\partial \mu} > \frac{\sigma-1}{\sigma}\beta \frac{\mathbb{E}\left(\frac{1}{1+\mu_a}|\mu=0\right)}{\mathbb{E}(1+\mu_a|\mu=0)} \frac{\partial \mathbb{E}[1+\mu_a|\mu=0]}{\partial \mu}. \tag{8}$$

Note if $\mu_a = \mu$ with certainty (no noise), Assumption 1 is satisfied. (The left hand side of (8) is one, while the right hand side is $\frac{\sigma-1}{\sigma}\beta < 1$.) The content of Assumption 1 is that for an arbitrary noise structure, the incentives of the central bank considering increasing μ from $\mu = 0$ are similar to those in the noiseless world.

Given Assumption 1, we can show that with noise, in any pure symmetric Markov equilibrium where the the central bank's type is known and permanently fixed, $\mu > 0$. To see this, note that simplifying (2) to allow for only one type of central bank delivers

$$P = \frac{\sigma}{\sigma-1}\frac{1}{\beta}\frac{\mathbb{E}[1+\mu_a|\mu]}{\mathbb{E}[\frac{1}{1+\mu_a}|\mu]}\alpha.$$

Next, without types, the continuation value of the central bank is itself a constant, which allows

[3] Suppose the cash-in-advance constraint does not bind for a given $\mu_a \geq 0$. Then, equation (7) implies

$$\frac{1}{1+\mu_a} = \frac{1}{c(\mu_a)}\frac{1}{\beta g} \Rightarrow \mathbb{E}\left[\frac{1}{1+\mu_a}|\mu\right] = \mathbb{E}\left[\frac{1}{c(\mu_a)}|\mu\right]\frac{1}{\beta g} \Rightarrow \mathbb{E}\left[\frac{1}{1+\mu_a}|\mu\right] = g\frac{1}{\beta g} \Rightarrow \mathbb{E}\left[\frac{\beta}{1+\mu_a}|\mu\right] = 1,$$

a contradiction.

the equilibrium condition for central bank optimization (5) to simplify to

$$\frac{\partial \mathbb{E}[\log(1+\mu_a)|\mu]}{\partial \mu} - \frac{\sigma-1}{\sigma}\beta \frac{\mathbb{E}[\frac{1}{1+\mu_a}|\mu]}{\mathbb{E}[1+\mu_a|\mu]} \frac{\partial \mathbb{E}[1+\mu_a|\mu]}{\partial \mu} = \gamma h'(\mu).$$

Note, however that Assumption 1 and that $h'(0) = 0$ guarantee that $\mu = 0$ violates (4). Thus, in any equilibrium where types are permanent, $\mu > 0$.

5 Existence

In this section, we consider whether pure strategy symmetric Markov perfect equilibria exist. Our main point is that without sufficient noise disconnecting a central bank's action to the observed outcome, pure strategy symmetric Markov equilibria may **not** exist.

Our results regarding pooling equilibria are presented in the form of two propositions. In the first, we show there cannot be a pure strategy symmetric Markov equilibrium where the types pool at the stationary reputation, (or where $\mu_1(\rho^*) = \mu_2(\rho^*)$, where $\rho^* \equiv \frac{\epsilon}{\delta+\epsilon}$). This rules out, in particular, Markov strategies where the types *always* pool (or where $\mu_1(\rho) = \mu_2(\rho)$ for all ρ).

In the second proposition, we show that there cannot be a pure strategy symmetric Markov equilibrium where $\mu_1(\rho) = \mu_2(\rho)$ for *any* ρ, unless $\mu_1(\rho) = \mu_2(\rho) = 0$. This rules out equilibria where the types *ever* pool, as long as both $\mu_1(\rho)$ and $\mu_2(\rho)$ are positive for all ρ.

At the end, we argue that it is difficult to sustain a separating pure strategy equilibria without sufficient noise disconnecting a central bank's target inflation from realized inflation, (and with sufficient patience).

First, we show there exists no Markov equilibria in pure strategies where the types pool at $\rho = \rho^*$.

Proposition 1. *Suppose a pure symmetric Markov strategy* $(P(\rho), \mu_1(\rho), \mu_2(\rho), c(\mu_a, \rho))$ *has* $\mu_1(\rho) = \mu_2(\rho) = \mu$ *for* $\rho = \rho^* \equiv \frac{\epsilon}{\delta+\epsilon}$. *If Assumption 1 holds,* $(P(\rho), \mu_1(\rho), \mu_2(\rho), c(\mu_a, \rho))$ *is not a symmetric Markov equilibrium.*

Proof. Towards a contradiction, consider that there is such an equilibrium. Note that $\rho^+(\mu_a, \rho^*) = \rho^*$ for all μ_a, since $\hat{\rho}(\mu_a, \rho) = \rho^*$ from $\mu_1(\rho^*) = \mu_2(\rho^*) = \mu$, and that $\rho^+(\rho^*, \mu_a) = (1-\delta)\rho^* + \epsilon(1-\rho^*) = \rho^*$. From (3), then,

$$\frac{P(\rho^*)c(\mu_a, \rho^*)}{1+\mu_a} = \min\left\{\frac{P(\rho^*)}{\beta g(\rho^*)}, 1\right\}.$$

This implies that starting from $\rho = \rho^*$, the cash-in-advance constraint always binds or never does.

Using the same argument as before, we can rule out that it never does, and it follows that

$$\frac{P(\rho^*)c(\mu_a, \rho^*)}{1+\mu_a} = 1.$$

From the definition of $g(\rho)$, we have

$$g(\rho^*) = \mathbb{E}\left[\frac{1}{c(\mu_a, \rho^*)}|\mu\right].$$

Using (2), we get that

$$P(\rho^*) = \frac{\alpha}{\beta}\frac{\sigma}{\sigma-1}\frac{\mathbb{E}[1+\mu_a|\mu]}{\mathbb{E}[1/(1+\mu_a)|\mu]}.$$

Now, given that $\rho^+(\rho^*)$ is independent of μ_a, the first order condition for type i is

$$\frac{\partial\mathbb{E}[\log(1+\mu_a)|\mu]}{\partial\mu} - \frac{\sigma-1}{\sigma}\beta\frac{\mathbb{E}[\frac{1}{1+\mu_a}|\mu]}{\mathbb{E}[1+\mu_a|\mu]}\frac{\partial\mathbb{E}[1+\mu_a|\mu]}{\partial\mu} \leq \gamma_i h_i'(\mu)$$

with equality if $\mu > 0$. This equation cannot be satisfied with $\mu = 0$, as it violates Assumption 1. Nor can it hold for $\mu > 0$, as $\gamma_1 \neq \gamma_2$. □

This proposition rules out that in an equilibrium, the types pool on any subset of ρ that contains ρ^*. We can strengthen this result, but focusing on *interior* equilibria.

Proposition 2. *Suppose a Markov pure strategy* $(P(\rho), \mu_1(\rho), \mu_2(\rho), c(\mu_a, \rho))$ *has* $\mu_1(\rho) = \mu_2(\rho) > 0$ *for some* $\rho \in [\epsilon, 1-\delta]$*. Then,* $(P(\rho), \mu_1(\rho), \mu_2(\rho), c(\mu_a, \rho))$ *is not a symmetric Markov equilibrium.*

Proof. Suppose $(P(\rho), \mu_1(\rho), \mu_2(\rho), c(\mu_a, \rho))$ has $\mu_1(\rho) = \mu_2(\rho) > 0$ for some $\rho \in [\epsilon, 1-\delta]$. Then, $\hat{\rho}(\rho, \mu_a, \mu_1(\rho), \mu_2(\rho)) = \rho$ and $\rho^+(\rho, \mu_a, \mu_1(\rho), \mu_2(\rho)) = (1-\delta)\rho + \epsilon(1-\rho)$. That is, each is independent of the realization of μ_a. Condition (5) can then be written as

$$\frac{\partial\mathbb{E}[\log(c(\mu_a, \rho)) - \alpha c(\mu_a, \rho)|\mu = \mu_i(\rho)]}{\partial\mu} = \gamma_i h'(\mu_i). \tag{9}$$

Since $\gamma_2 < \gamma_1$ and $h'(\mu) > 0$ for $\mu > 0$, equation (9) cannot simultaneously hold for both $i = 1$ and $i = 2$; thus, a contradiction is generated. □

Next, we argue (informally) that noise makes the existence of a separating equilibrium, for given values of β and ϵ easier.

To see this, note that if $\mu_1(\rho) \neq \mu_2(\rho)$ and there is no noise, or $\mu_a = \mu_i(\rho)$ with certainty, Bayesian updating requires that $\rho^+(\mu_1(\rho), \rho) = 1-\delta$ (the highest possible reputation) and

$\rho^+(\mu_2(\rho), \rho) = \epsilon$ (the lowest possible reputation). This implies that in order for the bad central bank not to wish to mimic the good central bank,

$$(1-\beta_2)[\log(c(\mu_2(\rho), \rho)) - \alpha c(\mu_2(\rho), \rho)] + \beta_2 V_2(\epsilon) \\ \geq (1-\beta_2)[\log(c(\mu_1(\rho), \rho)) - \alpha c(\mu_1(\rho), \rho)] + \beta_2 V_2(1-\delta).$$

Rearranged, this becomes

$$(1-\beta_2)[\log(c(\mu_2(\rho), \rho)) - \alpha c(\mu_2(\rho), \rho) - (\log(c(\mu_1(\rho), \rho)) - \alpha c(\mu_1(\rho), \rho))] \\ \geq \beta_2(V_2(1-\delta) - V_2(\epsilon)).$$

In words, the left hand side is the static gain to the bad central bank type of following its strategy $\mu_2(\rho)$ as opposed to mimicking the good central bank, while the right hand side is the cost, in terms of the forgone value of attaining the reputation of being a good central bank, of following strategy $\mu_2(\rho)$. As long as $V_2(1-\delta) > V_2(\epsilon)$, for high values of β_2 (either because β is high or because ϵ is low), this incentive condition becomes difficulty to satisfy.

So instead assume noise, so μ_a depends on but is not necessarily equal to $\mu_i(\rho)$. Then, the equivalent condition to (5) is

$$(1-\beta_2)(\mathbb{E}[\log(c(\mu_a, \rho)) - \alpha c(\mu_a, \rho)|\mu_2(\rho)] - \mathbb{E}[\log(c(\mu_a, \rho)) - \alpha c(\mu_a, \rho)|\mu_1(\rho)]) \\ \geq \beta_2(\mathbb{E}[V_2(\rho^+(\mu_a, \rho)|\mu_1(\rho))] - \mathbb{E}[V_2(\rho^+(\mu_a, \rho)|\mu_2(\rho))]).$$

Now, the effect on type $i = 2$'s reputation of mimicking type $i = 1$ is not so extreme. In fact, if μ_a were determined only by noise, tomorrow's reputation, ρ^+, would be independent of μ_a, the right hand side of (5) would be zero, and the condition would be satisfied merely by making type $i = 2$ have a static incentive to follow $\mu_2(\rho)$ rather than $\mu_1(\rho)$.

6 An Example Equilibrium

In the previous section, we established that without sufficient noise, it is difficult for pure strategy Markov equilibria to exist. In this section, we present an example where the central bank's target growth rate, μ_i, is sufficiently disconnected from the actual money growth rate that a pure strategy symmetric Markov equilibrium exists.

In the example, parameters are chosen so that if there were only one type, in equilibrium, the good type, type $i = 1$, would set μ_1 so that mean inflation was 2% each period, while the bad type, type $i = 2$, would set μ_2 so that mean inflation was 3% each period. That is, households are trying to determine if the central bank has a mean inflation target of 2% or 3%.

The determination of actual money growth, μ_a, is as follows: with probability μ_i, μ_a is determined by an exponential distribution with mean 0.04. With complementary probability $1 - \mu_i$, μ_a is determined by an exponential distribution with mean 0.01. Here, we choose the penalty coefficients for setting $\mu_i > 0$, γ_1 and γ_2 so that if it were common knowledge the central bank is type 1, it would set $\mu_1 = 0.3333$ (so inflation has a mean of 0.02), and if it were common knowledge the central bank is type 2, it would set $\mu_2 = 0.6666$ (so inflation has a mean of 0.03).

Filling out the game's parameters, we let $\alpha = 1$ (so the efficient level of consumption $c^* = 1$) and $\beta = 0.99$. For the central banks, the respective penalty coefficients are $\gamma_1 = 0.0173$ and $\gamma_2 = 0.0093$, $h(\mu_i) = 0.5\mu_i^2$, with switching probabilities $\epsilon = \delta = 0.02$.

Under these parameters, in a world where it is common knowledge that central banks are always the good ($i = 1$) type, in every period

$$P = 1.313, \quad \mu_1 = 0.333, \quad \mathbb{E}[\log(c) - \alpha c] = -1.0297.$$

In a world where it is common knowledge that central banks are always the bad ($i = 2$) type, in every period

$$P = 1.338, \quad \mu_2 = 0.666, \quad \mathbb{E}[\log(c) - \alpha c] = -1.0320.$$

For comparison, in the efficient allocation where $c = \frac{1}{\alpha} = 1$ every period, $\log(c) - \alpha c = -1$.

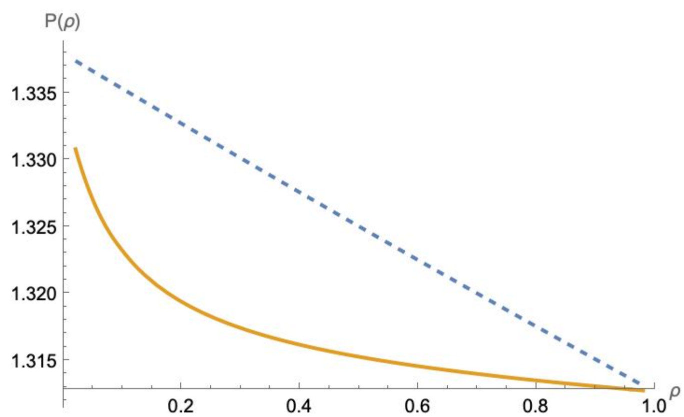


Figure 1: Price as a function of reputation

Prices as a function of reputation in the full game, where central banks switch types, are shown in Figure 1. In this figure, the lower curve represents prices set by sellers in our actual game. Here, households are always unsure of the type of the central bank. Central banks know this, and thus strategically set $\mu_1(\rho)$ and $\mu_2(\rho)$ to influence their future reputation. For reference, the top dashed line represents the price charged by sellers if they believe the central bank is the good type with probability ρ, but where this type is revealed just after the price is set and will never change again.

Under this assumption, a central bank of type i will always choose its static best response given the prices set. We include this to see the effect of reputation *directly*, as opposed to the effects of central banks *caring* about their reputation. For all figures, dashed lines represent the outcome in the reference game, while solid lines represent the outcome for the actual game.

Figure 2 displays $\mu_1(\rho)$ and $\mu_2(\rho)$ as a function of reputation, again for the full game and the reference game, where types are revealed once and for all just after prices are set. Here, type 2 takes a higher action $\mu_2(\rho)$ than type 1 for all ρ, but both types i set μ_i less than they would if they were unconcerned with protecting their reputation. Notably, in this example, both types choose their action to be the lowest for interior reputations, when Bayesian posteriors are most easily moved.

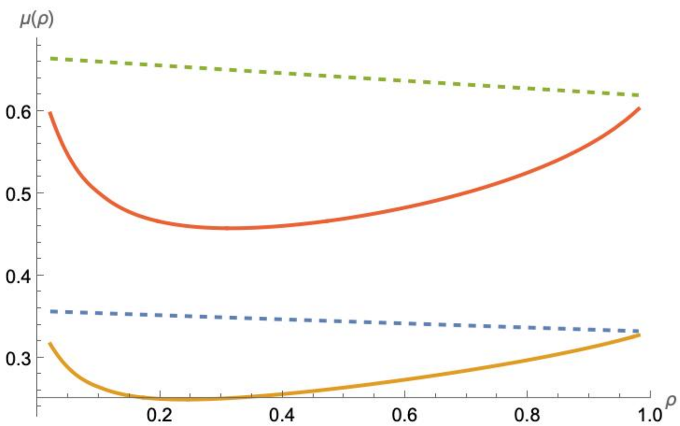
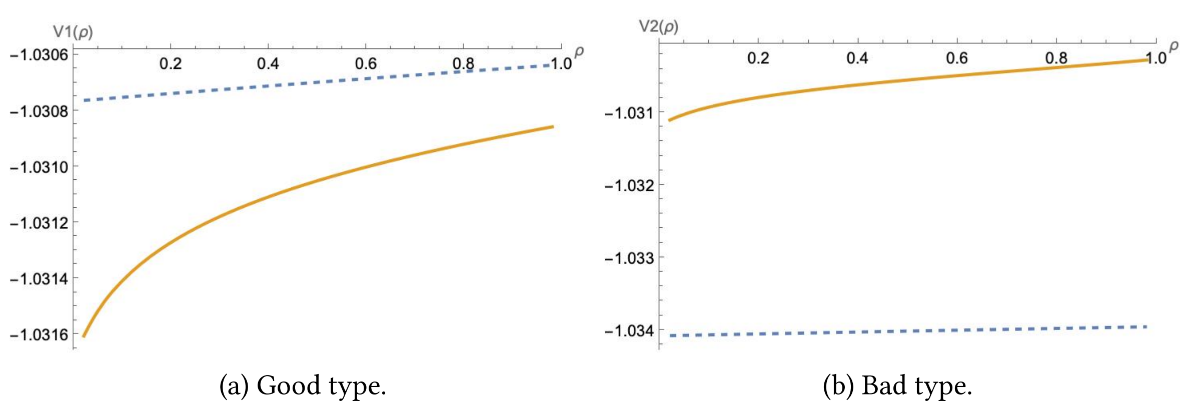


Figure 2: Central bank actions as functions of reputation.

Figure 3 displays the value functions for both the good and bad central bank types in panels (a) and (b) respectively. Both types value being seen as more likely to be the good type. Interestingly, the good type is made worse off from the continuing uncertainty of households regarding central bank type, while the bad type is made better off.



(a) Good type.

(b) Bad type.

Figure 3: Central bank payoff as functions of reputation.

In Figure 4, we present the expected discounted utility of households as a function of beginning of period reputation. Not surprisingly, a household's expected discounted utility is increasing in the probability of its facing a good central bank type. Furthermore, unless reputation is near its highest possible value, households are better off in the world with continuing uncertainty (yellow line) versus the world where all uncertainty is revealed after the initial price is set, again because continued uncertainty offsets the inflation temptations of both central bank types.

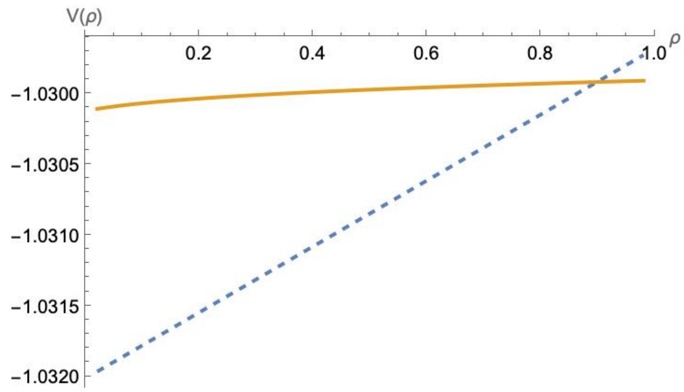


Figure 4: Household value function as a function of reputation.

A final issue is whether reputation considerations in this example persist. That is, nothing theoretically rules out the possibility that the central bank's reputation, ρ, is nearly always near one or zero.

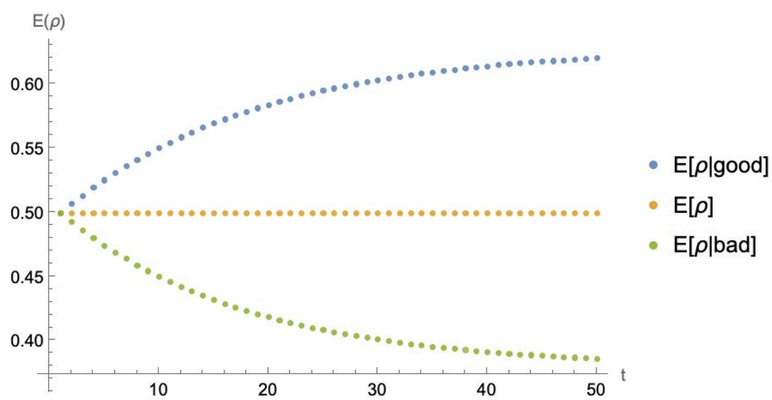


Figure 5: Expectation of ρ over time starting from $\rho = .5$.

Figures 5 and 6 demonstrate that at least for these parameters, our model exhibits a long lasting effect of reputation: reputations do not quickly go to either pole. Figure 5 gives the expected paths of reputation, starting from $\rho = .5$ conditional on being the good type (top path), bad type (bottom) path, and unconditionally (middle path). Note first that even conditional on being a good or bad type, expected reputation remains above $\rho = .38$ (for the bad type) and below $\rho = .62$

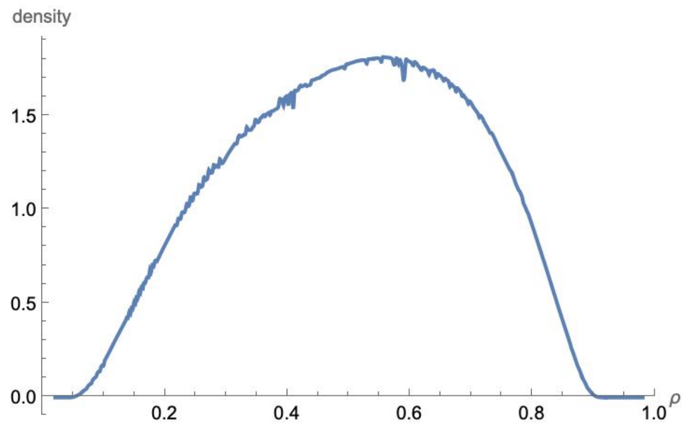


Figure 6: Ergodic distribution of reputation, ρ.

(for the good type). Noise and the fact that types possibly switch keeps expected reputations interior. Figure 6 displays the ergodic distribution of reputation, ρ, again displaying that interior reputations persist.

7 Conclusion

This paper presents a model of central banks whose actions noisily affect their reputation. In it, we argue that if there is sufficiently little discounting, a pure strategy Markov perfect equilibrium cannot exist if there isn't enough noise. However, we show computationally that if there is enough noise, a pure strategy Markov perfect equilibrium can exist with appealing characteristics: both good and bad central bank types inflate less than they otherwise would, especially when their reputation is middling, and this outcome improves household welfare.

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